Propositional Logic (cont.)



Course Code: CSC 1204 Course Title: Discrete Mathematics

Dept. of Computer Science Faculty of Science and Technology

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Lecture Outline



1.1 Propositional Logic

- Logic
- Propositional Logic
- Propositions
- Propositional Variables
- Compound Propositions
- Logical Operators
- Truth Value
- Truth Tables of Compound Propositions
- Conditional Statements
- Logic and Bit Operations

* We have already covered

Objectives and Outcomes



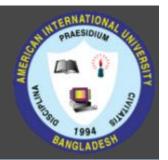
- Objectives: To understand how to construct a truth table for a compound proposition, to understand the conditional statement $p \rightarrow q$ and different equivalent expressions of $p \rightarrow q$, to understand bit operations.
- Outcomes: Students are expected to be able construct a truth table for a given compound proposition, be able to explain the conditional statement $p \rightarrow q$ and it's equivalent expressions, be able to perform Bit Operations.

Conditional Statements



- Let p and q be propositions.
- The conditional statement $p \rightarrow q$ is the proposition "if p, then q."
- The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.
- In the conditional statement $p \rightarrow q$, p is called hypothesis and q is called conclusion.
- This one is the English usage of "if, then" or "implies".
- The connective → is called the 'conditional connective'.
- A conditional statement is also called an implication.

Truth Table for Conditional Statement



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TABLE 5	The	Truth	Table f	or		
the Conditional Statement						
$p \rightarrow q$.						

p	\boldsymbol{q}	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Equivalent Expression of $p \rightarrow q$



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In terms of words, the proposition p \rightarrow q also reads:
    (a) if p, then q, or
        if p, q
        (The word "then" is sometimes omitted in English
        sentences)
    (b) p implies q
    (c) p is a sufficient condition for q, or
         a sufficient condition for q is p
    (d) q is a necessary condition for p, or
         a necessary condition for p is q
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Equivalent Expression of $p \rightarrow q$



- (e) p only if q
- (f) q if p , or q, if p
- (g) q whenever p
- (h) q when p
- (i) q unless ¬p

Remember!



- The hypothesis expresses a sufficient condition
- The conclusion expresses a necessary condition
- "but" is a logical synonym for "and"
- "when" / "whenever" means the same as "if"
- The hypothesis is the clause following "if"
- The conclusion is the clause following "then"
- "only if" clause is the conclusion
- If hypothesis, then conclusion

Example 7 (page 7)



Let p: "Maria learns discrete mathematics" and

q: "Maria will find a good job."

Express the statement $\mathbf{p} \rightarrow \mathbf{q}$ as a statement in English.

Solution:

- "If Maria learns discrete mathematics, then she will find a good job." There are many other ways to express this conditional statement in English.
- "Maria will find a good job when she learns discrete mathematics"
- "For Maria to get a good job, it is sufficient for her to learn discrete mathematics".
- "Maria will find a good job unless she does not learn discrete mathematics."



- Write each of these statements in the form "if p, then q" in English.
- a) It snows whenever the wind blows from the northeast. Ans. If the wind blows from the northeast, then it snows
- b) The apple trees will bloom **if** it stays warm for a week.

 Ans. **If** it stays warm for a week, **then** the apple trees will bloom



- c) That the Pistons win the championship **implies** that they beat the Lakers.
- Ans. If the Pistons win the championship, then they beat the Lakers.
- d) It is **necessary** to walk 8 miles to get to the top of Long's Peak.
- Ans. If you get to the top of Long's Peak, then you must have walked eight miles.



- e) To get tenure as a professor, it is **sufficient** to be world-famous.
- Ans. If you are world-famous, then you will get tenure as a professor.
- f) **If** you drive more than 400 miles, you will need to buy gasoline.

Ans. If you drive more than 400 miles, then you will need to buy gasoline.



- g) Your guarantee is good **only if** you bought your CD player less than 90 days ago.
- Ans. If your guarantee is good, then you must have bought your CD player less than 90 days ago.
- h) Jan will go swimming unless the water is too cold.

Ans. If the water is not too cold, then Jan will go swimming.



Converse, Contrapositive, and Inverse

- We can form some new conditional statements starting with a conditional statement $p \rightarrow q$
- The Converse of $p \rightarrow q$ is the proposition $q \rightarrow p$
- The Contrapositive of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$
- The *Inverse of p* \rightarrow *q* is the proposition $\neg p \rightarrow \neg q$

Examples of Converse, Contrapositive and Inverse



• Converse: $p \rightarrow q ==> q \rightarrow p$

Example: "If it is noon, then I am hungry."

Converse: "If I am hungry, then it is noon."

• Contrapositive: $p \rightarrow q ==> \neg q \rightarrow \neg p$

Example: "If it is noon, then I am hungry."

Contrapositive: "If I am not hungry, then it is not noon."

• Inverse: $p \rightarrow q ==> \neg p \rightarrow \neg q$

Example: "If it is noon, then I am hungry."

Inverse: "If it is not noon, then I am not hungry."



Bi-Conditional

- Let p and q be propositions.
- The bi-conditional statement p ↔ q is the proposition "p if and only if q."
- The bi-conditional statement p ↔ q is true when p and q have the same truth values, and is false otherwise.
- Bi-conditional statements are also called "bi-implications"
- Question: Which operator is the opposite of ↔?

Truth Table for Bi-conditional $p \leftrightarrow q$



TABLE 6	The	Truth	Table	for	the
Bicondition	ıal <i>p</i>	$\leftrightarrow q$.			

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example of Bi-conditional statement



Example 10 (Page 9): Let p be the statement "You can take the flight" and q be the statement "You buy a ticket". What is the statement for p ↔ q?

Solution:

"You can take the flight if and only if you buy a ticket"

How to **Construct** a **Truth Table** for a Compound Proposition?



- At first look at the number of propositions (e.g. *p*, *q*, *r*) in the given compound proposition.
- There will be 2^n number of rows in the truth table, where n is the number of propositions in the compound proposition.
- Draw the table. In the first row, write down the name of propositions (e.g. *p*, *q*, *r*) starting from left/first column.
- Construct the truth table step by step.

Example: Construct a truth table for $(p \oplus q) \oplus r$



P	\mathbf{q}	r	$p\oplus q$	$(p \oplus q) \oplus r$
Τ	T	T	F	Τ
\mathbf{T}	\mathbf{T}	\mathbf{F}	\mathbf{F}	\mathbf{F}
\mathbf{T}	\mathbf{F}	\mathbf{T}	${ m T}$	\mathbf{F}
\mathbf{T}	\mathbf{F}	\mathbf{F}	${ m T}$	${f T}$
\mathbf{F}	\mathbf{T}	\mathbf{T}	${ m T}$	\mathbf{F}
\mathbf{F}	\mathbf{T}	\mathbf{F}	${ m T}$	${f T}$
\mathbf{F}	\mathbf{F}	\mathbf{T}	\mathbf{F}	${f T}$
\mathbf{F}	\mathbf{F}	F	F	F

Precedence of Logical Operators



- Negation operator is applied before all other logical operators
- Conjunction operator takes precedence over disjunction operator
- Conditional and bi-conditional operators have lower precedence
- Parentheses are used whenever necessary

Precedence of Logical Operators



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TABLE 8 Precedence of

Logical Operators.

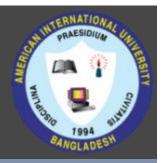
Operator	Precedence
П	1
< >	2 3
\rightarrow \leftrightarrow	4 5

Logic and Bit Operations



- bit ==> binary digit
- Boolean variable: either true or false
 - Can be represented by a bit
- **Bit String**: A *bit string* is a sequence of zero or more bits. The *length* of the string is the number of bits in the string.
- Example 20(p.15): 101010011 is a bit string of length nine

Bit Operations



Computer bit operations correspond to the logical connectives. By replacing true by a one (1) and false by a zero (0) in the truth tables for the operators Λ , V, and \bigoplus , the tables shown in Table 9 for the corresponding bit operations are obtained. We will also use the notation OR, AND, and XOR for the operators V, Λ , and \bigoplus respectively, as is done in various programming languages.

Table for Bit Operations



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TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Bit string and bit operation



DEFINITION 7

A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.

EXAMPLE 12

101010011 is a bit string of length nine.

We can extend bit operations to bit strings. We define the **bitwise** OR, **bitwise** AND, and **bitwise** XOR of two strings of the same length to be the strings that have as their bits the OR, AND, and XOR of the corresponding bits in the two strings, respectively. We use the symbols \vee , \wedge , and \oplus to represent the bitwise OR, bitwise AND, and bitwise XOR operations, respectively. We illustrate bitwise operations on bit strings with Example 13.

EXAMPLE 13

Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of the bit strings 01 1011 0110 and 11 0001 1101. (Here, and throughout this book, bit strings will be split into blocks of four bits to make them easier to read.)

Solution: The bitwise OR, bitwise AND, and bitwise XOR of these strings are obtained by taking the OR, AND, and XOR of the corresponding bits, respectively. This gives us

01 1011 0110 11 0001 1101 11 1011 1111 bitwise *OR* 01 0001 0100 bitwise *AND* 10 1010 1011 bitwise *XOR*



Books

• Discrete Mathematics and its applications with combinatorics and graph theory (7th edition) by Kenneth H. Rosen [Indian Adaptation by KAMALA KRITHIVASAN], published by McGraw-Hill

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- 2. Discrete Mathematical Structures, *Bernard Kolman*, *Robert C. Busby*, *Sharon Ross*, Prentice-Hall, Inc.
- 3. SCHAUM'S outlines Discrete Mathematics(2nd edition), by Seymour Lipschutz, Marc Lipson