Propositional Logic

Course Code: CSC 1204 Course Title: Discrete Mathematics

Dept. of Computer Science
Faculty of Science and Technology

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Lecturer:	*S.M. Abdur Rouf Bhuiyan [arouf@aiub.edu]*				

Lecture Outline

1.1 Propositional Logic

- Logic
- Propositional Logic
- Propositions
- Propositional Variables
- Compound Propositions
- Logical Operators
- Truth Value & Truth Table
- Truth Tables of Compound Propositions (next class)
- Conditional Statements (next class)
- Logic and Bit Operations (next class)

Objectives and Outcomes

- **Objectives**: To understand the importance of logic in mathematical reasoning, to understand proposition and propositional logic, symbol and usage of different types of logical operators.
- **Outcomes**: Students are expected to be able to apply logical operators and analyze logical propositions via truth tables, be able to construct a truth table for a given compound proposition.

Key Terms

- **Logic:** Logic is the discipline that deals with the methods of reasoning.
  - Logic is the basis of all mathematical reasoning
  - The rules of logic specify the meaning of mathematical statements

- **Propositional Logic**: The area of logic that deals with ***propositions*** is called the propositional logic.

Key Terms

- **Proposition:** A ***proposition*** is a declarative statement that's either **TRUE** or **FALSE**, but **not both**.

- Statements that are not propositions *include*
  - Questions
  - Commands

Key Terms

- **Propositional variable:** A variable that represents a proposition. The conventional letters used for propositional variables are *p, q, r, s, t,..*
- **Compound proposition:** A proposition constructed by combining two or more propositions using *logical operators* (**AKA** : *logical connectives*)
- **Logical Operators**: Operators used to combine propositions
- **Truth Value**: **The truth value** of a proposition is **true**, denoted by **T**, if it is a true statement and **false**, denoted by **F**, if it is a false statement. **Truth Value ==>** Either **True** or **False**
- **Truth Table:** A table displaying the truth values of propositions.

Proposition: Examples

Proposition	Not Proposition
3 + 2 = 32	Bring me coffee!
3 + 2 = 5	3 + 2
CSC 1204 is Katrina's favorite class.	CSC 1204 is her favorite class.
Every cow has four legs.	Do you like Cake?

Logical Operators

- **Logical Operators ==> unary, binary**
- **Unary**:
  - Negation
- **Binary**
  - Conjunction
  - Disjunction
  - Exclusive OR
  - Conditional/Implication
  - Bi-conditional

Logical Operators: Symbols & Usage

Operator	Symbol	Usage
Negation	$\neg$	NOT
Conjunction	$\wedge$	AND
Disjunction	$\vee$	OR
Exclusive or	$\oplus$	XOR
Conditional	$\rightarrow$	if, then
Bi-conditional	$\leftrightarrow$	iff

Propositional Logic : Negation

- Let *p* be a proposition. The *negation* of *p*, denoted by $\neg p$ (or $\overline{p}$), is the statement "It is not the case that *p*."
- The proposition $\neg p$ *is read "not p"*
- The truth value of the negation of *p*, $\neg p$, is the opposite of the truth value of *p*.

Truth table for Negation of a Proposition

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Propositional Logic : **Negation**

- Negation just turns *a **false*** proposition to ***true*** and the opposite for a true proposition.

- Example1: *p*: I am going to town

  ¬ *p*: I am not going to town; or,

  It is not the case that I am going to town

- Example2: *p :* "23 = 15 +7"

  p happens to be false, so ¬ *p* is true.

Conjunction

- Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q."
- The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.
- Conjunction corresponds to English "**AND**".

- Example: Liana is curious AND clever.

Truth Table for Conjunction

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction: Example

- **Example:** p : 'I am going to town'

  q : 'It is going to rain'

  $p \wedge q$: 'I am going to town and it is going to rain.'

- **Note:** **Both *p* and *q*** must be true to $p \wedge q$ be **true**

Disjunction

- Let p and q be propositions.
- The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q."
- The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.
- Disjunction is true when at least one of the components is true.
- Disjunction corresponds to English "**OR**".
- Example: Abdullah is brave OR intelligent.

Truth Table for Disjunction

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Examples of Conjunction & Disjunction

Let,

p : 5 < 9

q : 9 < 7.

Construct the propositions $p \wedge q$ and $p \vee q$.

Solution:

- The conjunction of the propositions p and q is the proposition

  $p \wedge q$: 5 < 9 and 9 < 7

- The disjunction of the propositions p and q is the proposition

  $p \vee q$: 5 < 9 or 9 < 7

Question: What are the truth values of $\boldsymbol{p \wedge q}$ *and* $\boldsymbol{p \vee q}$?

Exclusive Or

- Let p and q be propositions.
- The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is **true** when exactly one of p and q is true and is **false** otherwise.

Truth Table of Exclusive Or

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Books

- *Discrete Mathematics and its applications with combinatorics and graph theory (7th edition)* by Kenneth H. Rosen [Indian Adaptation by KAMALA KRITHIVASAN], published by McGraw-Hill

References

1. Discrete Mathematics, *Richard Johnsonbaugh*, Pearson education, Inc.
2. Discrete Mathematical Structures, *Bernard Kolman*, *Robert C. Busby*, *Sharon Ross,* Prentice-Hall, Inc.
3. *SCHAUM'S outlines Discrete Mathematics(2nd edition)*, by *Seymour Lipschutz*, *Marc Lipson*