Modeling of Wind Farms in the Load Flow Analysis

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***Abstract*—Two methods are proposed, for the simulation of wind farms with asynchronous generators in the load flow analysis. Both methods are based on the steady-state model of the induction machine. The first involves improving the conventional PQ bus, and the second involves modeling the generators in steady-state in the bus where the wind farm is located. The two sets of results are then compared.**

***Index Terms*—Induction machines, load flow analysis, wind energy.**

I. Introduction

In recent years, wind energy has become an important part of electrical generation in many countries and its importance is continuing to increase.

For this reason, and in order to investigate the effects the wind farms will produce on the grid, adequate models must be used.

One of the problems that wind energy will create in electrical power systems is the dependence of the injected power on the wind speed. The wind speed cannot be predicted, but the probability of a particular wind speed occurring can be estimated. This can be done if the probability distribution is known by assuming it to be a Weibull distribution [1] or a Rayleigh one, as recommended in [2]. Once the wind speed is known, the power injected into the grid can be calculated by means of the wind turbine (WT) power curve.

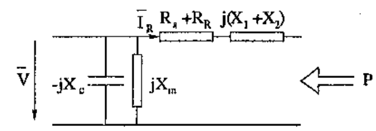
So, in order to assess the impact of wind energy on the steady-state security of electrical networks, the problem can be planned from a probabilistic point of view, by knowing the probability of injecting a determined power, if previously the probability of a given wind speed is known.

When a wind farm with asynchronous generation is to be included in a load flow analysis, the PQ and RX [3], [4] buses are the most commonly used.

When the conventional PQ bus model is used, the real and reactive powers have constant values, although some authors [5], [6] propose methods for modifying these values in order to represent loads depending either on the voltage or on the frequency.

When the PX bus model is used, the real power is known and the reactive power is calculated as a function of the magnetizing reactance of the generators. The PX bus is not studied in this paper.

Under the assumption of wind energy generation by means of asynchronous machines, the PQ bus can be improved, by taking some features relating to the WT into account. This is developed in the paper as one of the proposed methods.



P=f(U) if PQ node

$P=-I_R^2 R_R \frac{1-s}{s}$ if RX node

Fig. 1. Steady-state model of the induction machine.

A better approximation can be obtained with the other method, in which the induction machine is modeled by means of an RX bus based on the two following facts:

1) The machine can be simulated in steady-state as an impedance if its parameters and the slip are known.
2) The slip of the machine can be calculated if its power coefficient curve and the wind speed are known.

Both methods suppose prior knowledge of the WT features. The turbine's power curve is generally supplied by the manufacturer. When the induction generator parameters are not known, they must be estimated. As an example, a method is presented in [7] for estimating parameter values of the exact equivalent circuit of a three-phase induction motor.

II. PQ Model of an Asynchronous WT

A way to model a wind farm as a PQ bus is to assume a generated real power and a given power factor, with which the consumed reactive power is calculated.

Some improvements can be achieved if the steady-state model of the induction machine is taken into account. The model shown in Fig. 1 is assumed.

In this model, applying the conservation of complex power theorem (Boucherot's theorem) allows the following expression to be written for the reactive power consumed by the machine (positive when consumed):

$$Q = V^2 \frac{X_c - X_m}{X_c X_m} + X \frac{V_2 + 2RP}{2(R^2 + X^2)} - X \frac{\sqrt{(V^2 + 2RP)^2 - 4P^2(R^2 + X^2)}}{2(R^2 + X^2)} \quad (1)$$

where V is the voltage, P is the real power (positive when injected into the grid), X is the sum of the stator and rotor leakage reactances, X_c is the reactance of the capacitors bank, and$R$ is the sum of the stator and rotor resistances.

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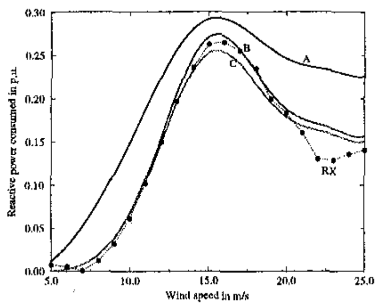

Fig. 2. Reactive power as a function of wind speed.

In [8] the following expression is proposed for the reactive power consumed by a WT as a function of its real power:

$$Q = -Q_0 - Q_1 P - Q_2 P^2 \tag{2}$$

where Q_0, Q_1, and Q_2 are experimentally obtained.

An approximation of (1) to (2) can be made by means of the McLaurin polynomial (see the Appendix), by taking into account the first two derivatives of (1), neglecting the resistance R for the sake of simplicity and finally writing the expression for the reactive power as:

$$Q \approx V^2 \frac{X_c - X_m}{X_c X_m} + \frac{X}{V^2} P^2. \tag{3}$$

The reactive power curve as a function of wind speed can be seen in Fig. 2, under the assumption that the voltage V equals 1 p.u. Line A corresponds to a PQ bus with a constant power factor of 0.96, line B to the reactive power obtained with (1) and line C to the reactive power obtained with (3). Line RX corresponds to the results obtained with the RX model that will be explained later.

If the wind speed is desired to be the input datum for the problem, the real power can be obtained as a function of it. This is done by means of the power curve for the turbine and next equation [1]:

$$P = \tfrac{1}{2} \rho A U^3 c_p \tag{4}$$

where A is the rotor area, ρ is the density of air, U is the wind speed and c_p is the power coefficient.

The power coefficient is obtainable as a function of the tip speed ratio, $\lambda = (\omega R/U)$, where ω is the rotor speed and R is its radius. The power and power coefficient curves are given in the Appendix.

III. RX Model of an Asynchronous WT

The other method proposed here consists of modeling the machine as an RX bus, following the next three steps:

1) Calculate the power that each WT can extract from the wind for a given wind speed and a given rotor speed, according to its power coefficient curve.
2) Calculate the power that each WT can generate, according to the results of the load flow analysis, and to the rotor speed given in step 1).
3) Compare both powers and look for the value of the slip, for which the electrical and the mechanical powers coincide, for the wind speed given.

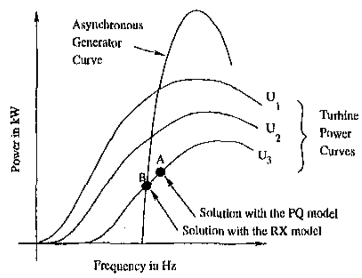

Fig. 3. Curves for the generator and the turbine.

The possible working points can be seen in Fig. 3. These points are where the generator curve crosses the turbine curves. The solution obtained using the PQ model is also shown in order to observe the different solutions given by each of the models.

The power curve of the WT used in the simulations has been obtained for a constant rotor speed of 34 rpm, which means a frequency of 50.29 Hz, given a gear ratio of 44.38, and also that the machine has four poles. This leads point A of Fig. 3 to be assumed as the solution given when using the PQ model, whilst point B is the solution given by the RX model. In both cases, the wind speed is considered to be U_3 ms^{-1}.

The RX model is based on the steady-state model of the induction machine, where it is represented by means of an impedance $\overline{Z} = R_s + jX_1 + (jX_m \| ((R_R/s) + jX_2))$ and a shunt capacitor with susceptance B_c. The magnetizing branch is here considered to be located between the stator and the rotor, which is a better approximation to the real machine.

The mechanical power developed by a WT can be calculated with (4).

A first, inexact way of working can be to fix the mechanical power and to assume that the slip can be calculated from the following equation:

$$s = \frac{-V^2 R_R + \sqrt{V^4 R_R^2 - 4 P R_R^2 (P X^2 + V^2 R_R)}}{2(P X_2 + V^2 R_R)}. \tag{5}$$

This equation is obtained, based on the steady-state model of the machine (Fig. 1), by applying Boucherot's theorem and assuming the stator resistance to be negligible.

The iterative process consists of the following: an initial value of the slip is considered at the beginning of the process, a suitable value of which is the machine's rated slip, or the value obtained from (5). With this value introduced in the expression for $\overline{Z}$, an initial load flow analysis can be carried out. With the results obtained, the mechanical power of the machine can be calculated as $P_m = -I_R^2 R_R((1-s)/s)$, where I_R is the rms value of the rotor current, calculated from the rms value of the stator current I_s as $\overline{I}_R = (jX_m/(R_R/s) + j(X_2 + X_m))\overline{I}_s$, where $\overline{I}_R$ and $\overline{I}_s$ are the phasors for the rotor and stator currents.

On the other hand, taking into account the wind speed and the slip of the machine, the value of the power coefficient can be calculated with the equation $c_p = f(\lambda)$ and the power extracted from the wind with (4). When these two powers are not equal, then a process of convergence for both values begins. In this case, the slip is modified:

$$s_k = s_{k-1} + \Delta s \tag{6}$$

where s_{k-i} and s_k are the present slip and the new slip (slip that must be taken into account in the next load flow analysis), respectively, and:

$$\Delta s = J^{-1} \Delta P_m \tag{7}$$

where ΔP_m is the difference between both powers. If the approximation $((1-s)/s) \approx (1/s)$ is assumed in the proximity of the working point, J can be calculated as:

$$J = R_R \left(\frac{S}{V}\right)^2 \frac{A+B}{s^2\left(\left(\frac{R_R}{s}\right)^2 + (X_2+X_m)^2\right)} \tag{8}$$

an equation where $S = \sqrt{P_g^2 + Q_c^2}$, and P_g and Q_c are the real power generated and the reactive power consumed by the machine. These powers can be written as $P_g = -(V^2/Z^2)\mathrm{Real}\{\overline{Z}\}$ and $Q_c = (V^2/Z^2)\mathrm{Imag}\{\overline{Z}\}$.

In (8), terms A and B are as follows:

$$A = -\frac{2X_m R_R}{s}\left(\left(\frac{R_R^2}{s}\right) + (X_2+X_m)^2\right)$$

$$B = -\left((X_m(X_2+X_m))^2 + \left(\frac{X_m R_R^2}{s}\right)\right) \cdot \left(-\frac{R_R}{s} + \left(\left(\frac{R_R}{s}\right)^2 + (X_2+X_m)^2\right)^2\right).$$

With the new values of s and J, the load flow analysis is repeated, and the process finishes when the mechanical power, P_{mk}, and the power taken out from the wind, P_k, are equal, or the difference between both is acceptable. This error is calculated as $\sum_{k=1}^{n}(P_{mk} - P_k)^2$, where n is the number of machines in the wind farm.

In short, the algorithm carried out to simulate the wind farm as an RX bus is as follows:

1) Begin with $s = s_{nom}$ in each machine, s_{nom} being the rated slip of the same. With this value, calculate the impedance $\overline{Z}$.
2) With these values, model the wind farm as an RX bus including the admittances of the machines in the admittance matrix and in the corresponding terms of the Jacobian.
3) As a result of the first power flow, the voltages in the buses can be obtained. With these, calculate the mechanical power of the machine with the expression $P_m = -I_R^2 R_R((1-s)/s)$.
4) With the value of s, calculate λ and c_p, and the power extracted from the wind with (4).
5) Compare both powers and, if they are not equal, recalculate s by means of (6) and go to step 2. If they are equal, the algorithm finishes.

TABLE I
NUMBER OF ITERATIONS AS A FUNCTION OF THE WIND SPEED AND THE INITIAL VALUE OF THE SLIP

$U(ms^{-1})$	$s_0 = -0.007$	$s_0 = 0$
5	3	2
10	4	6
15	3	5
20	4	4

IV. CONVERGENCE CHARACTERISTICS OF BOTH METHODS

The advantage of the PQ model is that the real power is calculated as a function of the wind speed for the first iteration of the power flow analysis, and from then on its value remains as a constant.

The reactive power depends on the real power and the bus voltage. Since the power is assumed constant, the only variable is the bus voltage. There are two possibilities for calculating the reactive power:

- The first of them consists of considering the voltage as having a constant value. In this case the specified real and reactive powers are known in the first iteration and remain unchanged. The bus behaves as a conventional PQ bus. With this simplification the error is not significant.
- The second way consists of calculating the reactive power as a function of the voltage. In this case the value of the specified reactive power must be updated in each iteration.

Either way, the convergence characteristics when the PQ bus model is used are similar to the case of the conventional PQ model.

When the RX model is used, there are two iterative processes. One of them is due to the load flow analysis and the other to the calculation of the slip of the asynchronous generator. From the point of view of the load flow analysis, the RX bus behaves as a PQ bus, where real And reactive powers are 0. Due to this, no additional load flow iterations are necessary. Nevertheless, obtaining the final slip usually involves between 2 and 6 iterations. In Table I the number of iterations can be seen for an example carried out with a wind farm containing 12 individually modeled WT's, under the assumption that the wind speed is the same in all the machines, and considering two different initial values for the slip. The value of -0.007 corresponds to the rated slip.

V. THE WIND FARM MODEL

The model of the entire wind farm must take all its machines into account.

In general, there can be four possible combinations relating to wind speed and machine type. The machines in the wind farm can be of the same kind or not. At the same time, the wind speed can be considered the same for all the machines of the wind farm or not. Additionally, the wind farm can be modeled as a PQ or as an RX bus.

The real power injected and the reactive power consumed by the wind farm are the sum of the real power injected and the reactive power consumed by all the machines. So, if the machines are of the same type and the wind speed is considered to be the same, the calculation of the power as a function of the wind

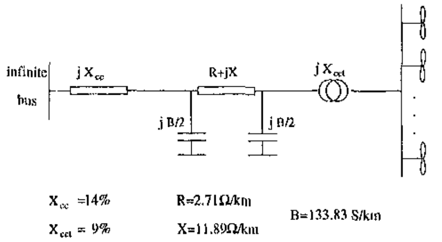

Fig. 4. Electrical network.

speed must be made only once in each iteration of the load flow analysis.

If there are different types of machines in the wind farm, with different power curves, the calculations must be made individually, and finally they must be added together if the PQ model is used, or the equivalent impedance must be calculated if the wind farm is modeled as an RX bus.

Even if the wind speed is considered constant for the whole wind farm, its layout can provoke different wind speeds to be considered for different WT's, especially if the direction of the wind is taken into account, and this direction coincides with the alignment of the WT's. When this occurs, a WT located behind another one, seen from the direction of the wind speed, will be affected by the wake created by the one upwind. In this case the effective wind speed for the WT downwind can be calculated following the model proposed in [9].

For the case simulated in this paper the following assumptions are made:

- The wind farm is composed of two rows of WT's, separated by a distance large enough to make sure that there is no interaction between both rows when the wind is perpendicular to them.
- The WT's in each row are near enough to make the interaction among several WT's important, due to wake effects. In this case, the model of [9] will be used.

In the case of the wind being perpendicular to the rows of turbines, and following the first of the stated assumptions, the wind speed is the same for all the machines. The probability of this circumstance occurring can be obtained by combining the wind speed distribution and the wind rose.

In the case of the wind being parallel to the rows of generators, the wind speed given is valid for the first WT, but for the other turbines, the approximation given in [9] is applied. So, for the second WT, the wind speed is calculated by means of next equation:

$$U_2 = U_1 K = U_1 \left(1 - (1 - \sqrt{1 - c_t}) \left(\frac{D}{D + 2kX} \right)^2 \right) \quad (9)$$

where U_1 is the wind speed for the first turbine, U_2 is the wind speed for the second turbine, c_t is the turbine thrust coefficient, D is the rotor diameter, X is the axial distance between both WT's and k is the wake decay constant. In [9] the wake decay constant is calculated as $k = (A/\ln(h/z_0))$, $A \approx 0.5$, where h is the hub height, and z_0 is the roughness length, defined in [2] as the *extrapolated height at which the mean wind speed becomes 0 if the vertical wind profile is assumed to have a logarithmic variation with height.* If the explained procedure is followed, the wind speed for a WT can be calculated as the product of the wind speed of the WT located in front of it and a constant equal to K.

TABLE II
REAL POWERS IN MW, REACTIVE POWERS IN MVAR AND VOLTAGES IN p.u. OBTAINED BY MEANS OF THE PQ AND RX BUS MODELS

	Real power		Reactive power		Voltage	
U (ms^{-1})	PQ	RX	PQ	RX	PQ	RX
7	-3.132	-3.102	0.140	0.013	1.004	1.004
9	-6.807	-6.696	0.647	0.560	1.003	1.003
11	-11.366	-11.036	1.837	1.785	1.000	1.000
13	-15.413	-14.843	3.395	3.449	0.995	0.995
15	-17.543	-16.859	4.392	4.603	0.992	0.991
17	-17.120	-16.640	4.182	4.467	0.992	0.992
19	-15.388	-14.929	3.377	3.494	0.995	0.995
21	-14.400	-13.550	2.957	3.211	0.996	0.997

TABLE III
POWER AND POWER COEFFICIENT OF THE TURBINE AS A FUNCTION OF WIND SPEED

$U(ms^{-1})$	$P(kW)$	c_p
5	13.89	0.25
6	35.12	0.36
7	62.75	0.41
8	96.75	0.42
9	136.15	0.42
10	180.35	0.40
11	227.33	0.38
12	271.61	0.35
13	308.27	0.31
14	335.39	0.27
15	350.86	0.23
16	352.98	0.19
17	342.41	0.15
18	324.20	0.12
19	307.76	0.10
20	295.85	0.08
21	288.00	0.07
22	282.03	0.05
23	277.20	0.05
24	272.81	0.04
25	271.06	0.03

As an example, for a wind farm where the rotors of the WT's have a length of 30 m, the distance between them is 75 m and the hub height is 31 m, if a value of $2 \cdot 10^{-3}$ is assumed for the roughness length, the value of k becomes 0.051. If the wind speed has a value of 15 ms^{-1} for the first WT, and a value of $c_t = 0.2$ is assumed, the wind speed of the second turbine can be found as the product of 15 ms^{-1} and the constant $K = 0.93$. This constant will be used to calculate the wind speeds for all turbines. So, for the ith turbine the wind speed can be approximated by $U_i = K^i U_1$.

VI. RESULTS

As an example, load flow analyses are carried out on the system in Fig. 4. The load flow algorithm used for the analysis is based on a Newton–Raphson method.

There are 50 identical stall-regulated WT's in the wind farm, where the generators are induction machines with the following parameters: $R_s = 0.00708\ \Omega$, $X_1 = 0.07620\ \Omega$,

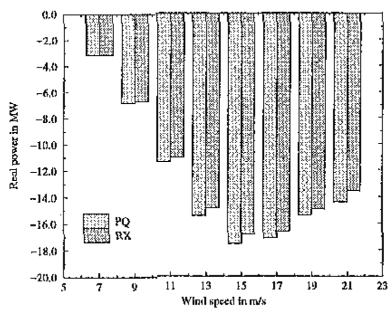

Fig. 5. Real powers with both models.

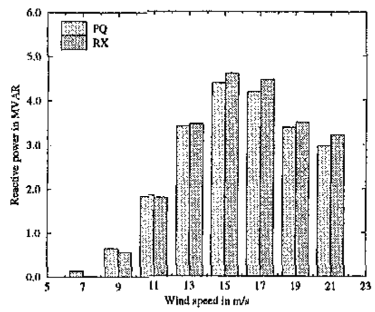

Fig. 6. Reactive powers with both models.

$X_m = 3.44979\ \Omega$, $X_2 = 0.23289\ \Omega$, and $R_R = 0.00759\ \Omega$. Its rated voltage is 660 V.

In Table III the powers the WT's can obtain from the wind for several wind speeds are given. These data are supplied assuming that the machine rotor speed is 34 rpm, so the function $c_p = f(\lambda)$ can easily be deduced.

The area swept by the blades of the WT's is 531 m^2 and the rated power is 330 kW.

The simulations are carried out for wind speeds going from 7 to 21 ms^{-1} in steps of 2 ms^{-1}.

The results for the real power, reactive power and voltage are given in Table II, and in Figs. 5–7. The results were obtained under the assumption that the wind speed is the same for all machines and the machines are idle compensated.

VII. Conclusions

Two models for wind farms have been developed in order to achieve better results in the load flow analysis than those given by the conventional models.

The first purpose of the paper was to demonstrate that, based on the steady-state of the induction machine, the RX model obtains the working point of this for each wind speed, and is the better approximation when the wind speed is the input datum.

However, the conventional PQ model can be improved, and good results can be obtained by means of it, with the advantage that it does not involve additional iterative processes.

The advantage of considering the wind speed as an input is that, knowing its frequency distribution (Weibull or Rayleigh distribution) in the location of the wind farm, a estimation can be made of the time that a given wind farm will produce a determined power, which can be of interest when, for example, studying steady-state security.

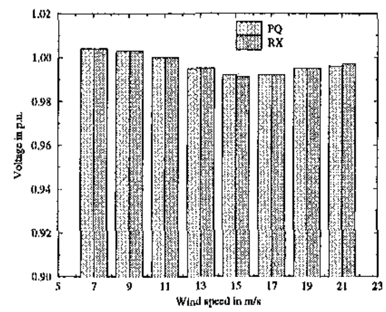

Fig. 7. Voltages obtained with both models.

TABLE IV
Constants for the Polynomial Approximation of the Power and Power Coefficient Curves of the WT Employed in the Simulations

constant	$P_m(kW) = f(U(ms^{-1}))$	$c_p = f(\lambda)$
a_0	-393.35	0.0699
a_1	468.91	-0.3765
a_2	-237.49	0.7400
a_3	65.71	-0.7216
a_4	-10.95	0.3892
a_5	1.16	-0.1214
a_6	-0.0804	0.0230
a_7	0.0035	-0.0027
a_8	$-9.49\ 10^{-5}$	0.0001

Appendix I
The Power Coefficient Curve

The manufacturer of a WT usually provides its power curve. From this curve and the rated rotor speed, the power coefficient curve can be obtained. For the WT employed in the simulations here, both curves are given in Table III, as functions of the wind speed.

The use of the power coefficient curve in the proposed algorithm for the RX model involves interpolating in those cases in which the calculated value of the tip speed ratio does not coincide with a value from the table. The coefficient can be alternatively calculated as:

$$c_p = \sum_{i=0}^{n} a_i \lambda^i \tag{10}$$

where the constants $a_i, \forall\, i = 0, \cdots, n$, are given in Table IV, assuming an eighth degree interpolation polynomial.

The curve of the mechanical power as a function of the wind speed can also be obtained by linear interpolation:

$$P_m = \sum_{i=0}^{n} a_i U^i \tag{11}$$

where the constants $a_i, \forall\, i = 0, \cdots, n$, are given in Table IV, assuming an eighth degree interpolation polynomial.

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