Chapter 1
Relationship Between Voltage and Active and Reactive Powers

The analytical link between voltage (V) and reactive power (Q) in electrical lines and loads is presented. Acceptable simplifications of equations are introduced to highlight dominant aspects of the V-Q link, which strongly impacts our understanding of grid voltage phenomena, voltage control, as well as performance required of protection solutions and design characteristics. An essential presentation of the main V-Q relationships in electrical lines often referred to by this book is provided.

1.1 Grid Short Lines

In order to establish the relationship between active and reactive power flow and voltage, one-line (Fig. 1.1a) and phasor (Fig. 1.1b) diagrams of a short line are analysed.

$\bar{v}_1$ and $\bar{v}_2$ are the phase voltages, and $\bar{\imath}_1$ and $\bar{\imath}_2$ are the currents at the line extremities. The supposed absence of the shunt admittance leads to $\bar{\imath} = \bar{\imath}_1 = \bar{\imath}_2$ in all points along the line. Denoting by φ the angle between $\bar{\imath}$ and $\bar{v}_2$, the components of the current $\bar{\imath}$ are $I_i = I\cos\varphi$ and $I_r = I\sin\varphi$. Assuming that voltage $\bar{v}_1$ is constant and $\bar{v}_2$ is the phase origin, the complex voltage drop $\Delta\bar{v} = \bar{Z}\,\bar{\imath}$ has two components:

$$\Delta u = RI_i + XI_r, \quad \delta u = XI_i - RI_r, \tag{1.1}$$

where $\bar{\imath} = I_i - jI_r$ for inductive loads, and Δu, δu are the longitudinal and transversal components of the voltage drop.

Let us denote by $\bar{S} = 3\bar{S}_2$ the three-phase complex power and by $\bar{S}_2 = V_2\left(I_i + jI_r\right) = P_2 + jQ_2$ the single-phase complex power. Introducing the active power P_2 and the reactive power Q_2, Eq. (1.1) become

$$\Delta u = \frac{RP_2 + XQ_2}{V_2}, \; \delta u = \frac{XP_2 - RQ_2}{V_2}. \tag{1.2}$$

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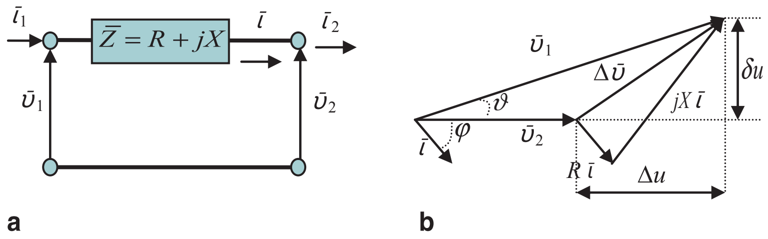

Fig. 1.1 Short line model: **a** one-line diagram; **b** phasor diagram

Because generally $R << X$ for transmission lines, it turns out that

$$\Delta u \approx \frac{XQ_2}{V_2},\ \delta u \approx \frac{XP_2}{V_2}. \tag{1.3}$$

Therefore, the voltage drop Δu is mainly determined by the reactive power flow along the line. That is, the magnitude difference between $\bar{\upsilon}_1$ and $\bar{\upsilon}_2$ depends mainly on the reactive power transits. Instead, the active power substantially affects the phase difference between $\bar{\upsilon}_1$ and $\bar{\upsilon}_2$. According to this, the flow of reactive power has to be reduced first in order to contain the voltage drop. In practice, this is possible when the reactive power generation occurs near the consumption area.

Starting from relation (1.3) and taking into account the fact that V_1/V_2 is close to 1 per unit (p.u.), it follows that

$$\frac{\Delta u}{V_2} \approx \frac{\Delta u}{V_1} \approx \frac{XQ_2}{V_1^2} \approx \frac{Q_2}{S_{2cc}}, \tag{1.4}$$

where $S_{2cc} = V_1^2/X$ is the short-circuit power at node 2.

Characteristic V-Q of the system is therefore given by expression

$$V_2 \approx V_1\left(1 - \frac{Q_2}{S_{2cc}}\right). \tag{1.5}$$

This means the voltage at node 2 will depend on the amount of reactive power injected into it by node 1 as well as on the weakness of node 2.

From the above considerations, the following points concerning receiving bus and sending bus are clear:

- Voltage magnitudes are largely determined by reactive power flows;
- Reactive power flow on the line increases the voltage difference between sending and receiving buses;
- The stronger the short-circuit power in a given bus, the less reactive power flow from the line reduces its voltage with respect to the sending bus value.

1.1.1 Reactive Power Transfer

Because of the relevance of reactive power on voltage value, it is important to highlight the basic equation describing reactive power transfer along a line. Referring again to Fig. 1.1 and considering a pure inductive line, the complex power at bus 2 is given by

$$\overline{S}_2 = \overline{V}_2 \overline{I}_2^{\,*}, \tag{1.6}$$

with

$$\overline{I_2} = \frac{\overline{V}_2 - \overline{V}_2}{jX}.$$

From Fig. 1.1, considering the angle ϑ between the two voltage phasors $(\overline{V}_1 \angle \overline{V}_2)$, we find that current I_2 becomes

$$\overline{I_2} = \frac{V_1 \cos\vartheta + jV_1 \sin\vartheta - V_2}{jX}.$$

and

$$\overline{I_2}^{\,*} = \frac{V_1 \sin\vartheta}{X} + \frac{j(V_2 - V_1 \cos\vartheta)}{X}.$$

Therefore,

$$\overline{S}_2 = \overline{V}_2 \overline{I_2}^{\,*} = V_2 \left[\frac{V_2 \sin\vartheta}{X} - \frac{j(V_2 - V_2 \cos\vartheta)}{X} \right].$$

Writing as

$$\overline{S}_2 = P_2 + jQ_2 = \frac{V_2 V_2 \sin\vartheta}{X} + \frac{j\left(V_2 V_2 \cos\vartheta - V_2^2\right)}{X}.$$

with

$$P_2 = \frac{V_2 V_1 \sin\vartheta}{X}. \tag{1.7}$$

and

$$Q_2 = \frac{V_1 V_2 \cos\vartheta - V_2^2}{X}. \tag{1.8}$$

Analogously, the reactive power at the sending bus V_1 is given by

$$Q_2 = \frac{V_1^2 - V_1 V_2 \cos \vartheta}{X}. \quad (1.9)$$

From the above equations the following observations about the receiving bus are clear:

- The reactive powers at extreme edges of a line are inversely proportional to line reactance value. The same law of dependence applies to the active powers;
- Line reactive power flow increases if voltage at sending bus increases or if the voltage at the receiving buses decreases;
- Small magnitude variations of extreme line edge voltages (values around 1 p.u.) and $\cos\vartheta$ (values around 1) combined significantly impact the amount and direction of line reactive power flow. This point illustrates the critical effect that changing voltages and/or voltage vector angles has on reactive power flow control, especially in manual operation;
- The active power flow is, more than reactive power, influenced by load angle variation due to the high value of the slope of $\sin\vartheta$ around $\vartheta = 0.0$.

1.1.2 Losses

Minimisation of reactive power transport is also motivated by the reduction of the Joule losses on the line. In fact, these losses are expressed by the equation

$$\Delta P_L = 3RI^2 = 3R\frac{S_2^2}{\left(\sqrt{3}V_2\right)^2} = R\frac{P_2^2 + Q_2^2}{V_2^2}, \quad (1.10)$$

where not only the active power but also the reactive power flow contributes to line losses.

Because the thermal limit is defined by the admissible current for any network element, the reactive power transfer also reduces the amount of active power transmission flow. Loss minimisation does, therefore, require reactive power compensation as well as system operation at the highest voltage values.

From the above considerations, the following statement concerning reactive power line transfer is apparent:

- Reactive power flow on the line increases losses.

1.2 Reactive Loads

We further consider load voltage. Figure 1.2 shows two basic reactive loads: inductive type ($+jX$), shown in Fig. 1.2a, and capacitive type ($-jX$), shown in Fig. 1.2b.

Considering the inductive load (a) and because the current is π/2-delayed with respect to the voltage:

$$\bar{V} = jX(-jI) = XI = V.$$

Since $I = Q/V$, the right side becomes $XQ/V = V$, so

$$V^2 = XQ.$$

At small variations (i.e., differentiating),

$$2V\Delta V = X\Delta Q,$$

so

$$\Delta V = X\frac{\Delta Q}{2V}.$$

Or

$$\frac{\Delta V}{V} = X\frac{\Delta Q}{2V^2}. \tag{1.11}$$

- According to (1.11), the injection of reactive power into an inductive load determines the load's voltage increase. Obviously, for a given amount of reactive power absorbed by the load, the higher the short circuit power of the load bus the less the voltage increase effect.

Analogously, considering the capacitive load (Fig. 1.2b) with the reactive power delivered by the load:

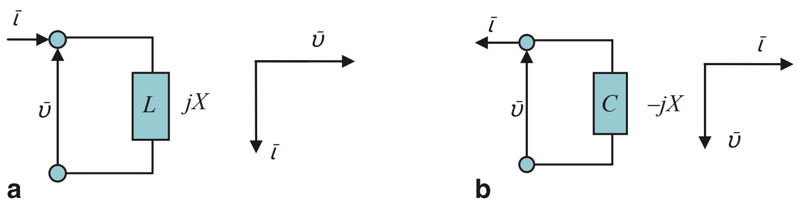

Fig. 1.2 One-line diagram: **a** inductive load ($+jX$); **b** capacitive load ($-jX$)

$$\bar{V} = -jX(+jI) = XI = V, \text{ so again, } V^2 = XQ.$$

As before,

$$2V\Delta V = X\Delta Q, \text{ giving } \Delta V = X\frac{\Delta Q}{2V},$$

and, again,

$$\frac{\Delta V}{V} = X\frac{\Delta Q}{2V^2}. \tag{1.12}$$

- According to equation (1.12) and Fig. 1.2b, it is clear that the voltage of a capacitive load increases when the load injects reactive power into the grid.

In extreme synthesis, reactive power injection into a load bus increases or reduces the bus voltage depending on the inductive or capacitive nature of the load seen by the bus. In a predominant inductive grid, as a real power system is, the evaluation of the positive or negative effects on a given bus voltage by the reactive power injection on that load bus would require a comparison between the line voltage drop (due to the reactive power flow) and the load bus voltage increase (due to the reactive power injection).

Because transmission line reactance is very small in comparison with the loads seen by the transmission buses and due to the fact that real loads are of reactive nature basically, the obvious conclusions follow:

- Transmission network voltage increase is properly operated by injecting reactive power into the load buses (inductive loads) from nearest resources. The opposite control determines the voltage lowering.
- The higher the voltage in the transmission grid the lower the control effort to sustain voltages; this is because of capacitive effect of the lines.

1.3 Grid Medium-Long Length Lines

Examining the medium/long line case, admittance can no longer be neglected as was the assumption in the short line case (Fig. 1.1). We refer now to the line Γ scheme (see Fig. 1.3), where the shunt parameters are modelled by a concentrated capacity *C*.

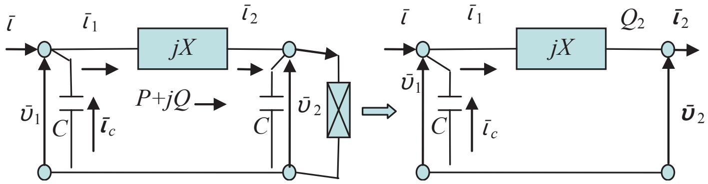

Fig. 1.3 One-line diagram Γ scheme

Equations (1.3) and (1.4) also describe the scheme in Fig. 1.3b, assuming $\bar{\iota}_1 = \bar{\iota} + \bar{\iota}_c$. So, the same conclusions as before can be reached when shunt admittance is considered. Obviously, when including shunt admittance, Q_2 is lower due to the contribution ωCV_2^2 to the load reactive power. On the other hand, Q_2 will also depend on the additional contribution of shunt admittance at the V_1 side:

$$Q_2 = Q_1 + \omega CV_1^2 - \frac{\omega LS_{02}^2}{V_2^2},$$

where $\bar{S}_2 = 3S_{02}$ indicates the complex power at a bus of voltage V_2 and current $I_2 = S_{02} / V_2$; Q_1 is the reactive power input due to $\bar{\iota}$.

In this case it is interesting to note that the reactive power balance between the amounts produced by the line and absorbed by the line reactance is

$$Q_L = \omega CV_1^2 - \frac{\omega LS_2^2}{V_2^2}.$$

If S_{02} assumes the value of the line "natural power",

$$S_{02} = P_N = V_2^2\sqrt{\frac{C}{L}} = \frac{V_2^2}{Z_C},$$

then

$$Q_L = \omega CV_1^2 - \omega CV_2^2.$$

If $V_1 = V_2$ then Q_L =0.0; but Q_2 =0.0 as well. Therefore, from the above conditions there exists an active power P_N injected into the load that makes the reactive balance zero.

Under these conditions in fact, the power transmitted on the line is at constant voltage magnitude and unitary power factor. If transmitted power S is higher than natural power P_N, as it is for high loaded overhead lines, then the line absorbs the reactive power.

For cables, the term ωCV^2 is predominant and the reactive power generation overcomes the absorption. For cable lines, the admissible maximum thermal power is always lower than natural power.

In conclusion:

- Shunt admittance gives, in general, a significant contribution to voltage support, partially compensating local loads, except at particular operating conditions.
- The higher the line voltage, the larger the reactive power production by the shunt admittance.

Without a doubt, the shunt admittance reactive power contribution must be considered in a power system voltage analysis, but only in terms of the reactive power resources determining the operating point; shunt admittance is not to be used for real-time voltage control. In other words, voltage control generally would not be operated by the continuous switching of operator lines on and off.

1.4 Grid as a Combination of Loads and Lines

In large real electrical grids, usually characterised by the prevailing inductive nature of the loads, the effects of the active (P) and reactive (Q) power injections into the system buses are generally seen in terms of voltage (V) and frequency (f) variations.

Voltage variations in system buses (vector dV) are usually described in terms of differential equations through linearised models making use of matrices $\partial V/\partial P$ and $\partial V/\partial Q$, otherwise called *sensitivity matrices*, denoted by S_{VP} and S_{VQ}, respectively. The voltage variation in a given network bus corresponds, as seen before, to active, but it mostly corresponds to reactive power flow changes on the concurrence lines in that bus. Therefore, considering overall grid buses, the matrix equation describing the dependence of vector dV on injection vectors dP and dQ is here after shown as

$$dV = \frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial Q} dQ = S_{VP}\, dP + S_{VQ} dQ. \tag{1.13}$$

Coefficients of the sensitivity matrices obviously depend on load and line characteristics and show, at each bus and for a given local injection, the resultant effect of local and remote changes either in voltages, reactive power flows or line losses.

Numerically speaking and from a voltage control perspective, the most relevant matrix is $\partial V/\partial Q$, whose coefficients at a given column indicate the voltage variation contribution at each grid bus corresponding to the injection into the bus linked to the selected column of a unitary amount of reactive power.

With respect to grid voltage controllability, it is important one recognise the dependence of bus voltage on reactive power injections. In fact, active power variations are basically managed in connection with the production plan, which in turn follows contractual agreements. Therefore, it is not usually required that active power production deviate toward contributing to voltage support, unless an emergency condition accompanied by extreme security risk justifies it.

From this perspective, changes in active power production or load shedding finalised to grid voltage control is a not economical, and it is a far from ethical grid operation praxis during grid normal operating conditions. In fact, the truer and more effective variable to be used, giving the best outcome for power system voltage control, is the reactive power. This confirms the power system operator's need for adequate controllable reactive power reserves and for an adequate control system, one that is aimed at achieving a proper and safe power system voltage control.

Further information on the power system linearised model and its use in the design of grid voltage control systems is provided in Chap. 4.

References

1. Elgerd OI (1971) Electric energy systems theory: an introduction. McGraw-Hill, New York
2. Weedy BM (1979) Electric power systems, 3rd edn. Wiley, New York
3. Kundur P (1994) Power system stability and control. McGraw-Hill, New York
4. Miller TJE (1982) Reactive power control in electric systems. Wiley, New York
5. Eremia M, Shahidehpour M (2013) Handbook of electrical power system dynamics—modeling, stability, and control. Wiley, New York