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237: Homework 5

November 24, 2009

## 6.3

3

a)

$$y'' = ty$$

$$y_1 = y$$

$$y_2 = y'$$

$$y_1' = y_2$$

$$y_2' = ty$$

b)

$$y'' = 2ty' - 2y$$

$$y_1 = y$$

$$y_2 = y'$$

$$y_1' = y_2$$

$$y_2' = 2ty_2 - 2y_1$$

c)

$$y'' = ty' + y$$

$$y_1 = y$$

$$y_2 = y'$$

$$y_1' = y_2$$

$$y_2' = ty_2 + y_1$$

## 6.6

2

a)

$$y' = y - y^2$$

Fixed points at  $y = 0, 1$ . 0 is unstable, 1 is stable.

$$f_y = 1 - 2y$$

$$f_y(0) = 1$$

$$f_y(1) = -1$$

Assuming -1 is small, then not stiff.

b)

$$y' = 10(y - y^2)$$

Fixed points at  $y = 0, 1$ . 0 is unstable, 1 is stable.

$$f_y = 10(1 - 2y)$$

$$f_y(0) = 10$$

$$f_y(1) = -10$$

Stiff.

c)

$$y' = -10 \sin(y)$$

Fixed points at  $y = 2n\pi$  and  $y = (2n + 1)\pi, \forall n \in \mathbb{Z}$

$y = 2n\pi$  stable,  $\forall n \in \mathbb{Z}$

$y = (2n + 1)\pi$  unstable,  $\forall n \in \mathbb{Z}$

$$f_y = -10 \cos(y)$$

$$y = 2n\pi \Rightarrow f_y = -10, \forall n \in \mathbb{Z}$$

$$y = (2n + 1)\pi \Rightarrow f_y = 10, \forall n \in \mathbb{Z}$$

Stiff.

1)

$$\begin{bmatrix} 2 & c \\ c & 1 \end{bmatrix} \begin{bmatrix} \theta'_1 \\ \theta'_2 \end{bmatrix} = \begin{bmatrix} -2 \sin(\theta_1 - s(\theta'_2)^2) \\ -\sin(\theta_2) + s(\theta'_1)^2 \end{bmatrix}$$

$$\theta_1 = u_1$$

$$\theta_2 = u_2$$

$$\theta'_1 = u_3$$

$$\theta'_2 = u_4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & c \\ 0 & 0 & c & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}' = \begin{bmatrix} u_3 \\ u_4 \\ -2 \sin(\theta_1 - s(\theta'_2)^2) \\ -\sin(\theta_2) + s(\theta'_1)^2 \end{bmatrix}$$

2) The default initial condition results in semi-periodic motion.

3) There is some sort of perturbation happening, which is most likely due to roundoff error in the calculations.

6) The stability plot is assymetric since there are multiple possibilities for the position of the second bob for symmetric positions of the first bob.