

THESIS ABSTRACT

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Noncommutative Projective Schemes were introduced by Michael Artin and J.J. Zhang in their 1994 paper of the same name as a generalization of projective schemes to the setting of not necessarily commutative algebras over a commutative ring. In this work, we study the derived category of quasi-coherent sheaves associated to a noncommutative projective scheme with a primary emphasis on the triangulated equivalences between two such categories.

We adapt Artin and Zhang's noncommutative projective schemes for the language of differential graded categories and work in $\mathrm{Ho}(\mathrm{dgc}at_k)$, the homotopy category of differential graded categories, making extensive use of Bertrand Toën's Derived Morita Theory. For two noncommutative projective schemes, X and Y , we associate differential graded enhancements, $\mathcal{D}(X)$ and $\mathcal{D}(Y)$, of the respective derived categories of quasi-coherent sheaves. Under appropriate cohomological conditions, we provide a noncommutative geometric description of the subcategory, $\mathbf{R}\underline{\mathrm{Hom}}_c(\mathcal{D}(X), \mathcal{D}(Y))$, of the internal Hom category in $\mathrm{Ho}(\mathrm{dgc}at_k)$. As an immediate application, we show that, under these conditions, any triangulated equivalence between the derived categories induces an equivalence of Fourier-Mukai type, with kernel an object of the derived category of quasi-coherent sheaves on the appropriate analogue of the product.