MATH 115
EXAM 01

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. Unless otherwise stated, all supporting work is required. It is advised, although not required, that you check your answers. You may **not** use any calculators.

Name: Answer Key

Problem	Points Earned	Points Possible
1		10
2		10
3		20
4		20
5		20
6		20
Bonus		10
Total		100

Date: September 24, 2014.

1. Problems

1. *Solve the following absolute value equation.*

$$5|y+1| - 20 = 0.$$

$5|y+1| - 20 = 0$

$\Rightarrow 5|y+1| = 20 = 5 \cdot 4$

$\Rightarrow |y+1| = 4$

$\Rightarrow 4 = y+1$ or $4 = -(y+1) = -y-1$

$\Rightarrow \boxed{y = 3}$ or $\boxed{y = -1-4 = -5}$

2. *Solve the equation*

$$\frac{6}{x-5} + \frac{7}{x+5} = \frac{13x-5}{x^2-25}.$$

$\underline{x \neq \pm 5} \Rightarrow x^2 - 25 = (x+5)(x-5) \neq 0$

$$\frac{6}{x-5} + \frac{7}{x+5} = \frac{13x-5}{x^2-25}$$

$$\Rightarrow (x^2-25)\left(\frac{6}{x-5} + \frac{7}{x+5}\right) = \left(\frac{13x-25}{x^2-25}\right)(x^2-25)$$

$$\Rightarrow \quad 6(x+5) + 7(x-5) = 13x - 5$$

$$\Rightarrow \quad 6x + 30 + 7x - 35 = 13x - 5$$

$$\Rightarrow \quad 13x - 5 = 13x - 5$$

This is an identity, so the equation is true for all real numbers except $x = 5$ and $x = -5$.

3. *Find all solutions (real and complex) to the equations*

(a)

$$x^2 - 3x - 18 = 0.$$

$x^2 - 3x - 18 = (x-6)(x+3) = 0$

$\Rightarrow x = 6$ or $x = -3$.

(b)

$$x^2 + x + 1 = 0.$$

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2}$$

4. *Solve the following inequalities. Write the solution set in interval notation and graph it.*

(a)

$$4x - 7 < -5(x - 4).$$

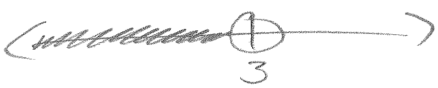
$4x-7<-5(x-4)$

$\Rightarrow 4x-7<-5x+20$

$\Rightarrow 4x+5x< 20+7$

$\Rightarrow 9x<27$

$\Rightarrow x<3$



(b)

$$1 < \left|\frac{x-6}{5}\right|$$

$1<\left|\frac{x-6}{5}\right|$

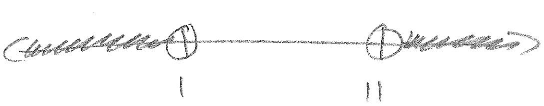
$\Rightarrow 1<\frac{|x-6|}{5}$

$\Rightarrow 5<|x-6|$

$\Rightarrow 5<x-6$ or $x-6<-5$

$\Rightarrow 11<x$ or $x<1$

$(-\infty,1)\cup(11,\infty)$



5. *Consider the two points* $(3,6)$ *and* $(6,2)$.

(a) Find the distance between these points.

$$\begin{aligned} d((3,6),(6,2)) &= \sqrt{(3-6)^2+(6-2)^2} \\ &= \sqrt{(-3)^2+(4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

(b) Find the midpoint of the line segment connecting these points.

$$m = \left(\frac{3+6}{2}, \frac{2+6}{2}\right) = \left(\frac{9}{2}, \frac{8}{2}\right) = \left(\frac{9}{2}, 4\right).$$

(c) Recall that the equation of a circle centered at (h,k) *of radius* r *is*

$$(x-h)^2+(y-k)^2=r^2.$$

Using this equation and the previous two parts, give the equation of a circle passing through $(3,6)$ *and* $(6,2)$.

Use the circle of radius $\frac{d}{2} = 5/2$ centered at the midpoint $m = (\frac{9}{2}, 4)$. This has equation

$$\left(x-\frac{9}{2}\right)^2 + (y-4)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}.$$

6. *Given the two points* $(0, 7)$ *and* $(1, -7)$

(a) Compute the slope of the line between these points.

$$m = \frac{-7-7}{1-0} = \frac{-14}{1} = -14$$

(b) Write the equation of the line between these points in Slope-Intercept form.

$$y = -14x + 7.$$

7 (Bonus). *Given a quadratic equation*

$$ax^2 + bx + c = 0 \tag{1}$$

the quadratic formula gives the two solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{2}$$

By completing the square in Equation (1), *derive Formula* (2).

$ax^2 + bx + c = 0$

$\Rightarrow \quad ax^2 + bx = -c$

$\Rightarrow \quad x^2 + \left(\frac{b}{a}\right)x = -\frac{c}{a}$

$\Rightarrow \quad x^2 + \left(\frac{b}{a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$

$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$

$\Rightarrow \quad x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$