MATH 122

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Calculus for Business Administration and Social Sciences

Outline

1. 2.1: Instantaneous Rate of Change

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Definition

Definition 1

The *instantaneous rate of change* of f at a is defined to be the limit of the average rates of change of f over successively smaller intervals around a.
This is also known as the *derivative of* f *at* a.

Example

The quadratic

$$s(t) = -4.9t^2 + 9.8t$$

models the position of an object thrown vertically into the air with an initial velocity of 9.8 m/s.

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models the position of an object thrown vertically into the air with an initial velocity of 9.8 m/s. What is the instantaneous rate of change at the vertex, where $t = 1$?

Example (cont.)

Here are some values:

t	$\frac{f(t)-f(1)}{t-1}$

EXAMPLE (CONT.)

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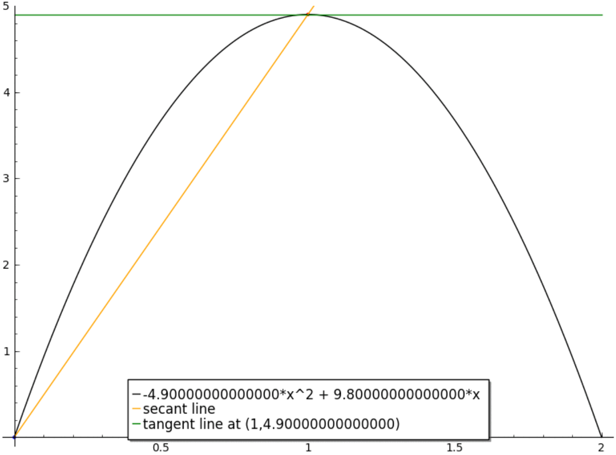
So, we would guess that the instantaneous rate of change is 0 at $t = 1$.

Animation


5
4
3
2
1
0.5
1
1.5
2
-4.90000000000000*x^2 + 9.80000000000000*x
secant line
tangent line at (1,4.90000000000000)

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- In this case, we can define a function $f'(x)$ that outputs the instantaneous rate of change of f at x.
- We call $f'(x)$ the *derivative function*.

Definition 3

- We can regard $f'(x_0)$ as a velocity by viewing it as the slope of a line passing through $(x_0, f(x_0))$.

The Tangent Line

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- We can regard $f'(x_0)$ as a velocity by viewing it as the slope of a line passing through $(x_0, f(x_0))$.
- We call the line

$$y - f(x_0) = f'(x_0)(x - x_0)$$

the *line tangent to f at* $(x_0, f(x_0))$.

Linearization

- Since we defined $f'(x_0)$ by a limit,

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- This means f locally looks like a line!

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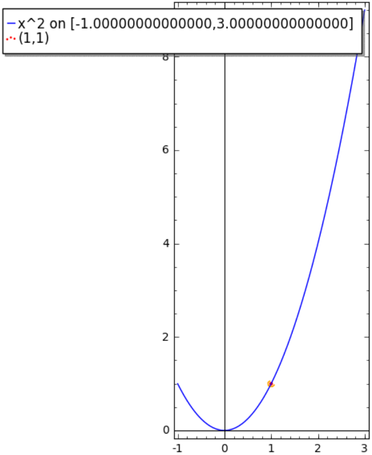

Non-Differentiable Function

Consider the absolute value function

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- So the derivative at $(0, 0)$ is **not** defined: it's -1 if we approach from left to right, and 1 if right to left.

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- $f'(x) \leq 0$, then f is decreasing on (a, b),
- $0 \leq f'(x)$, then f is increasing on (a, b),
- $f'(x) = 0$, then f is constant on (a, b).