MATH 122

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Calculus for Business Administration and Social Sciences

Outline

1. 3.1: Derivatives of Polynomials
   - Constants
   - Linearity
   - Power Rule

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Derivative of Constants

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- Therefore $f'(x) = 0$.

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- $$\frac{\mathrm{d}}{\mathrm{dx}}(af(x))$$

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Derivative of a Power Functions

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Remark 1

The derivative of a linear function is

$$\frac{\mathrm{d}}{\mathrm{d}x}(mx + b) = m\frac{\mathrm{d}}{\mathrm{d}x}x + \frac{\mathrm{d}}{\mathrm{d}x}(b) = m$$

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The derivative is

$$\begin{aligned} p'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}(a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0) \\ &= a_n \frac{\mathrm{d}}{\mathrm{d}x}(x^n) + a_{n-1} \frac{\mathrm{d}}{\mathrm{d}x}(x^{n-1}) + \cdots \\ &+ a_2 \frac{\mathrm{d}}{\mathrm{d}x}(x^2) + a_1 \frac{\mathrm{d}}{\mathrm{d}x}(x) \end{aligned}$$

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&= a_n \frac{d}{dx}(x^n) + a_{n-1} \frac{d}{dx}(x^{n-1}) + \cdots \\
&+ a_2 \frac{d}{dx}(x^2) + a_1 \frac{d}{dx}(x) \\
&= n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \cdots + 2 a_2 x + a_1.
\end{aligned}$$

Example

Differentiate the following

1. $A(t) = 3t^5$
2. $r(p) = p^5 + p^3$
3. $f(x) = 5x^2 - 7x^3$
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4. $g'(t) = t/2$.

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$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^3 - 2x^2 - 5x + 7) \\ &= \frac{d}{dx}(x^3) - \frac{d}{dx}(2x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(7) \end{aligned}$$

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Exponentials

The derivative of e^x is

$$\frac{d}{dx}(e^x) = e^x.$$

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