Math 122

Blake Farman [1]

[1] University of South Carolina, Columbia, SC USA

Calculus for Business Administration and Social Sciences

Outline

Outline

1. 3.3: The Chain Rule
2. 3.4: The Product and Quotient Rules

THE CHAIN RULE

Let f and g be differentiable functions such that $f \circ g(x)$ is well-defined.

The Chain Rule

Let f and g be differentiable functions such that $f \circ g(x)$ is well-defined. The derivative of the composition is given by

$$(f \circ g)'(x) = f' \circ g(x) \cdot g'(x).$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$(f \circ g)(t)$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t)$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t) = P_0 e^{\ln(a)t}$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t) = P_0 e^{\ln(a)t} = P_0 (e^{\ln(a)})^t$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t) = P_0 e^{\ln(a)t} = P_0 (e^{\ln(a)})^t = P_0 a^t$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t) = P_0 e^{\ln(a)t} = P_0 (e^{\ln(a)})^t = P_0 a^t = P(t).$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t) = P_0 e^{\ln(a)t} = P_0(e^{\ln(a)})^t = P_0 a^t = P(t).$$

Hence

$$P'(t) = f' \circ g(t) \cdot g'(t)$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t) = P_0 e^{\ln(a)t} = P_0 (e^{\ln(a)})^t = P_0 a^t = P(t).$$

Hence

$$\begin{aligned} P'(t) &= f' \circ g(t) \cdot g'(t) \\ &= P_0 e^{\ln(a)t} \cdot \ln(a) \end{aligned}$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t) = P_0 e^{\ln(a)t} = P_0 (e^{\ln(a)})^t = P_0 a^t = P(t).$$

Hence

$$\begin{aligned} P'(t) &= f' \circ g(t) \cdot g'(t) \\ &= P_0 e^{\ln(a)t} \cdot \ln(a) \\ &= P_0 a^t \cdot \ln(a) \end{aligned}$$

The Derivative of Arbitrary Exponentials

Let $P(t) = P_0 a^t$.
Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$$(f \circ g)(t) = f(\ln(a)t) = P_0 e^{\ln(a)t} = P_0 (e^{\ln(a)})^t = P_0 a^t = P(t).$$

Hence

$$\begin{aligned} P'(t) &= f' \circ g(t) \cdot g'(t) \\ &= P_0 e^{\ln(a)t} \cdot \ln(a) \\ &= P_0 a^t \cdot \ln(a) \\ &= \ln(a) P(t). \end{aligned}$$

EXAMPLE

Differentiate $(x+5)^2$.

Example

Differentiate $(x+5)^2$.

$$\frac{d}{dx}(x+5)^2 = 2(x+5)^1 \cdot \frac{d}{dx}(x+5)$$

Example

Differentiate $(x+5)^2$.

$$\begin{aligned} \frac{d}{dx}(x+5)^2 &= 2(x+5)^1 \cdot \frac{d}{dx}(x+5) \\ &= 2(x+5)(1) \end{aligned}$$

Example

Differentiate $(x+5)^2$.

$$\begin{aligned}
\frac{d}{dx}(x+5)^2 &= 2(x+5)^1 \cdot \frac{d}{dx}(x+5) \\
&= 2(x+5)(1) \\
&= 2(x+5)
\end{aligned}$$

Example

Differentiate $(x+5)^2$.

$$\begin{aligned} \frac{d}{dx}(x+5)^2 &= 2(x+5)^1 \cdot \frac{d}{dx}(x+5) \\ &= 2(x+5)(1) \\ &= 2(x+5) \\ &= 2x+10. \end{aligned}$$

Example

Find the derivative of e^{3x}.

Example

Find the derivative of e^{3x}.

$$\frac{d}{dx} e^{3x} = e^{3x} \cdot \frac{d}{dx}(3x)$$

Example

Find the derivative of e^{3x}.

$$\begin{aligned} \frac{d}{dx} e^{3x} &= e^{3x} \cdot \frac{d}{dx}(3x) \\ &= e^{3x} \cdot 3 \end{aligned}$$

EXAMPLE

Find the derivative of e^{3x}.

$$\begin{aligned}\frac{d}{dx}e^{3x} &= e^{3x}\cdot\frac{d}{dx}(3x)\\ &= e^{3x}\cdot 3\\ &= 3e^{3x}.\end{aligned}$$

Example

Differentiate $\ln\left(2t^2+3\right)^2$

Example

Differentiate $\ln\left(2t^2+3\right)^2$

$$\frac{d}{dx}\left(\ln\left(2t^2+3\right)^2\right) = 2\ln\left(2t^2+3\right)^1 \cdot \frac{d}{dx}\ln\left(2t^2+3\right)$$

Example

Differentiate $\ln\left(2t^2+3\right)^2$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(2t^2+3\right)^2\right) &= 2\ln\left(2t^2+3\right)^1\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(2t^2+3\right)\\ &= 2\ln\left(2t^2+3\right)\cdot\frac{\frac{\mathrm{d}}{\mathrm{d}x}\left(2t^2+3\right)}{2t^2+3}\end{aligned}$$

Example

Differentiate $\ln\left(2t^2+3\right)^2$

$$\begin{aligned}
\frac{d}{dx}\left(\ln\left(2t^2+3\right)^2\right) &= 2\ln\left(2t^2+3\right)^1 \cdot \frac{d}{dx}\ln\left(2t^2+3\right) \\
&= 2\ln\left(2t^2+3\right) \cdot \frac{\frac{d}{dx}\left(2t^2+3\right)}{2t^2+3} \\
&= 2\ln\left(2t^2+3\right) \cdot \frac{4t}{2t^2+3}
\end{aligned}$$

Example

Differentiate $\ln\left(2t^2+3\right)^2$

$$
\begin{aligned}
\frac{d}{dx}\left(\ln\left(2t^2+3\right)^2\right) &= 2\ln\left(2t^2+3\right)^1 \cdot \frac{d}{dx}\ln\left(2t^2+3\right) \\
&= 2\ln\left(2t^2+3\right) \cdot \frac{\frac{d}{dx}\left(2t^2+3\right)}{2t^2+3} \\
&= 2\ln\left(2t^2+3\right) \cdot \frac{4t}{2t^2+3} \\
&= \frac{8t\ln\left(2t^2+3\right)}{2t^2+3}
\end{aligned}
$$

Example

- The amount of gas, G, in gallons, consumed by a car depends on the distance, s, traveled in miles, which in turn depends on the time traveled, t.

Example

- The amount of gas, G, in gallons, consumed by a car depends on the distance, s, traveled in miles, which in turn depends on the time traveled, t.
- If the car consumes 0.05 gallons for each mile traveled and the car is traveling 30 mph, then how fast is the gas being consumed?

Example

- The amount of gas, G, in gallons, consumed by a car depends on the distance, s, traveled in miles, which in turn depends on the time traveled, t.
- If the car consumes 0.05 gallons for each mile traveled and the car is traveling 30 mph, then how fast is the gas being consumed?
-

$$\frac{\mathrm{d}}{\mathrm{dt}}(G \circ s(t))$$

Example

- The amount of gas, G, in gallons, consumed by a car depends on the distance, s, traveled in miles, which in turn depends on the time traveled, t.
- If the car consumes 0.05 gallons for each mile traveled and the car is traveling 30 mph, then how fast is the gas being consumed?
- $$\frac{\mathrm{d}}{\mathrm{dt}}(G \circ s(t)) = G' \circ s(t) \cdot s'(t)$$

Example

- The amount of gas, G, in gallons, consumed by a car depends on the distance, s, traveled in miles, which in turn depends on the time traveled, t.
- If the car consumes 0.05 gallons for each mile traveled and the car is traveling 30 mph, then how fast is the gas being consumed?
- $$\begin{aligned} \frac{\mathrm{d}}{\mathrm{dt}}(G \circ s(t)) &= G' \circ s(t) \cdot s'(t) \\ &= 0.05 \frac{\text{gal}}{\text{mile}} \cdot 30 \frac{\text{miles}}{\text{hour}} \end{aligned}$$

Example

- The amount of gas, G, in gallons, consumed by a car depends on the distance, s, traveled in miles, which in turn depends on the time traveled, t.
- If the car consumes 0.05 gallons for each mile traveled and the car is traveling 30 mph, then how fast is the gas being consumed?
- $$\begin{aligned} \frac{\mathrm{d}}{\mathrm{dt}}(G \circ s(t)) &= G' \circ s(t) \cdot s'(t) \\ &= 0.05\frac{\text{gal}}{\text{mile}} \cdot 30\frac{\text{miles}}{\text{hour}} \\ &= 1.5\frac{\text{gal}}{\text{hour}}. \end{aligned}$$

Product Rule

If f and g are differentiable functions, then

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).$$

Quotient Rule

Assume that f and g are differentiable functions and $f(x)/g(x)$ is well-defined.

Quotient Rule

Assume that f and g are differentiable functions and $f(x)/g(x)$ is well-defined. Using the Product and Chain Rules we can compute the derivative of the quotient:

Quotient Rule

Assume that f and g are differentiable functions and $f(x)/g(x)$ is well-defined. Using the Product and Chain Rules we can compute the derivative of the quotient:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)g(x)^{-1}\right)$$

Quotient Rule

Assume that f and g are differentiable functions and $f(x)/g(x)$ is well-defined. Using the Product and Chain Rules we can compute the derivative of the quotient:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)g(x)^{-1}\right)\\ &= f'(x)g(x)^{-1} + f(x)(-1)g(x)^{-2}g'(x)\end{aligned}$$

Quotient Rule

Assume that f and g are differentiable functions and $f(x)/g(x)$ is well-defined. Using the Product and Chain Rules we can compute the derivative of the quotient:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)g(x)^{-1}\right) \\
&= f'(x)g(x)^{-1} + f(x)(-1)g(x)^{-2}g'(x) \\
&= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g(x)^2}
\end{aligned}$$

Quotient Rule

Assume that f and g are differentiable functions and $f(x)/g(x)$ is well-defined. Using the Product and Chain Rules we can compute the derivative of the quotient:

$$\begin{aligned}
\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) &= \frac{d}{dx}\left(f(x)g(x)^{-1}\right) \\
&= f'(x)g(x)^{-1} + f(x)(-1)g(x)^{-2}g'(x) \\
&= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g(x)^2} \\
&= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.
\end{aligned}$$

Example

Differentiate

(A) x^2e^{2x} (B) $t^3\ln(t+1)$ (C) $(3x^2+5x)e^x$

Example

Differentiate

(a) $x^2 e^{2x}$ (b) $t^3 \ln(t+1)$ (c) $(3x^2+5x)e^x$

$$\frac{d}{dx}(x^2 e^{2x})$$

Example

Differentiate

(a) $x^2 e^{2x}$ (b) $t^3 \ln(t+1)$ (c) $(3x^2+5x)e^x$

$$\frac{d}{dx}(x^2 e^{2x}) = 2xe^{2x} + x^2(2e^{2x})$$

Example

Differentiate

(a) x^2e^{2x} (b) $t^3\ln(t+1)$ (c) $(3x^2+5x)e^x$

$$\begin{aligned}\frac{d}{dx}(x^2e^{2x}) &= 2xe^{2x}+x^2(2e^{2x})\\ &= 2xe^{2x}(1+x)\end{aligned}$$

Example

Differentiate

(A) $x^2 e^{2x}$ (B) $t^3 \ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(t^3 \ln(t+1)\right)$$

EXAMPLE

Differentiate

(A) $x^2 e^{2x}$ (B) $t^3 \ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\frac{d}{dt}(t^3 \ln(t+1)) = 3t^2 \ln(t+1) + t^3 \left(\frac{1}{t+1}\right)$$

Example

Differentiate

(A) x^2e^{2x} (B) $t^3\ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\begin{aligned}\frac{d}{dt}(t^3\ln(t+1)) &= 3t^2\ln(t+1)+t^3\left(\frac{1}{t+1}\right)\\ &= 3t^2\ln(t+1)+\frac{t^3}{t+1}.\end{aligned}$$

Example

Differentiate

(A) $x^2 e^{2x}$ (B) $t^3 \ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\frac{d}{dx}(3x^2+5x)e^x$$

Example

Differentiate

(A) $x^2 e^{2x}$ (B) $t^3 \ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\frac{d}{dx}(3x^2+5x)e^x = (6x+5)e^x + (3x^2+5x)e^x$$

Example

Differentiate

(A) $x^2 e^{2x}$ (B) $t^3 \ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\begin{aligned} \frac{d}{dx}(3x^2+5x)e^x &= (6x+5)e^x + (3x^2+5x)e^x \\ &= e^x(6x+5+3x^2+5x) \end{aligned}$$

Example

Differentiate

(A) x^2e^{2x} (B) $t^3\ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\begin{aligned}\frac{d}{dx}(3x^2+5x)e^x &= (6x+5)e^x+(3x^2+5x)e^x\\ &= e^x(6x+5+3x^2+5x)\\ &= e^x(3x^2+11x+5).\end{aligned}$$

Example

Differentiate $\frac{e^{2t}}{t}$.

Example

Differentiate $\frac{e^{2t}}{t}$.

$$\frac{d}{dt}\frac{e^{2t}}{t}$$

EXAMPLE

Differentiate $\frac{e^{2t}}{t}$.

$$\frac{d}{dt}\frac{e^{2t}}{t} = \frac{2e^{2t}t - e^{2t}(1)}{t^2}$$

Example

Differentiate $\frac{e^{2t}}{t}$.

$$\frac{d}{dt}\frac{e^{2t}}{t} = \frac{2e^{2t}t - e^{2t}(1)}{t^2}$$
$$= \frac{(2t-1)e^{2t}}{t^2}$$

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

EXAMPLE

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(A) Find the revenue as a function of the quantity sold.

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(A) Find the revenue as a function of the quantity sold.

(B) How does revenue vary with respect to quantity?

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(A) Find the revenue as a function of the quantity sold.

(B) How does revenue vary with respect to quantity?

$$R(q) = p(q) \cdot q$$

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(A) Find the revenue as a function of the quantity sold.

(B) How does revenue vary with respect to quantity?

$$R(q) = p(q) \cdot q = 80e^{-0.003q}q$$

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(A) Find the revenue as a function of the quantity sold.

(B) How does revenue vary with respect to quantity?

$$R(q) = p(q) \cdot q = 80e^{-0.003q}q = 80qe^{-0.003q}.$$

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(A) Find the revenue as a function of the quantity sold.
(B) How does revenue vary with respect to quantity?

$$R(q) = p(q) \cdot q = 80e^{-0.003q}q = 80qe^{-0.003q}.$$

$$\frac{d}{dq}R(q) = \frac{d}{dq}(80qe^{-0.003q})$$

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(a) Find the revenue as a function of the quantity sold.
(b) How does revenue vary with respect to quantity?

$$R(q) = p(q) \cdot q = 80e^{-0.003q}q = 80qe^{-0.003q}.$$

$$\begin{aligned} \frac{d}{dq}R(q) &= \frac{d}{dq}(80qe^{-0.003q}) \\ &= 80\frac{d}{dq}qe^{-0.003q} \end{aligned}$$

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(A) Find the revenue as a function of the quantity sold.

(B) How does revenue vary with respect to quantity?

$$R(q) = p(q) \cdot q = 80e^{-0.003q}q = 80qe^{-0.003q}.$$

$$\begin{aligned}
\frac{d}{dq}R(q) &= \frac{d}{dq}(80qe^{-0.003q}) \\
&= 80\frac{d}{dq}qe^{-0.003q} \\
&= 80(e^{-0.003q} + q(-0.003)e^{-0.003q})
\end{aligned}$$

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

(A) Find the revenue as a function of the quantity sold.

(B) How does revenue vary with respect to quantity?

$$R(q) = p(q) \cdot q = 80e^{-0.003q}q = 80qe^{-0.003q}.$$

$$\begin{aligned}
\frac{d}{dq}R(q) &= \frac{d}{dq}(80qe^{-0.003q}) \\
&= 80\frac{d}{dq}qe^{-0.003q} \\
&= 80(e^{-0.003q} + q(-0.003)e^{-0.003q}) \\
&= 80e^{-0.003q}(1 - 0.003q).
\end{aligned}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right)$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right) = \frac{10(x^3+1) - 5x^2(3x^2)}{(x^3+1)^2}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right) = \frac{10(x^3+1) - 5x^2(3x^2)}{(x^3+1)^2}$$
$$= \frac{10x^4 + 10x - 15x^2}{(x^3+1)^2}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\begin{aligned} \frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right) &= \frac{10(x^3+1)-5x^2(3x^2)}{(x^3+1)^2} \\ &= \frac{10x^4+10x-15x^2}{(x^3+1)^2} \\ &= \frac{-5x^4+10}{(x^3+1)^2}. \end{aligned}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\frac{1}{1+e^x}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\frac{1}{1+e^x} = \frac{0(1+e^x)-(1)e^x}{(1+e^x)^2}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\begin{aligned} \frac{d}{dx}\frac{1}{1+e^x} &= \frac{0(1+e^x)-(1)e^x}{(1+e^x)^2} \\ &= \frac{-e^x}{(1+e^x)^2}. \end{aligned}$$

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\frac{e^x}{x^2}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\frac{e^x}{x^2} = \frac{e^x x^2 - e^x(2x)}{x^4}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\frac{e^x}{x^2} = \frac{e^x x^2 - e^x(2x)}{x^4}$$
$$= \frac{e^x(x^2-2x)}{x^4}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\begin{aligned} \frac{d}{dx}\frac{e^x}{x^2} &= \frac{e^x x^2 - e^x(2x)}{x^4} \\ &= \frac{e^x(x^2-2x)}{x^4} \\ &= \frac{e^x(x)(x-2)}{x^4} \end{aligned}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\begin{aligned}
\frac{d}{dx}\frac{e^x}{x^2} &= \frac{e^x x^2 - e^x(2x)}{x^4} \\
&= \frac{e^x(x^2-2x)}{x^4} \\
&= \frac{e^x(x)(x-2)}{x^4} \\
&= \frac{e^x(x-2)}{x^3}.
\end{aligned}$$

Example

Assume

Example

Assume

- $f(2) = 1$,

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

EXAMPLE

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$, (B) $k'(2)$.

$$h'(2) =$$

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$h'(2) = f'(2)g(2) + f(2)g'(2)$$

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$\begin{aligned} h'(2) &= f'(2)g(2) + f(2)g'(2) \\ &= 5(3) + 1(6) \end{aligned}$$

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$\begin{aligned} h'(2) &= f'(2)g(2) + f(2)g'(2) \\ &= 5(3) + 1(6) \\ &= 21. \end{aligned}$$

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$k'(2) =$$

Example

Assume

- $f(2) = 1$,
- $g(2) = 3$,
- $f'(2) = 5$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$, (B) $k'(2)$.

$$k'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2}$$

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$\begin{aligned} k'(2) &= \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} \\ &= \frac{5(3) - 1(6)}{3^2} \end{aligned}$$

EXAMPLE

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$
\begin{aligned}
k'(2) &= \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} \\
&= \frac{5(3) - 1(6)}{3^2} \\
&= \frac{15 - 6}{9}
\end{aligned}
$$

Example

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$k'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} = \frac{5(3) - 1(6)}{3^2} = \frac{15 - 6}{9} = \frac{9}{9}$$

EXAMPLE

Assume

- $f(2) = 1$,
- $f'(2) = 5$,
- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$, (B) $k'(2)$.

$$
\begin{aligned}
k'(2) &= \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} \\
&= \frac{5(3) - 1(6)}{3^2} \\
&= \frac{15 - 6}{9} = \frac{9}{9} = 1.
\end{aligned}
$$