1.

$$\left\{ \frac{\left[\frac{12x^2-3x-42}{10x^2-43x-24}\right]}{\left[\frac{36x^2+11x-12}{18x^2+x-4}\right]} \right\} \div \left\{ \frac{\left[\frac{-3x^2-6x+24}{4x^2+7x+12}\right]}{\left[\frac{20x^2-131x+168}{49-16x^2}\right]} \right\}$$

Solution. First start by factoring the polynomials; this will help reduce the size of the problem. The factorization of the polynomial is given by the roots of the polynomial, so we can use the quadratic equation

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

to make the factoring easier.

(1) Factorization of $12x^2 - 3x - 42$:

First, we factor out the largest common divisor of the coefficients, which in this case is 3:

$$12x^2 - 3x - 42 = 3(4x^2 - x - 14).$$

The solutions to $4x^2 - x - 14 = 0$ are given by the quadratic formula,

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-14)}}{2(4)} = \frac{1 \pm \sqrt{1+224}}{8} = \frac{1 \pm \sqrt{225}}{8} = \frac{1 \pm 15}{8}$$

and are $x = (1+15)/8 = 16/8 = 2$ or $x = (1-15)/8 = -14/8 = -7/4$. The factorization of $4x^2 - x - 14$ is then

$$4 \cdot (x-2) \cdot (x - (-7/4)) = (x-2) \cdot 4 \cdot (x + 7/4) = (x-2)(4x+7)$$

and hence

$$12x^2 - 3x - 42 = 3(x-2)(4x+7).$$

(2) Factorizatin of $10x^2 - 43x - 24$:

There are no common divisors of the coefficients, so we can skip straight to the roots. The solutions to $10x^2 - 43x - 24 = 0$ are

$$x = \frac{-(-43) \pm \sqrt{(-43)^2 - 4(10)(-24)}}{2(10)} = \frac{43 \pm \sqrt{1849 + 960}}{20} = \frac{43 \pm \sqrt{2809}}{20} = \frac{43 \pm 53}{20}$$

so either $x = 96/20 = 24/5$ or $x = -10/20 = -1/2$. Thus the factorization is

$$\begin{aligned} 10x^2 - 43x - 24 &= 10 \cdot (x - 24/5) \cdot (x - (-1/2) \\ &= 5 \cdot (x - 24/5) \cdot 2 \cdot (x + 1/2) \\ &= (5x-24)(2x+1). \end{aligned}$$

(3) Factorization of $36x^2 + 11x - 12$:

Again, the coefficients have no common divisors (11 is prime and doesn't divide either 12 or 36). Applying the quadratic equation in the same manner gives

$$x = \frac{-11 \pm \sqrt{11^2 - 4(36)(-12)}}{2(36)} = \frac{-11 \pm \sqrt{121 + 1728}}{72} = \frac{-11 \pm 43}{72}$$

and thus $x = 32/72 = 4/9$ or $x = -54/72 = -3/4$. Therefore the factorization is

$$36x^2 + 11x - 12 = 36 \cdot (x - 4/9)(x + 3/4) = 9(x - 4/9) \cdot 4(x + 3/4) = (9x - 4)(4x + 3).$$

(4) Factoriztion of $18x^2 + x - 4$:

Applying the quadratic equation we get

$$x = \frac{-1 \pm \sqrt{1^2 - 4(18)(-4)}}{2(18)} = \frac{-1 \pm 17}{36}$$

so $x = 16/36 = 4/9$ or $x = -18/36 = -1/2$

$$18x^2 + x - 4 = 18(x - 4/9)(x + 1/2) = (9x - 4)(2x + 1).$$

(5) Factorization of $-3x^2 - 6x + 24$:

We have $-3x^2 - 6x + 24 = -3(x^2 + 2x - 8) = -3(x + 4)(x - 2)$.

(6) Factorization of $4x^2 + 7x + 12$:

Checking the discriminant of the polynomial we have

$$7^2 - 4(4)(12) = 49 - 16(12) = 49 - 192 < 0$$

so it has no roots and thus is irreducible.

(7) Factorization of $20x^2 - 131x + 168$:

The coefficients have no common divisors, so we use the quadratic formula

$$x = \frac{131 \pm \sqrt{131^2 - 4(20)(168)}}{40} = \frac{131 \pm \sqrt{3721}}{40} = \frac{131 \pm 61}{40}$$

so $x = 192/40 = 24/5$ or $x = 70/40 = 7/4$. Therefore

$$20x^2 - 131x + 168 = 20(x - 24/5)(x - 7/4) = (5x - 24)(4x - 7).$$

(8) Factorization of $49 - 16x^2$:

We note that $49 = 7^2$ and $16 = 4^2$ so we can factor this polynomial as

$$49 - 16x^2 = 7^2 - 4^2x^2 = 7^2 - (4x)^2 = (7 - 4x)(7 + 4x).$$

Next we rewrite the original expression as a single fraction. The first step is to rewrite the division inside the curly braces as multiplication by the reciprocal:

$$\left\{\frac{\left[\frac{12x^2-3x-42}{10x^2-43x-24}\right]}{\left[\frac{36x^2+11x-12}{18x^2+x-4}\right]}\right\} \div \left\{\frac{\left[\frac{-3x^2-6x+24}{4x^2+7x+12}\right]}{\left[\frac{20x^2-131x+168}{49-16x^2}\right]}\right\} = \frac{\left\{\frac{12x^2-3x-42}{10x^2-43x} \cdot \frac{18x^2+x-4}{36x^2+11x-12}\right\}}{\left\{\frac{-3x^2-6x+24}{4x^2+7x+12} \cdot \frac{49-16x^2}{20x^2-131x+168}\right\}}.$$

The next step is to change the division into multiplication by the reciprocal:

$$\frac{(12x^2-3x-42)(18x^2+x-4)(49-16x^2)(-3x^2-6x+24)}{(10x^2-43x)(36x^2+11x-12)(20x^2-131x+168)(4x^2+7x+12)}.$$

Now we have something we can reduce. Replacing each of the polynomials by its factorization we get

$$\frac{3(4x+7)(x-2)(9x-4)(2x+1)(4x^2+7x+12)(5x-24)(4x-7))}{(5x-24)(2x+1)(9x-4)(4x+3)(-3(x+4)(x-2))(7-4x)(7+4x)}$$

and then cancelling the common factors in the numerator and the denominator gives

$$\begin{aligned}
\frac{3(4x^2+7x+12)(4x-7)}{(-3)(7-4x)(4x+3)(x+4)} &= \frac{(4x^2+7x+12)(4x-7)}{-(7-4x)(4x+3)(x+4)} \\
&= \frac{(4x^2+7x+12)(4x-7)}{(4x-7)(4x+3)(x+4)} \\
&= \frac{4x^2+7x+12}{(4x+3)(x+4)}.
\end{aligned}$$