Math 122

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Calculus for Business Administration and Social Sciences

Outline

1. 1.7: Exponential Growth and Decay
   - Doubling Time and Half-Life
   - Financial Applications
   - Continuously Compounding Interest

Definition

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- The *half-life* of an exponentially decaying quantity is the time required for the quantity to be reduced by a factor of one half.

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Computing Doubling Time/Half-Life

To approximate the value of the doubling time with a calculator:

$$d = log_a(2) = \frac{\ln(2)}{\ln(a)}$$

and

$$h = -\log_a(2) = -\frac{\ln(2)}{\ln(a)}.$$

EXAMPLE

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(A) What was the radiation level 24 hours later?

(B) How long will it take for the radiation levels to decay to the maximum acceptable radiation level of 0.6 millirems/hour set by the EPA?

Example (Cont.)

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(B) Solve the equation below for t:

$$\begin{aligned} 0.6 &= 2.4e^{-0.004t} \\ \Rightarrow e^{-0.004t} &= \frac{2.4}{0.6} = \frac{1}{4} \end{aligned}$$

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Therefore, it will take approximately $346.57/24 = 14.4$ days.

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Therefore

$$P(t) \approx 19.5e^{0.28t}.$$

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Compounding Interest (Cont.)

Consider the table:

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So at the end of the year, the balance will be $P_0\left(1+\frac{r}{n}\right)^n$.

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So at the end of the year, the balance will be $P_0\left(1+\frac{r}{n}\right)^n$. Continuing this way, the account balance after t years will be

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Remark 1 ("Rule of 70")

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$$d = \frac{\ln(2)}{\ln\left(1 + \frac{r}{100}\right)} \approx \frac{.7}{r/100} = \frac{70}{r}.$$

The method above is discrete.

Continuously Compounding Interest

The method above is discrete. If instead, we wish to compound interest at every instant, we get *continuously compounding interest*,

$$P(t) = P_0 e^{rt}.$$

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&\approx 8 \text{ years.}
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