MATH 115
EXAM 04

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. Unless otherwise stated, all supporting work is required. You may **not** use any calculators.

Name: Answer Key

Problem	Points Earned	Points Possible
1		20
2		20
3		20
4		20
5		20
Total		100

Date: November 24, 2014.

1. Problems

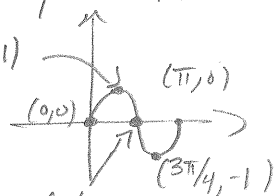
1. *Find the period, frequency, phase shift, and amplitude of*

$$y = 4\sin(2x - \pi) - 1,$$

then graph one period.

$4\sin(2x-\pi)-1 = 4\sin(2(x-\pi/2))+(-1)$

① $y=\sin(2x)$

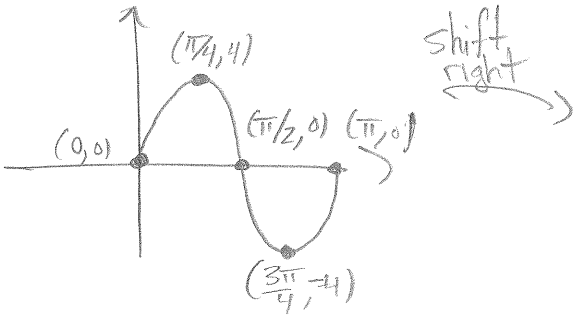


$A=4$
$B=2$
$C=\pi/2$
$D=-1$

Period: $2\pi/2=\pi$
Amplitude: 4
Frequency: $1/\pi$
Phase: $\pi/2$

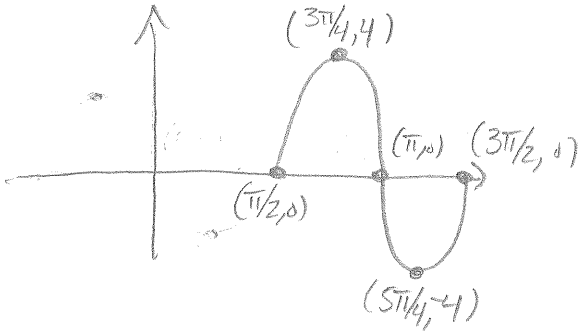
stretch vert.

② $y=4\sin(2x)$



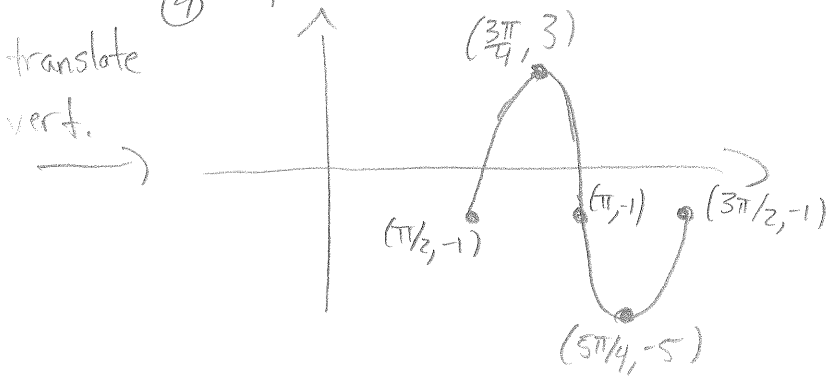
shift right

③ $y=4\sin(2(x-\pi/2))$



④ $y=4\sin(2x-\pi)-1$

translate vert.



2. *Find the period, frequency, phase shift, and amplitude of*

$$y = 3\cos\left(\frac{x}{2}\right) + 2,$$

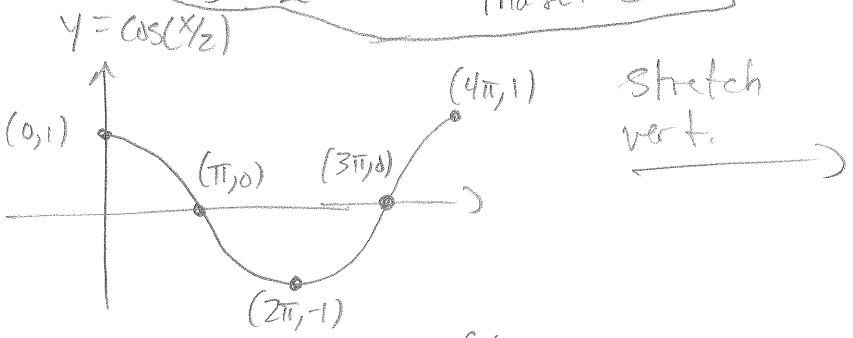
then graph one period.

$A = 3$ Amplitude: 3

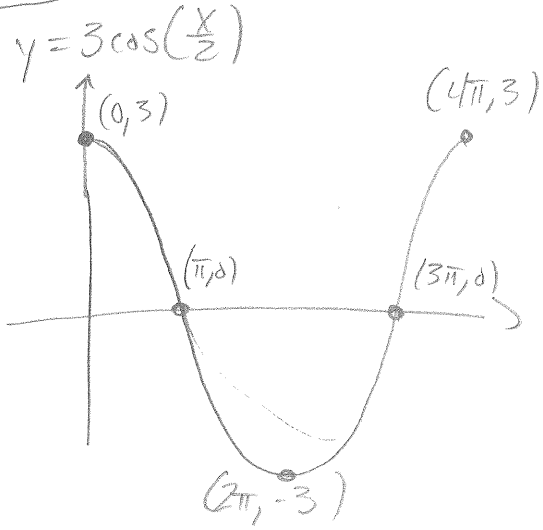
$B = \frac{1}{2}$ Period: $2\pi/(1/2) = 4\pi$

$C = 0$ Freq: $1/4\pi$

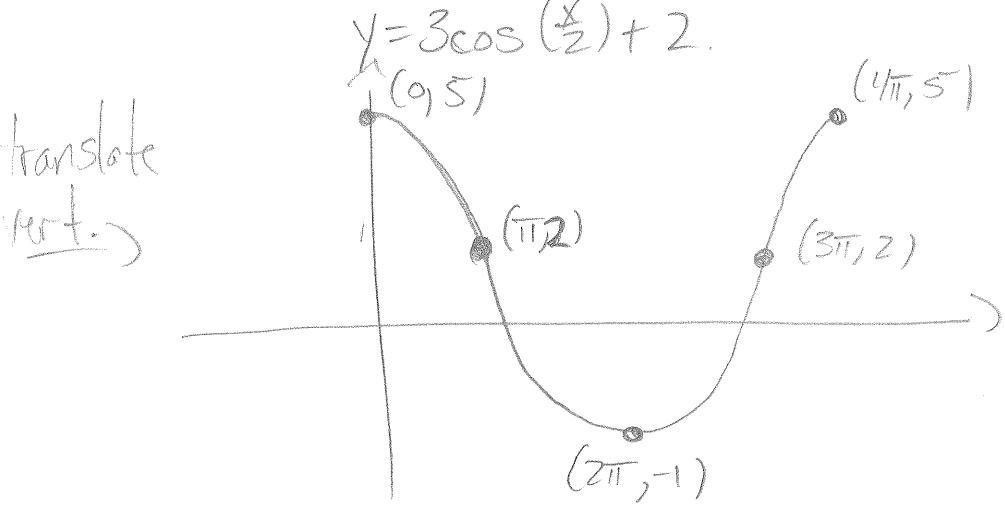
$D = 2$ Phase: 0.



stretch vert.



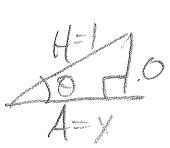
translate vert.



3. *Show that*

$$\tan(\arccos(x)) = \frac{\sqrt{1-x^2}}{x}.$$

Let $\theta = \arccos(x)$.



$\Rightarrow \cos(\theta) = \frac{A}{H} = \frac{x}{1}$

$\Rightarrow \theta^2 + x^2 = 1$

$\Rightarrow \theta^2 = 1 - x^2$

$\Rightarrow \theta = \sqrt{1-x^2}$

$\tan(\theta) = \frac{O}{A} = \frac{\sqrt{1-x^2}}{x}$ ∎

4. *Determine whether the following functions are even, odd, or neither.* **Justify your answers.**

(a) $f(x) = \sin(x^2)$

$f(-x) = \sin((-x)^2) = \sin(x^2) = f(x)$

$\Rightarrow f$ is even.

(b) $g(x) = 1 + \sec(x)$

$g(-x) = 1 + \sec(-x) = 1 + \frac{1}{\cos(-x)} = 1 + \frac{1}{\cos(x)} = 1 + \sec(x) = g(x)$

$\Rightarrow g$ is even.

(c) $h(x) = x\cos(x)$

$h(-x) = (-x)\cos(-x) = -x\cos(x) = -h(x)$

$\Rightarrow h$ is odd.

(d) $k(x) = \sin(x) - 1$

$k(-x) = \sin(-x) - 1 = -\sin(x) - 1 \neq -k(x) = -\sin(x) + 1$

So k is neither.

5. *Given* $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$, $\cos(\pi/6) = \sqrt{3}/2$, *and* $\sin(\pi/6) = 1/2$, *compute the values of each of the following expressions. You will find a list of potentially useful trigonometric identities on the following page. [Hint:* $1/4$ *and* $1/6$ *have common denominator* 12*]*

(a) $\sin\left(\dfrac{5\pi}{12}\right)$.

$$\frac{5\pi}{12} = \frac{3\pi}{12} + \frac{2\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$$

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin(\pi/4)\cos(\pi/6) + \sin(\pi/6)\cos(\pi/4)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}\sqrt{3} + \sqrt{2}}{4}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

(b) $\cos\left(\dfrac{5\pi}{12}\right)$.

$$\cos(5\pi/12) = \cos(\pi/4 + \pi/6)$$

$$= \cos(\pi/4)\cos(\pi/6) - \sin(\pi/4)\sin(\pi/6)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{2}\sqrt{3} - \sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}.$$