MATH 115
EXAM 02

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. Unless otherwise stated, all supporting work is required. It is advised, although not required, that you check your answers. You may **not** use any calculators.

Name: ______________________________

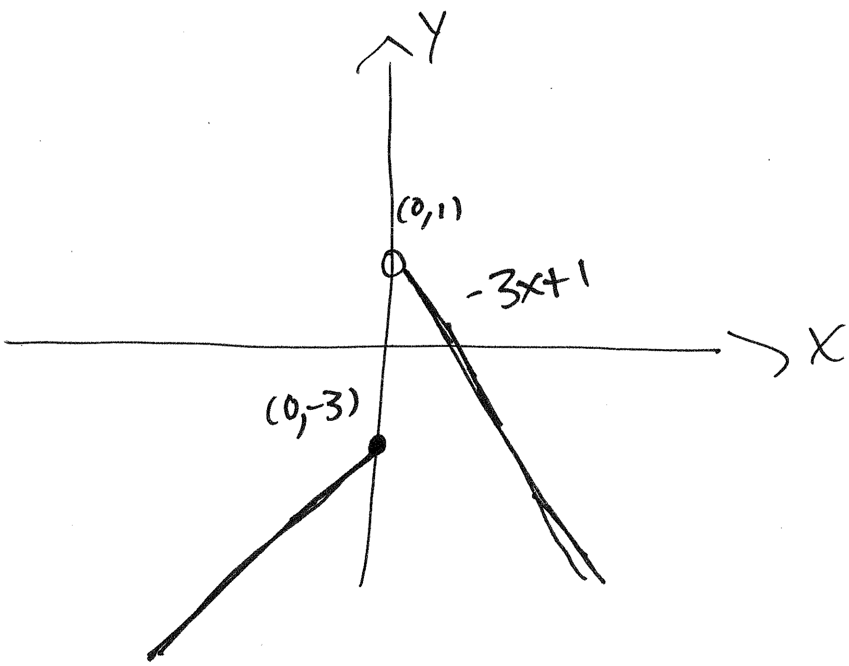
Problem	Points Earned	Points Possible
1		20
2		20
3		20
4		20
5		20
Bonus		10
Total		100

Date: October 15, 2014.

1. Problems

1. *Graph the function*

$$f(x) = \begin{cases} -3x+1 & \text{if } x > 0, \\ 2x-3 & \text{if } x \leq 0. \end{cases}$$



$$\sqrt{12} = \sqrt{4\cdot 3} = \sqrt{4}\cdot\sqrt{3} = 2\cdot\sqrt{3}$$

2. *Let* $f(x) = \underset{a}{3}x^2 + \underset{b}{6}x + \underset{c}{2}$.

(a) Find the solutions to $f(x) = 0$.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-6 \pm \sqrt{6^2-4(3)(2)}}{2(3)}$$
$$= \frac{-6 \pm \sqrt{36-24}}{6}$$
$$= \frac{-6 \pm \sqrt{12}}{6} = \frac{-6 \pm 2\sqrt{3}}{6} = \frac{-3 \pm \sqrt{3}}{3}.$$

(b) Write $f(x)$ *in vertex form.*

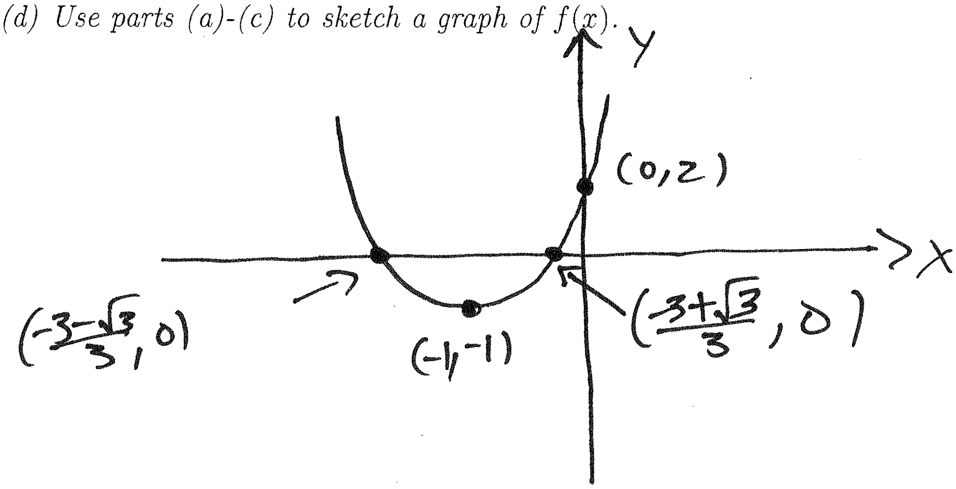
$$3x^2+6x+2 = 3(x^2+2x)+2$$
$$= 3(\underbrace{x^2+2x+1}-1)+2$$
$$= 3((x+1)^2-1)+2$$
$$= 3(x+1)^2-3+2 = 3(x+1)^2-1.$$

(c) Find the coordinates of the y-intercept for $f(x)$.

$$f(0) = 3\cdot(0)^2+6\cdot(0)+2$$
$$= 0+0+2$$
$$= 2.$$

$(0,2)$.

(d) Use parts (a)-(c) to sketch a graph of $f(x)$.



3. *Find and simplify the difference quotient for* $f(x) = x^2 - x$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-(x+h)-(x^2-x)}{h}$$

$$= \frac{\cancel{x^2}+2xh+h^2\cancel{-x}-h\cancel{-x^2}\cancel{+x}}{h}$$

$$= \frac{2xh+h^2-h}{h}$$

$$= \frac{h(2x+h-1)}{h}$$

$$= 2x+h-1.$$

4. *Let* $f(x) = 3x - 1$ *and* $g(x) = x^2 + 1$.

(a) Compute the composition $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x))$$
$$= f(x^2+1)$$
$$= 3(x^2+1) - 1$$
$$= 3x^2 + 3 - 1$$
$$= 3x^2 + 2.$$

(b) Compute the composition $(g \circ f)(x)$

$$(g \circ f)(x) = g(f(x))$$
$$= g(3x-1)$$
$$= (3x-1)^2 + 1$$
$$= (3x)^2 + 2(-1)(3x) + (-1)^2 + 1$$
$$= 9x^2 - 6x + 1 + 1$$
$$= 9x^2 - 6x + 2.$$

5. *Find the inverse of the function*

$$f(x) = 2x + 1.$$

$y = 2x + 1$ switch x & y

$x = 2y + 1$ solve for y

$x - 1 = 2y$

$y = \frac{x-1}{2}$ replace y by $f^{-1}(x)$

$f^{-1}(x) = \frac{x-1}{2}.$

6 (Bonus). *Let $f(x) = \sqrt{1-x^2}$. Determine the domain of this function. Is this function invertible? If so, compute its inverse.*

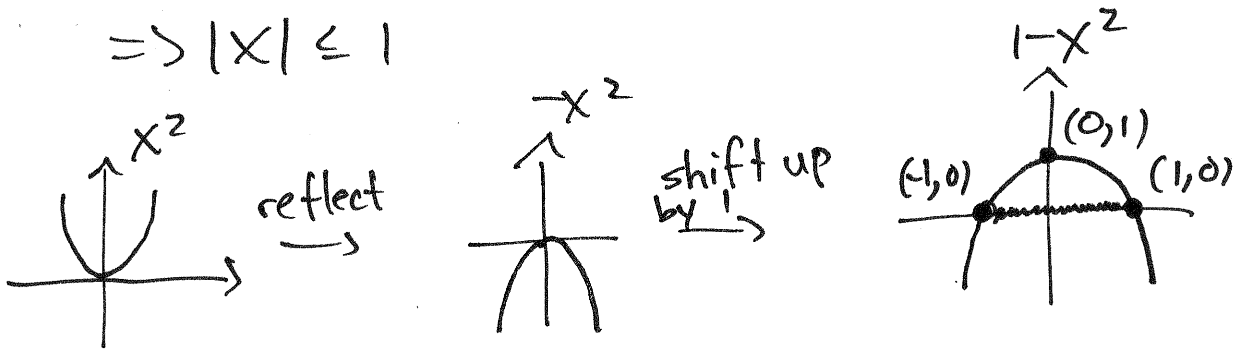
Domain of $\sqrt{x}$ is $\{x \in \mathbb{R} \mid x \geq 0\}$

Domain of $\sqrt{1-x^2}$ is $\{x \mid 1-x^2 \geq 0\}$

$0 \leq 1-x^2 = (1-x)(1+x)$

$\Rightarrow x^2 \leq 1$

$\Rightarrow |x| \leq 1$

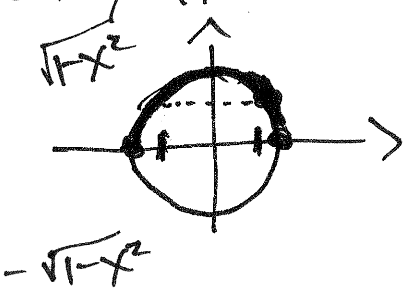


Domain of $\sqrt{1-x^2}$ $[-1, 1]$.

$y = \sqrt{1-x^2}$ on $[-1, 1]$

$\Rightarrow y^2 = 1-x^2$

$\Rightarrow y^2 + x^2 = 1$ circle of radius one about $(0,0)$



This function is not invertible because it fails the horizontal line test.

On the interval $[0,1]$, $f(x)=\sqrt{1-x^2}$

$$f\circ f(x) = f(f(x))$$
$$= f\left(\sqrt{1-x^2}\right)$$
$$= \sqrt{1-\left(\sqrt{1-x^2}\right)^2}$$
$$= \sqrt{1-(1-x^2)}$$
$$= \sqrt{1-1+x^2}$$
$$= \sqrt{x^2} = |x|$$
$$= x$$ because x was assumed to be ~~posi~~ non-negative

On $[0,1]$, f is its own inverse.