

COMPILER CONSTRUCTION

Grammars and Parsing

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Spring 2017

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Chapter 4

Grammars and Parsing



Why Grammar?

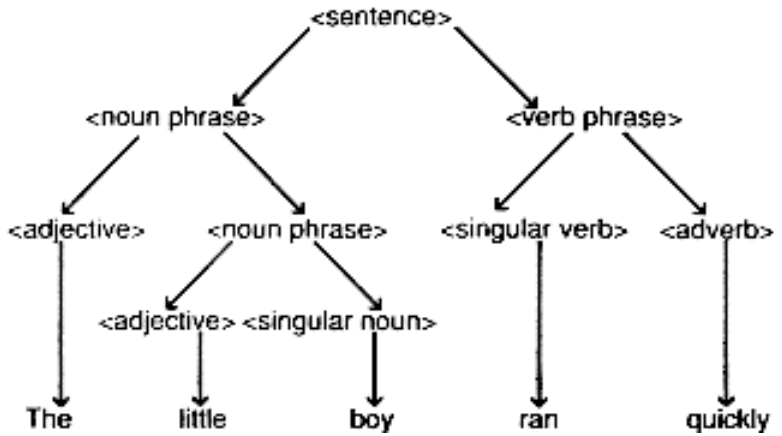
- Grammar rules of **natural languages**, such as English or Chinese
  - **Define proper sentence structure**, e.g., defining phrases in terms of subjects, verbs, and objects; and phrases and conjunctions
  - Served as a tool for **diagnosing malformed sentences** (**validity check**)
- It is possible to construct sentences in a natural language
  - that are grammatically correct
  - but still make no sense



Why Grammar? (Cont'd)

- A structured sentence can be diagrammed
  - to show how its components conform to a language's grammar
  - Grammars can also explain what is absent or superfluous in a malformed sentence
- **Ambiguity** of a sentence
  - Can often be explained by providing **multiple diagrams for the same sentence**

``The little boy ran quickly" can be diagrammed as:





Grammars for Programming Languages

- Modern programming languages
  - contain a **grammar in their specification** as a guide to those who teach, study, or use the language

- A compiler front-end for the language
  - Scans for tokens in input stream based on the **regular sets**
  - Parses the structures formed by the tokens using the **grammar** that specifies a programming language's syntax

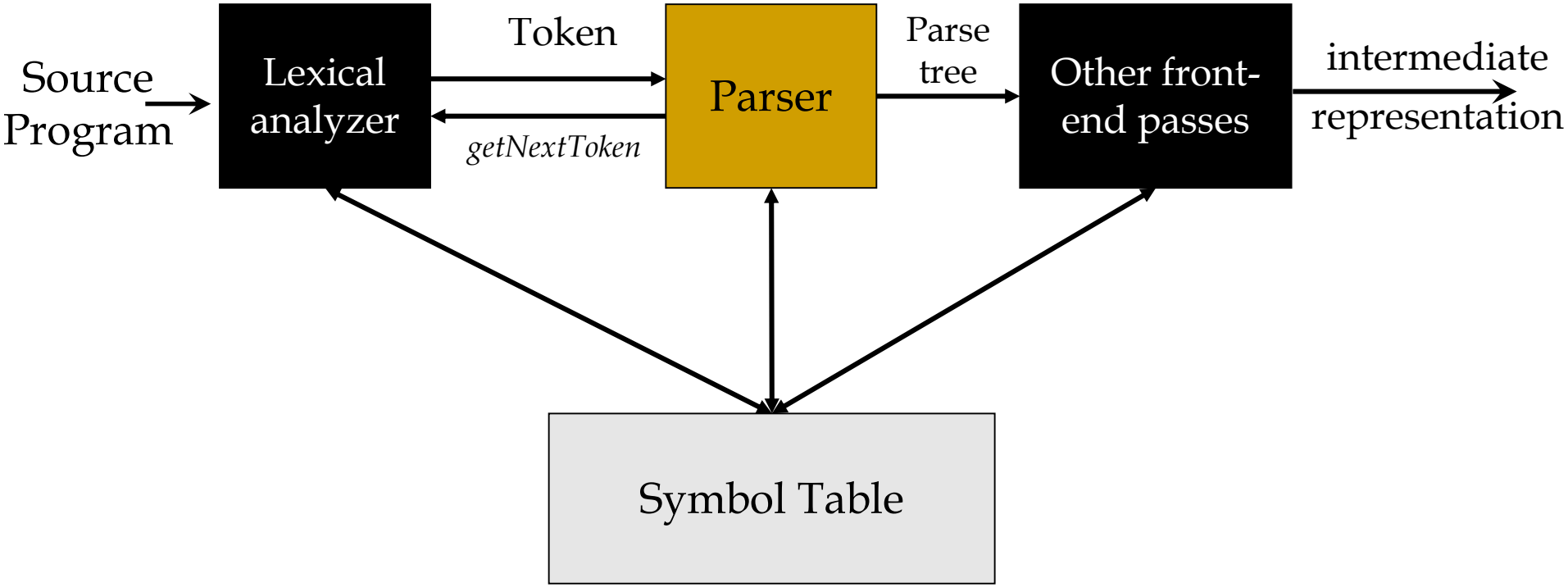


This Chapter is for CFGs

- We discuss the basics of **context-free grammars** (CFGs) in Ch. 2
- In Ch. 4 we
  - **formalize the definition and notation for CFGs** and
  - present algorithms that analyze such grammars in preparation for the parsing techniques covered in Ch. 5 and 6



The Role of the Parser





Context-Free Grammar

- Context-free grammar is a 4-tuple
  $G = < \Sigma, N, P, S >$ where
  - Σ is a finite set of **terminal alphabet**, which is the set of tokens produced by the scanner
  - N is a finite set of **nonterminal alphabet**
  - P is a finite set of **productions** of the form
    $A \rightarrow \beta$
    where $A \in N$ and $\beta \in (N \cup \Sigma)^*$
  - $S \in N$ is a designated **start symbol**, which initiates all derivations



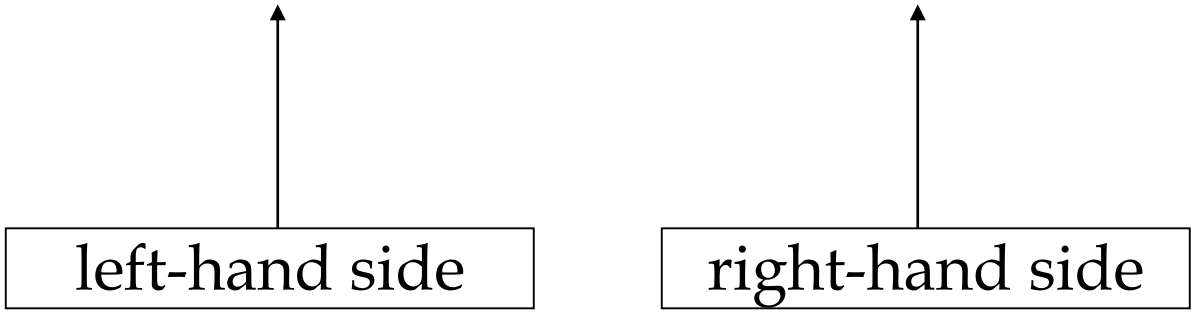
Conventions

- Terminals
  - $a, b, c, \ldots \in \Sigma$
  - More example: **0**, **1**, **+**, *****, **id**, **if**
- Nonterminals
  - $A, B, C, \ldots \in N$
  - More example: *expr*, *term*, *stmt*
- Grammar symbols
  - $X, Y, Z \in (N \cup \Sigma)$
- Strings of grammar symbols (sentential form)
  - $\alpha, \beta, \gamma \in (N \cup \Sigma)^*$
- The head of the first production is often the start symbol



Production Rules

- A **grammar** consists of a set of **production rules** and **a start symbol** (left symbol of first rule)
- A **production rule** consists of two parts: a left-hand side and a right-hand side
  - ex: expression → expression '+' term





Production Rules (Cont.)

- The **left-hand side** (LHS) is the **name** of the syntactic construct
- The **right-hand side** (RHS) shows a **possible form** of the syntactic construct

- E.g., there are **two possible forms** (rules) derived by **the name** "expression":

  expression → expression '+' term (rule 1)

  expression → expression '-' term (rule 2)



Production Rules (Cont.)

- LHS must be a single **non-terminal** symbol (or *non-terminal*)
- RHS of a production rule can contain zero or more **terminals and non-terminals**

- A **terminal symbol** (or *terminal*) is a grammar symbol
  - that cannot be rewritten
  - Is also an end point of the production process, also called **token**
  - Use lower-case letters such as a and b
- A **non-terminal symbol** (or *non-terminal*) is able to be rewritten
  - Use upper-case letters such as A, B, and S
- Non-terminal and terminal together are called **grammar symbols (or vocabulary)**



Different Forms of Production Rules

- There is often more than one way to rewrite a given nonterminal
- For example, when multiple productions share the same LHS symbol, it could be presented in one of the two forms

$$A \rightarrow \alpha$$
$$A \rightarrow \beta$$
$$\vdots$$
$$A \rightarrow \zeta \text{ (Zeta)}$$

$$A \rightarrow \alpha \mid \beta \mid \cdots \mid \zeta$$



Derivations of Production Rules

- Derivation
  - The rewriting step replaces a **nonterminal** by the RHS of one of **its** production rules

- Example:
  - If $A \rightarrow \zeta$ is a production, then $\alpha A \beta \Rightarrow \alpha \zeta \beta$ denotes one step of a derivation using this production
  - The one-step derivation can be denoted as $\alpha A \beta \Rightarrow \alpha \zeta \beta$
  - The sequence of replacement a derivation of ζ from A



Notation for Derivations

- $\Rightarrow$ derives in one step
- $\Rightarrow^+$ derives in one or more steps
- $\Rightarrow^*$ derives in zero or more steps
- $\Rightarrow_{lm}$ refers to **leftmost derivation**, which expands nonterminals left to right
- $\Rightarrow_{rm}$ refers to **right most derivation**, which expands nonterminals right to left

- Example:

  $\alpha A \beta \Rightarrow \alpha \zeta \beta$

  - is leftmost derivation, if α does not contain a nonterminal
  - is rightmost derivation, if β does not contain a nonterminal



More about the Grammar

- We say that α is a **sentential form** of G
  - if S ⇒* α, where S is the start symbol of a grammar G
  - Note that a sentential form may contain both terminals and nonterminals, and may be empty

- A **sentence** of *G* is a sentential form with no nonterminals
- The language generated by a grammar is its set of sentences
  - Hence, a string of terminals *w* is in L(G), the language generated by G, if and only if *w* is a sentence of G (or $S \Rightarrow^* w$)



Example of Rule Derivations

- Given the production rules:
  - expr → ‘(‘expr op expr ‘)’
  - expr → ‘1’
  - op → ‘+’
  - op → ‘*’



Example of Rule Derivations (Cont'd)

- Derivation of the string (1*(1+1))
  - expr
  - '(' expr op expr ')'
  - '(' '1' op expr ')'
  - '(' '1' '*' expr ')'
  - '(' '1' '*' '(' expr op expr ')' ')'
  - '(' '1' '*' '(' '1' op expr ')' ')'
  - '(' '1' '*' '(' '1' '+' expr ')' ')'
  - '(' '1' '*' '(' '1' '+' '1' ')' ')'

expr → '(' expr op expr ')'
expr → '1'
op → '+'
op → '*'

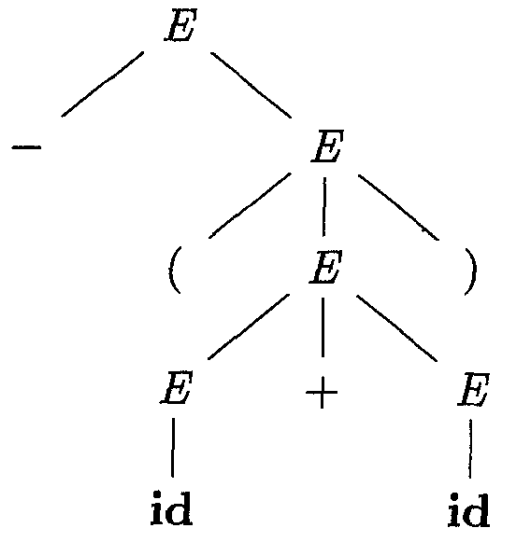
  - Each of the strings is a **sentential form**
  - The *strings* refer to expr, '(' expr op expr ')', '(' '1' op expr ')', '(' '1' '*' expr ')', etc.
- It forms a **leftmost derivation**, in which the leftmost nonterminal is always rewritten in each sentential form



A CFG Example

- G = < Σ, N, P, S >
  - Σ = { **+**, *, **(**, **)**, **-**, **id** }
  - N = { E }
  - P = $E \rightarrow E$ **+** E
    $E \rightarrow E$ * E
    $E \rightarrow$ **(** E **)**
    $E \rightarrow$ **-** E
    $E \rightarrow$ **id**
  - S = E

Parse tree for –(id + id)



- Is the string **–(id + id)** a sentence of G?

  Yes, there is a derivation of the given string

  → $E \Rightarrow_{lm} -E \Rightarrow_{lm} -(E) \Rightarrow_{lm} -(E+E) \Rightarrow_{lm} -(\mathbf{id}+E) \Rightarrow_{lm} -(\mathbf{id}+\mathbf{id})$

- The strings *E, - E, - (E) , . . . , - (id + id)* are all sentential forms of this grammar
  - We write $E \Rightarrow^{*}$ - (**id** + **id**) to indicate that - (**id** + **id**) can be derived from E



You would …

- Refer to Section 4.1 for more information
  - Examples of leftmost and right most derivations in Section 4.1.1 and 4.1.2
  - Parse Trees in Section 4.1.3



Properties of CFGs

- Some grammars have one or more of the following problems that preclude their use
  - (**Reduced Grammar**) The grammar may include useless symbols
  - (**Ambiguity**) The grammar may allow multiple, distinct derivations (parse trees) for some input string
  - (**Faulty Language Definition**) The grammar may include strings that do not belong in the language, or
    - the grammar may exclude strings that are in the language



We are …

- going to know more about the ambiguity next
- You would …
  - read Section 4.2.1 and 4.2.3 by yourself



Ambiguity

- A grammar that produces more than one parse tree for some sentence is said to be ***ambiguous***
- Hence, an ambiguous grammar is one that produces
  - more than one leftmost derivation or
  - more than one rightmost derivation for the same sentence



Example of Ambiguous Grammar

- Given the sentence: **id + id * id**
- Production rules:

$$E \rightarrow E + E \mid E * E \mid (E) \mid \mathbf{id}$$

- Two possible derviations:

$$\begin{aligned} E &\rightarrow E + E \\ &\rightarrow \mathbf{id} + E \\ &\rightarrow \mathbf{id} + E * E \\ &\rightarrow \mathbf{id} + \mathbf{id} * E \\ &\rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id} \end{aligned}$$

$$\begin{aligned} E &\rightarrow E * E \\ &\rightarrow E + E * E \\ &\rightarrow \mathbf{id} + E * E \\ &\rightarrow \mathbf{id} + \mathbf{id} * E \\ &\rightarrow \mathbf{id} + \mathbf{id} * \mathbf{id} \end{aligned}$$



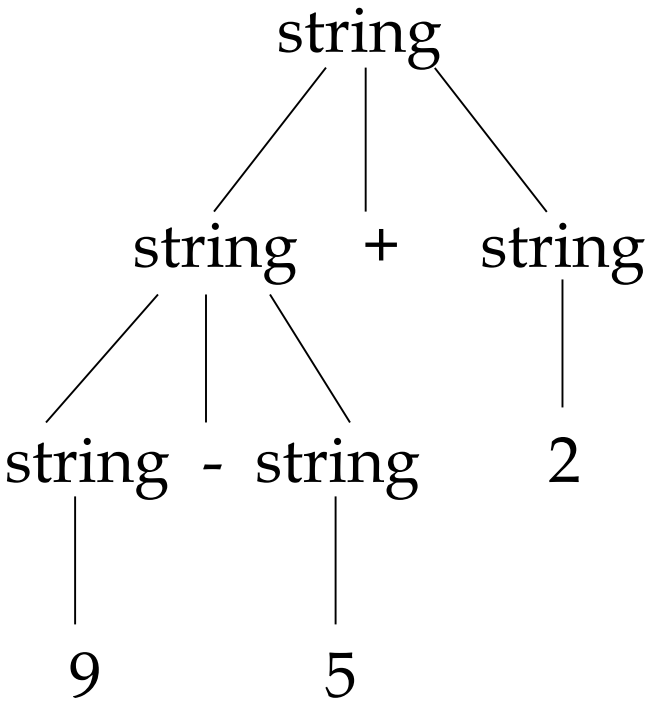
Another Example

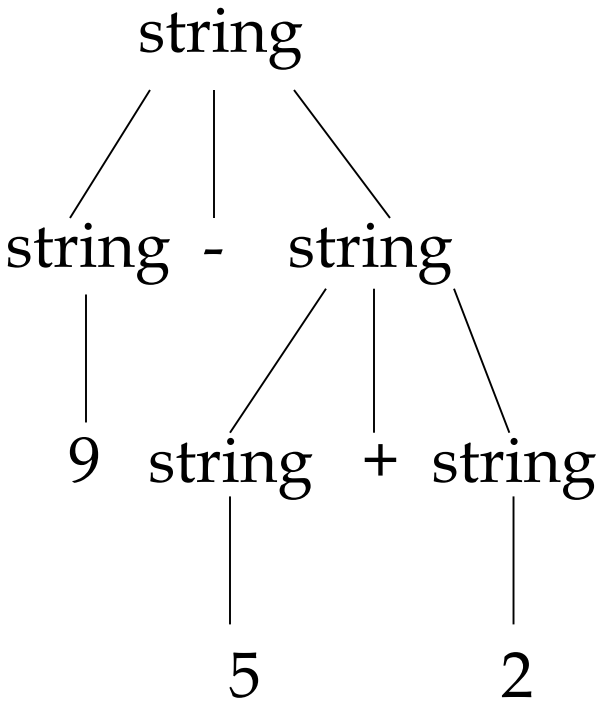
- Given the sentence: **9-5+2**
- Production rules:

string → string + string
| string – string
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9



Another Example (Con'td)







To Deal with Ambiguity

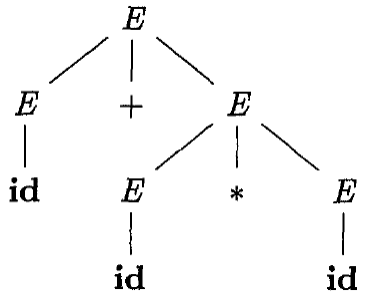
- Disambiguating rules are used to throw away undesirable parse trees

- Two options to deal with ambiguity:
  - Enforce precedence and associativity of the existing rules
  - Rewrite the grammar (rules)

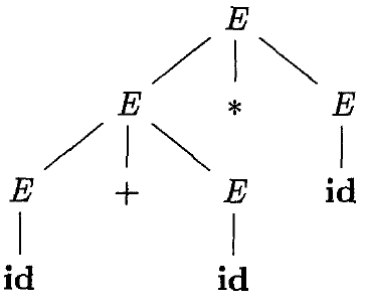


Precedence and Associativity to Resolve Conflicts

- The grammar G with the rules
  $E \rightarrow E + E \mid E * E \mid (E) \mid \mathbf{id}$
- G is **ambiguous** because
  - it does not specify the associativity or precedence of the operators + and *



- The following grammar G′ with the rules
  $E \rightarrow E + T \mid T$
  $T \rightarrow T * F \mid F$
  $F \rightarrow (E) \mid \mathbf{id}$



- G′ is **unambiguous** grammar that generates the same language
  - but gives + lower precedence than * (as the top figure), and makes both operators left associative
- There are parsers for both handling unambiguous and ambiguous grammars respectively



Example

- Given the sentence: **id+id*id**
- The following grammar G′ with the rules

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \mathbf{id}$$

- The corresponding derivations:

$$\begin{aligned} E &\rightarrow E{+}T \\ &\rightarrow T{+}T \\ &\rightarrow F{+}T \\ &\rightarrow \mathit{id}{+}T{*}F \\ &\rightarrow \mathbf{id}{+}F{*}F \\ &\rightarrow \mathbf{id}{+}\mathbf{id}{*}F \\ &\rightarrow \mathbf{id}{+}\mathbf{id}{*}\mathbf{id} \end{aligned}$$



Extended Grammars

- Backus-Naur Form (BNF)
  - is a **formal grammar** for expressing context-free grammars
- The single grammar rule format:
  - Non-terminal → zero or more grammar symbols

- Actually, BNF **extends** the grammar notation defined above, with syntax for defining **optional** and **repeated symbols**



Optional Symbols

- Optional symbols are enclosed in square brackets
- In the production rule

  $A \rightarrow \alpha [X1 \ldots Xn] \beta$

  - the symbols X1 . . .Xn are **entirely present or absent** between the symbols of α and β
  - Refer to Fig. 4.4 for the algorithm to transform a BNF grammar into standard form



Repeated Symbols

- Repeated symbols are enclosed in braces. In the production
- In the production rule

  $B \rightarrow \gamma \{ X1 \ldots Xm \} \delta$

  - the entire sequence of symbols X1 . . .Xm can be **repeated zero or more times**
  - Refer to Fig. 4.4 for the algorithm to transform a BNF grammar into standard form, which is accepted by parsers



Example of BNF

- The extensions are useful in representing many programming language constructs
- For example, in Java
  - Declarations can optionally include modifiers, such as **final**, **static**, and **const**, and
  - each declaration can include a ***list*** of identifiers
  - A production specifying a Java-like declaration could be as follows:
    Declaration→[**final**] [**static**] [**const**] Type identifier { , **identifier** }
  - Possible declarations:
    int a
    int a,b,c
    static int a
- This declaration insists that the modifiers be ordered as shown



Parsers and Recognizers

- Compilers are expected to
  - verify the **syntactic validity** of their **inputs** with respect to a **grammar** that
  - defines the programming language's syntax

- **A recognizer** is an algorithm determines if $x \in L(G)$, given a grammar G and an input string x
- **A parser** is a module that determines the string's validity and its structure (or parse tree)
  - The process of finding the structure in the flat stream of tokens is called **parsing**



Two Parsing Approaches

- Top-down parsers
  - Left-scan, Leftmost derivation
  - Best-known parser in this category, called **LL** parsers
- Bottom-up parsers
  - Left-scan, Rightmost derivation in reverse
  - Best-known parser in this category, called **LR** parsers



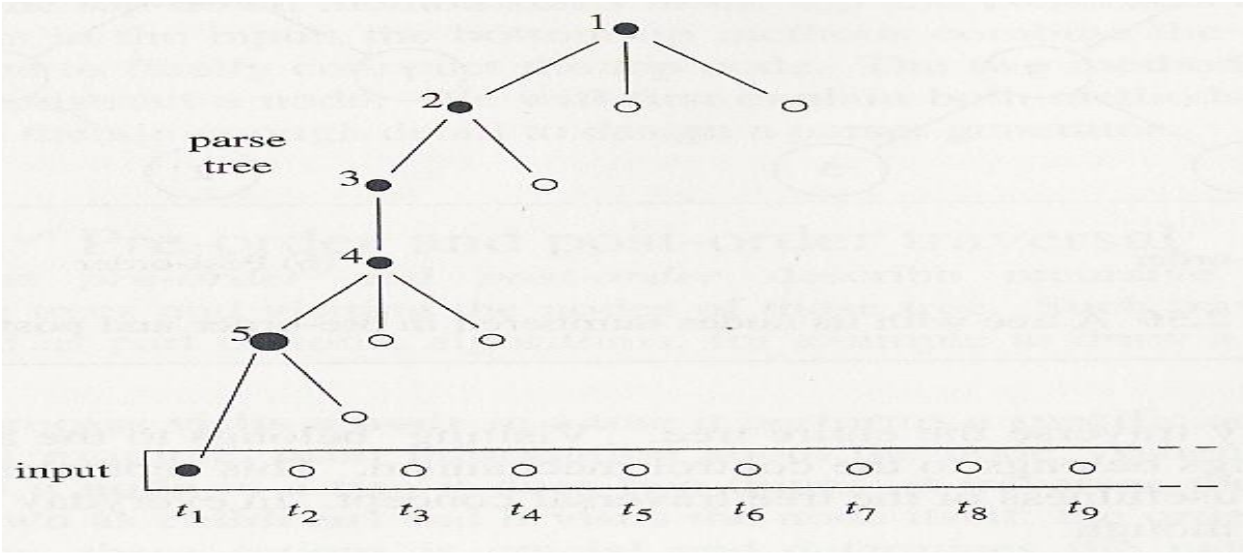
Top-Down Parsers

- A parser is considered **top-down**
  - if it generates a parse tree by starting at the root of the tree (the start symbol),
  - expanding the tree by applying productions in a **depth-first** manner
  - It corresponds to a **preorder traversal** of the parse tree
- Top-down parsing techniques are **predictive**
  - because they always predict the production that is to be matched before matching actually begins



Illustration of Top-Down Parsing

- A top-down parser begins by constructing the top node of the parse tree, which is the start symbol





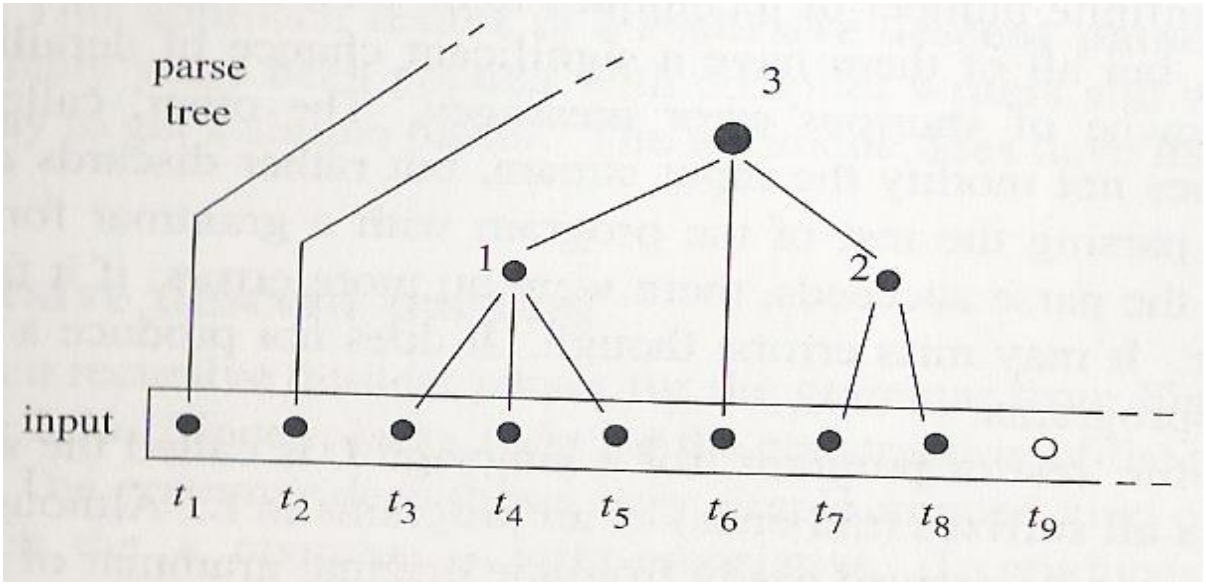
Bottom-Up Parsers

- The **bottom-up** parsers generate a parse tree by
  - starting at the tree's leaves and working toward its root
  - A node is inserted in the tree only after its children have been inserted
- A bottom-up parse corresponds to a **postorder traversal** of the parse tree



Illustration of Bottom-Up Parsing

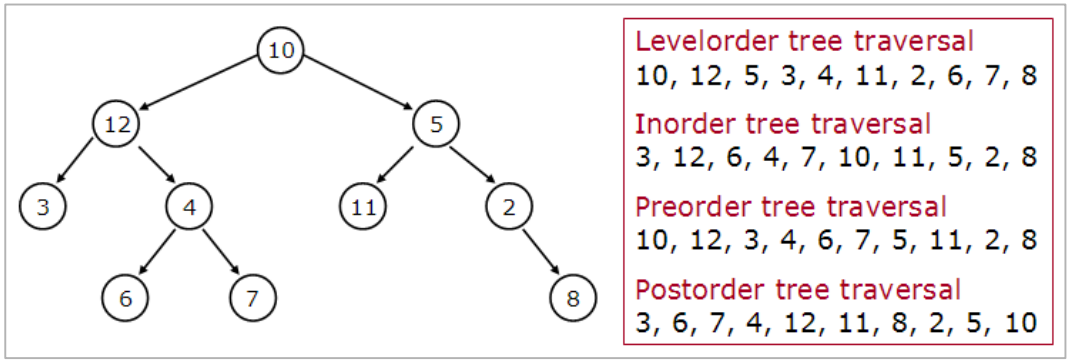
- The bottom-up parsing method constructs the nodes in the parse tree in post-order





To Refresh Your (and My) Memory

- Several ways for tree traversals



Levelorder tree traversal
10, 12, 5, 3, 4, 11, 2, 6, 7, 8

Inorder tree traversal
3, 12, 6, 4, 7, 10, 11, 5, 2, 8

Preorder tree traversal
10, 12, 3, 4, 6, 7, 5, 11, 2, 8

Postorder tree traversal
3, 6, 7, 4, 12, 11, 8, 2, 5, 10

← Top-down parsers

← Bottom-up parsers

- When traversing a node N in pre-order
  - the process first visits the node N and then traverses N's subtrees in left-to-right order
- When traversing a node N in post-order
  - the process first traverses N's subtrees in left-to-right order and then visits the node N

Quick note
Preorder: Root, Left, Right
Postorder: Left, Right, Root

Courtesy of http://www.namhpham.com/2015/10/post-order-tree-traversal-non-recursive.html



Example of the Parsers

- An example grammar generates the skeletal block structure of a hypothetical programming language

1 Program → begin Stmts end $
2 Stmts → Stmt ; Stmts
3 | λ
4 Stmt → simplestmt
5 | begin Stmts end

- The Fig. 4.5 and 4.6 illustrate a top-down and bottom-up parse of the given string:

begin simplestmt ; simplestmt ; end $

Top-Down Parsing Example

Legend:

1. Each box shows one step of the parse, with the particular rule denoted by bold lines between a parent (the rule's LHS) and its children (the rule's RHS)
2. Solid, non-bold lines indicate rules that have already been applied
3. Dashed lines indicate rules that have not yet been applied

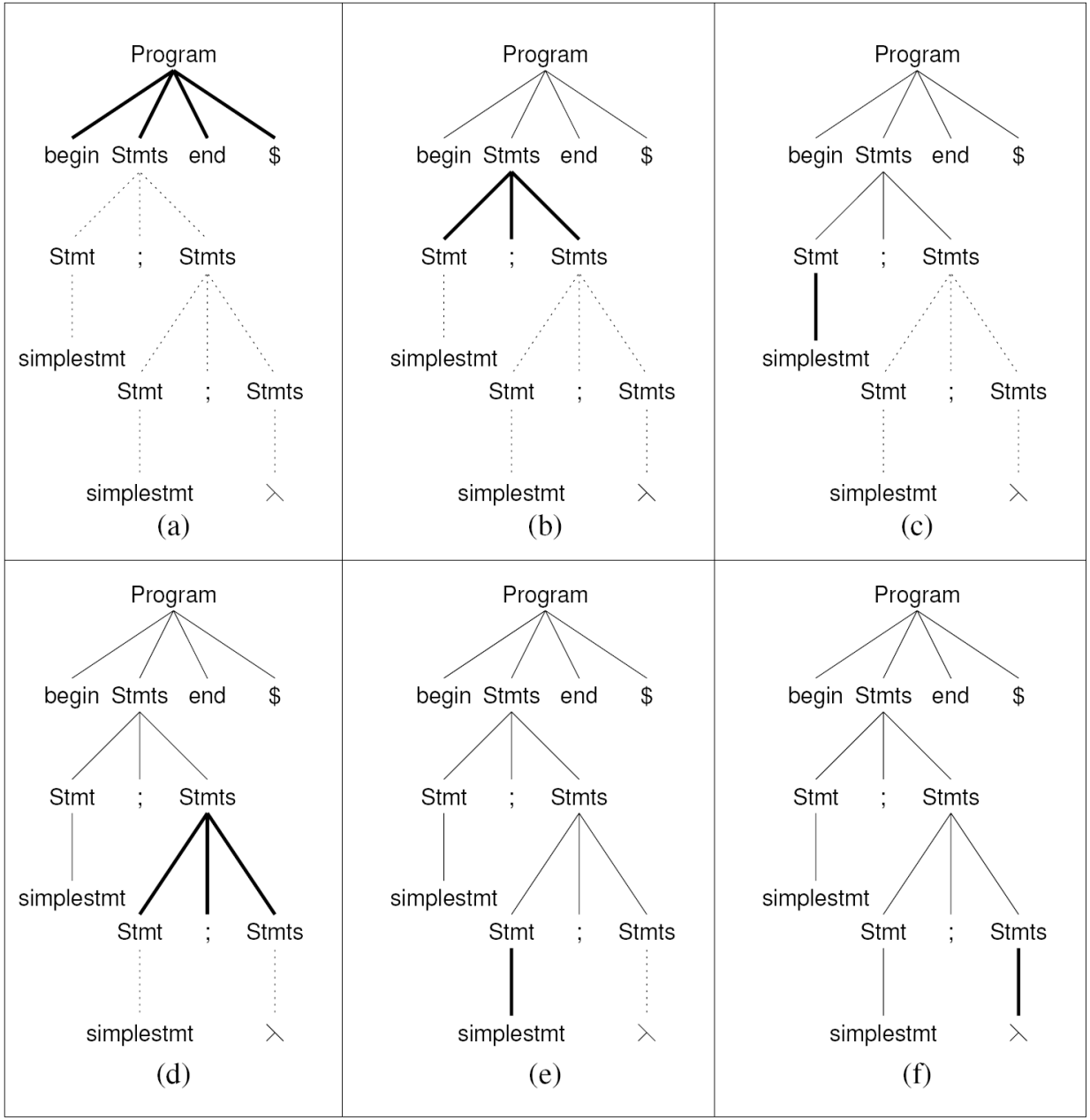
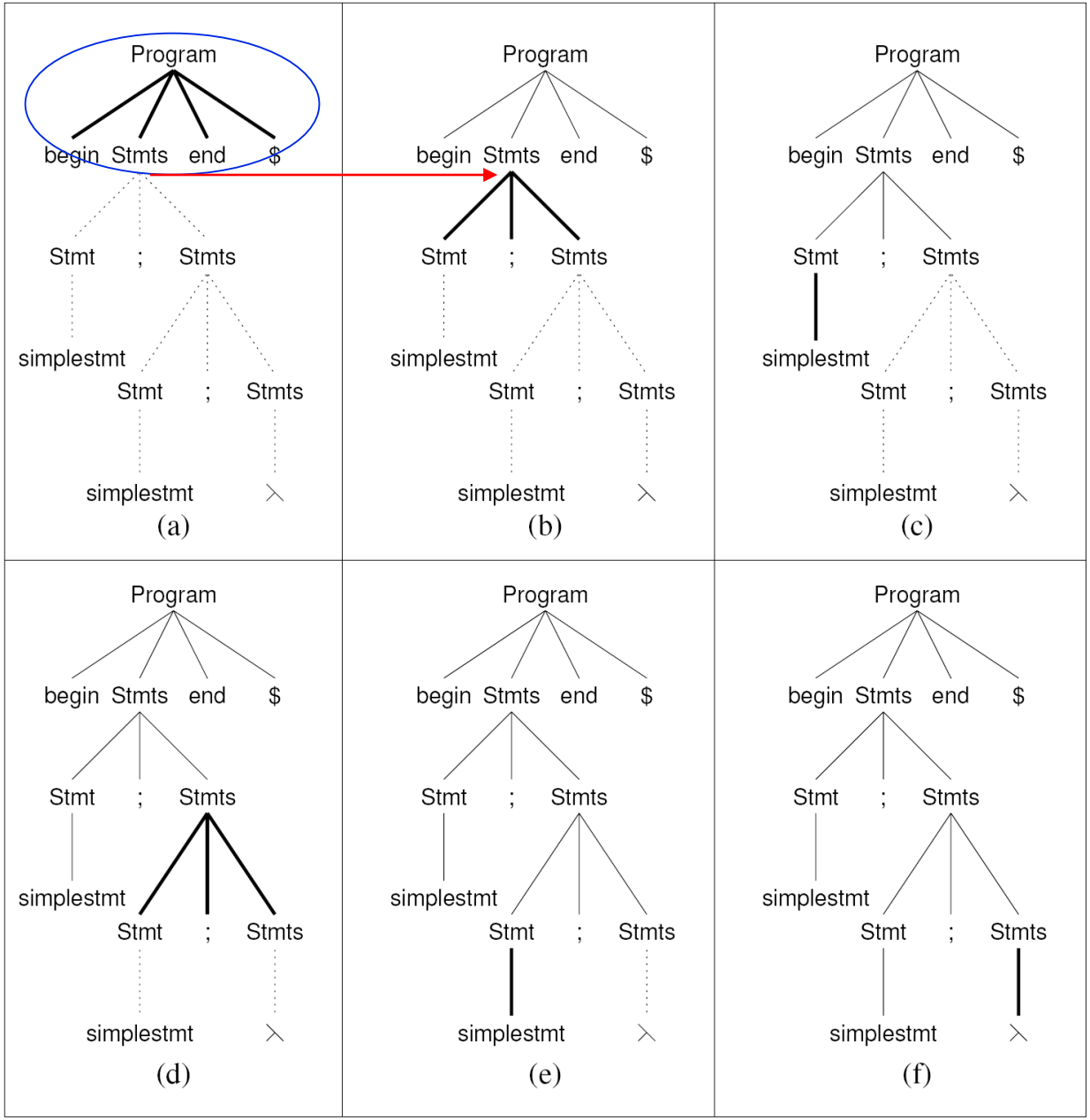


Figure 4.5: Parse of "begin simplestmt ; simplestmt ; end $" using the top-down technique. Legend explained on page 126.

Top-Down Parsing Example'

- Fig. 4.5(a) shows the rule **Program→begin Stmts end $** applied as the first step of a top-down parse



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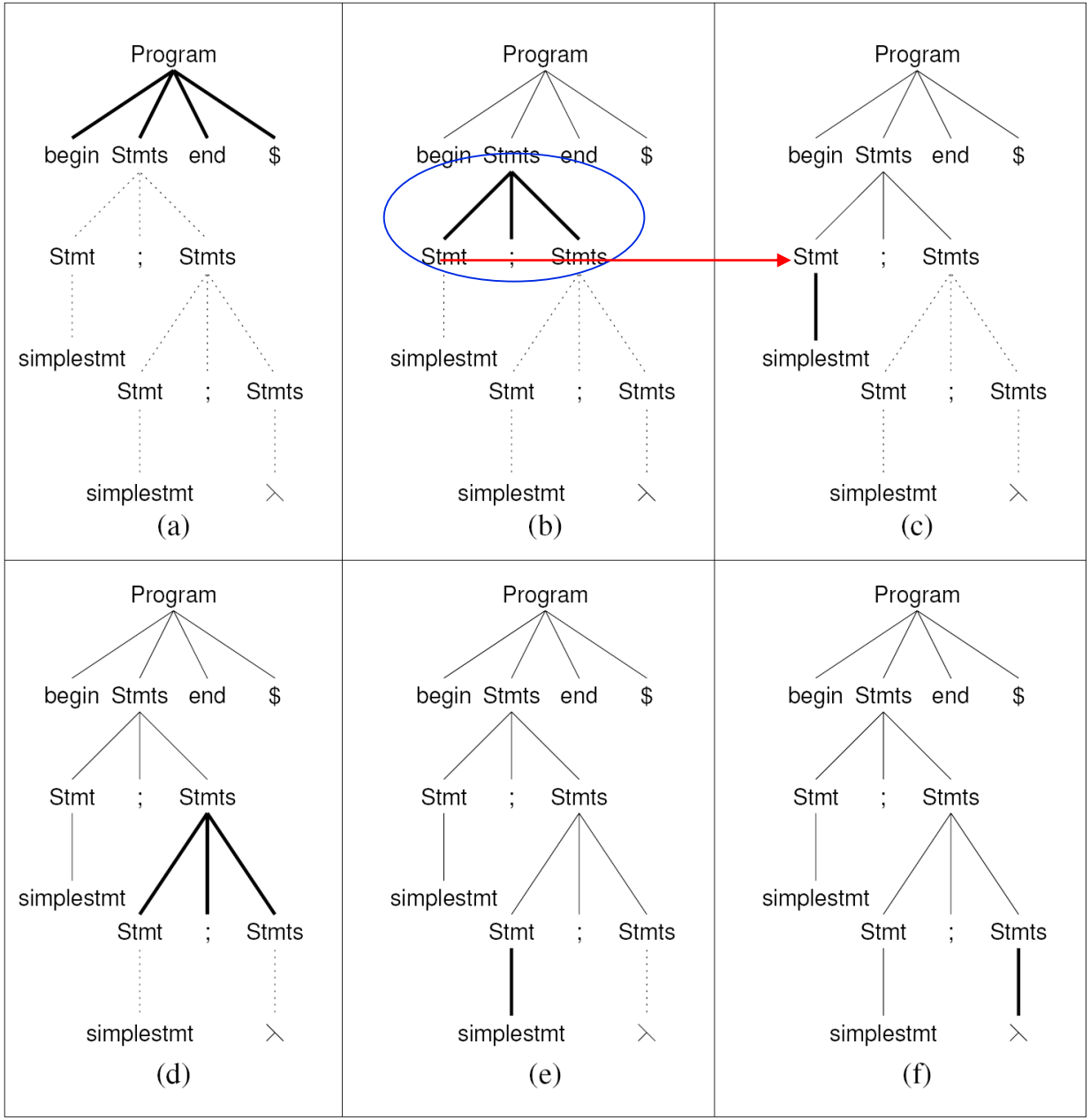

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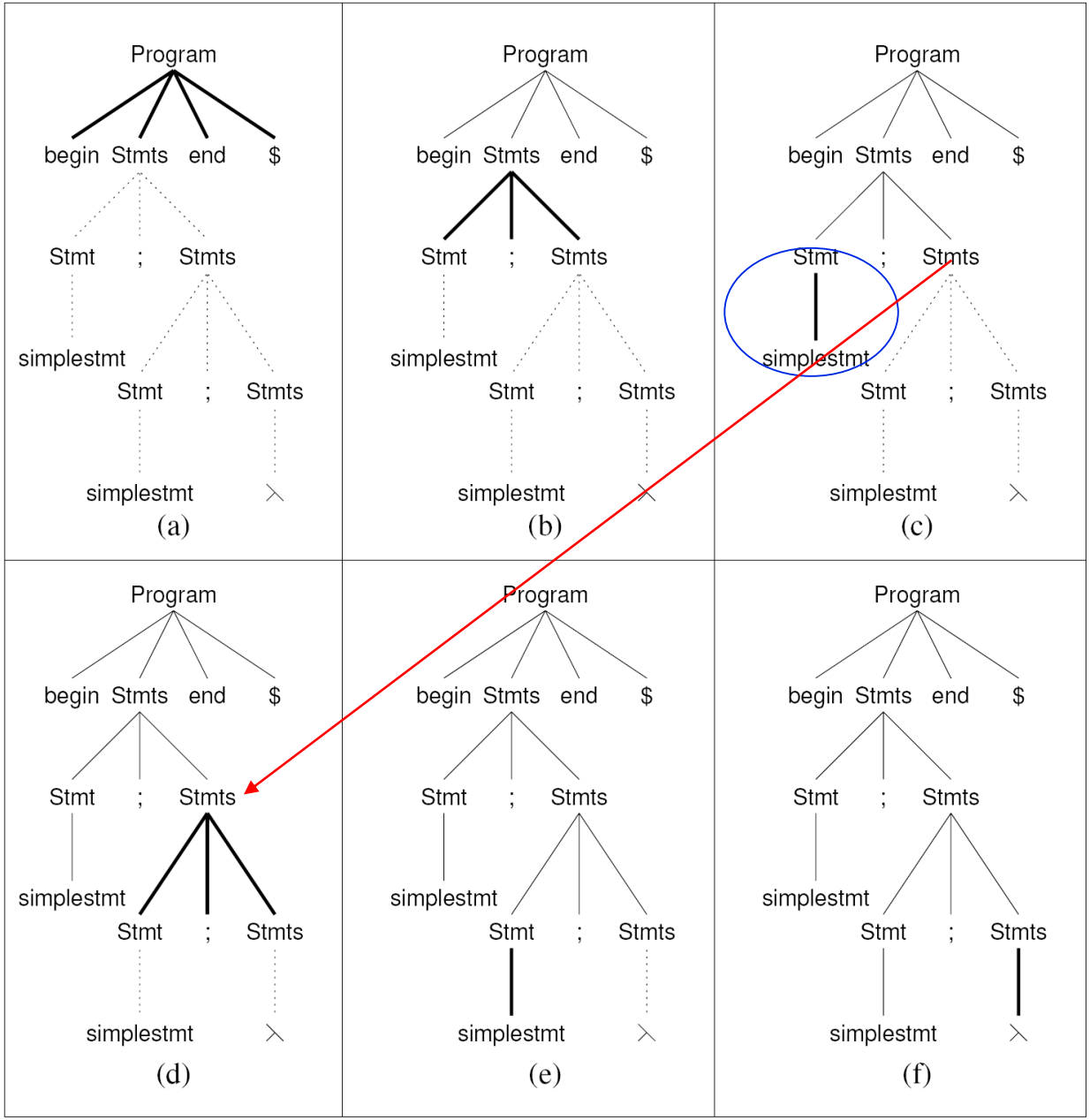

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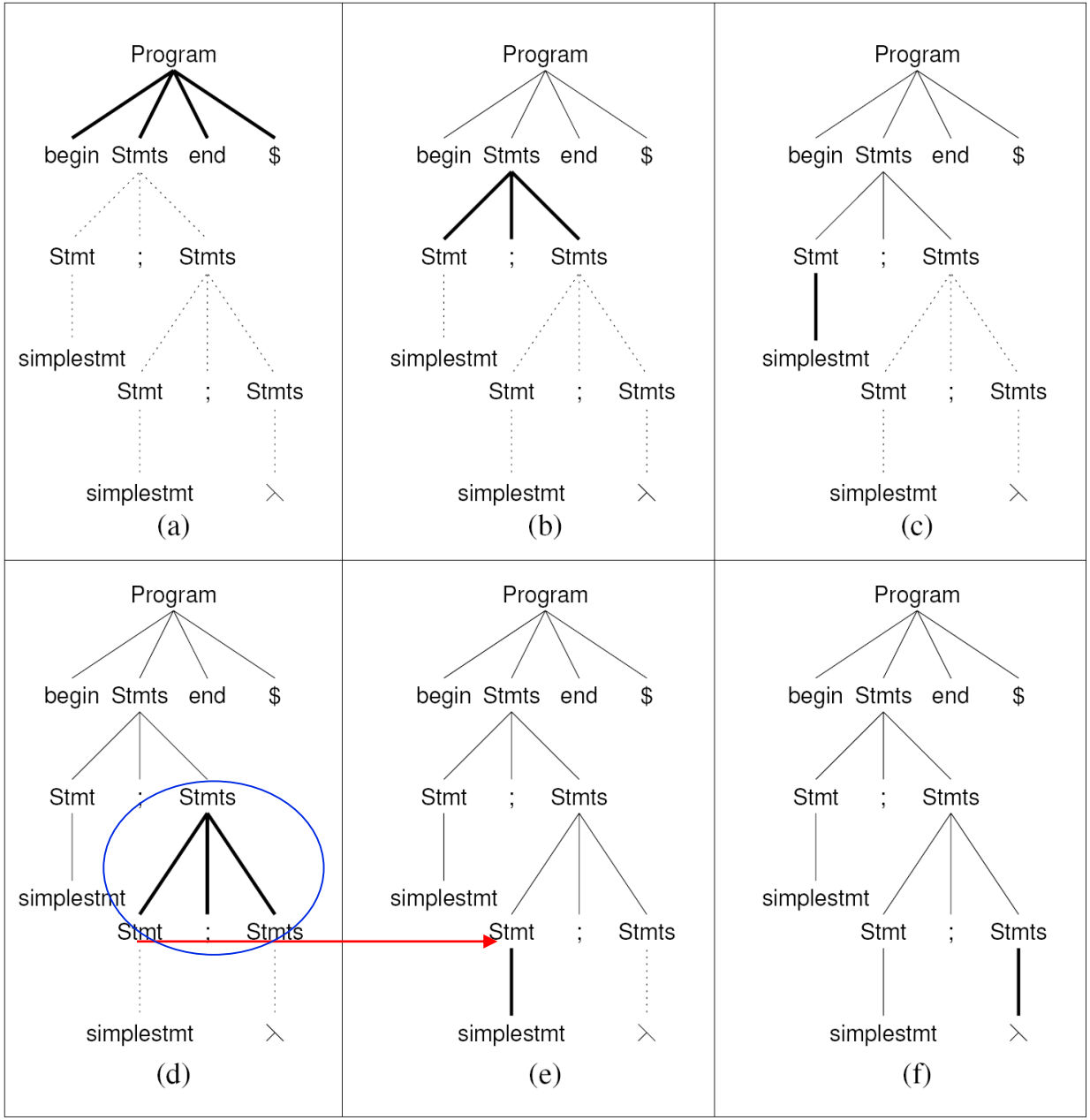

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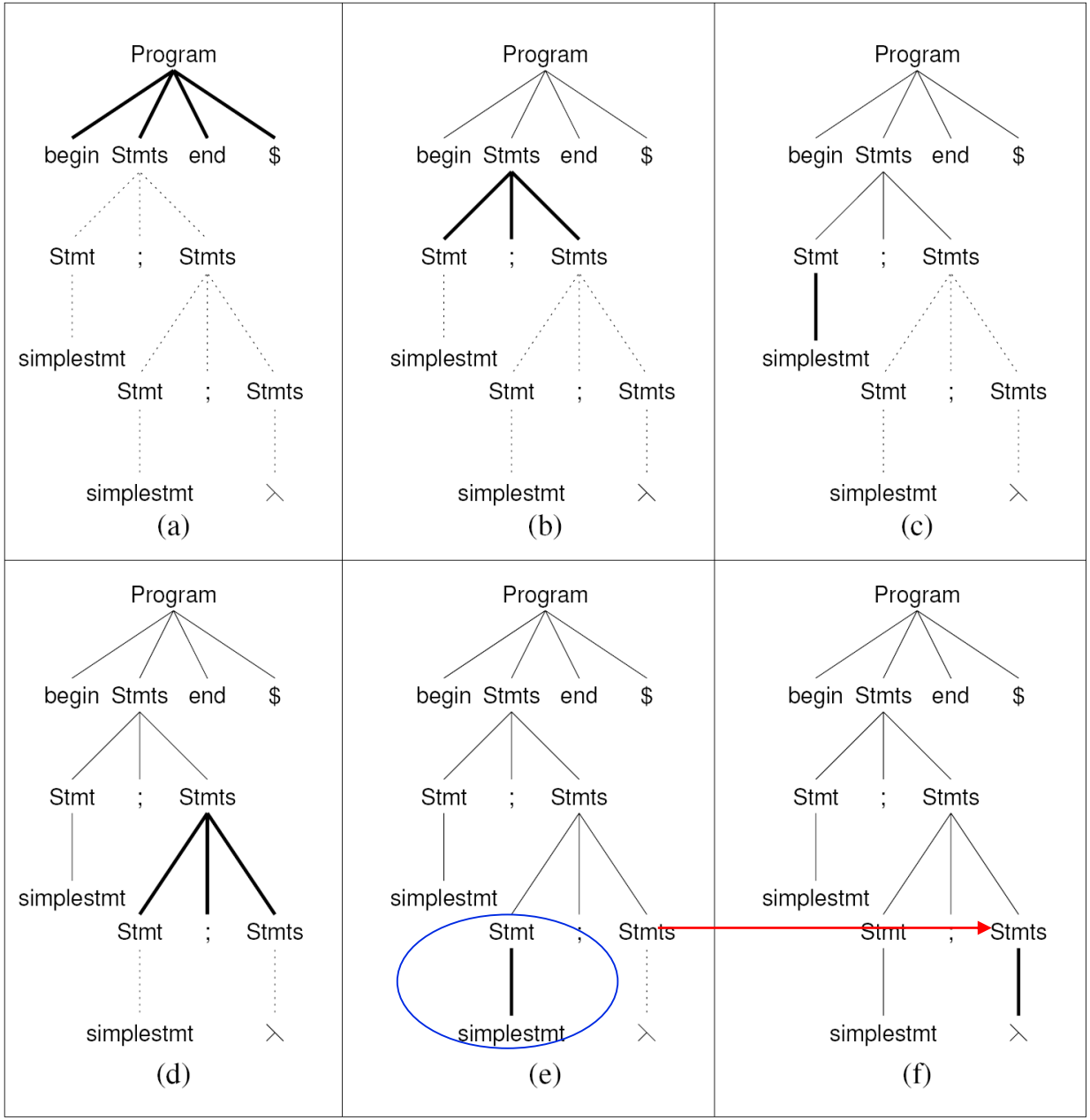

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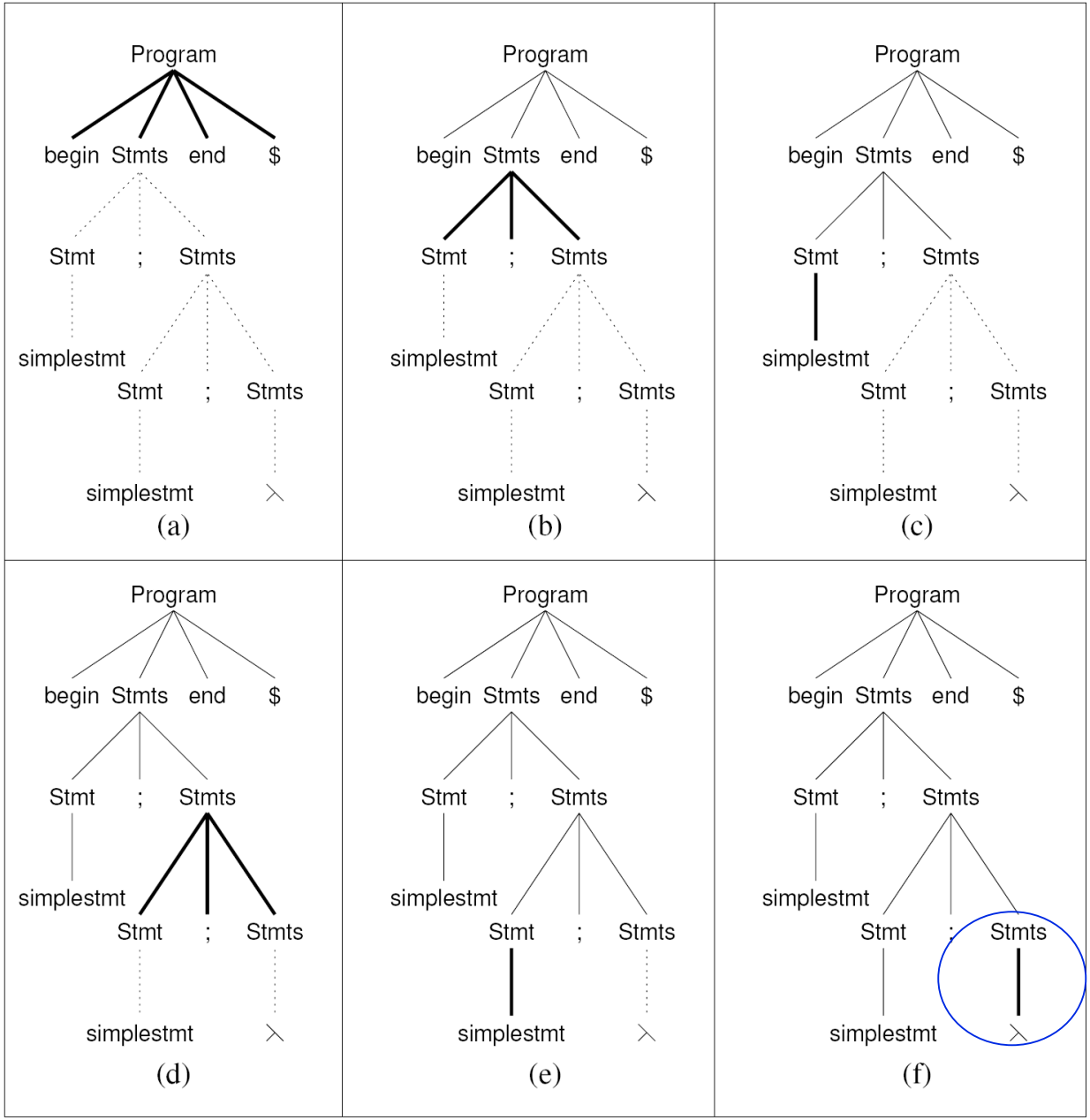

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You would …

- Practice the bottom-up parsing in Fig. 4.6
- If you get stuck
  - I assume you forget to read Section 4.1.1 and 4.1.2 ☺
  - Please carefully read the sections first and try it again



LL and LR Parsers

- **LL** and **LR** reflect
  - **how the input is processed** and **which kind of parse is produced**
  - The first character (L) states that the token sequence is processed from left to right
  - The second letter (L or R) indicates whether a leftmost or rightmost parse is produced
- The parsing technique can be further characterized by the **number of lookahead symbols**
  - i.e., symbols beyond the current token that the parser may consult to make
  - parsing choices
  - LL(1) and LR(1) parsers are the most common, requiring only one symbol of lookahead



Summary

- There is a many-to-one relationship between derivations and parse trees
  - A parse tree ignores variations in the order in which symbols in sentential forms are replaced
- While the parsing sequences of top-down and bottom-up parsing are different, two parsing techniques construct the same parse tree, as shown in Fig. 4.5 and 4.6



Why FIRST and FOLLOW?

- FIRST and FOLLOW
  - are the two important functions for the construction of top-down and bottom-up parsers
- FIRST and FOLLOW allow the parsers to choose which production to apply
- During panic-mode error recovery, sets of tokens produced by FOLLOW can be used as synchronizing tokens



FIRST

- FIRST(α)
  - refers to the set of **terminals** that begin strings derived from α
  - where α is any string of grammar symbols

  Example:
    - If $A \Rightarrow^* cy$, then *c* is in FIRST(A)
    - If $\alpha \Rightarrow^* \lambda$, then then λ is also in FIRST (α)
- Given A ⇒ a | b
  - FIRST(A) is the union of FIRST (a) and FIRST(b),
  - where FIRST (a) and FIRST(b) are disjoint sets



FIRST (Cont'd)

- *Compute FIRST(X)
  - X is grammar symbol
  - We apply the following rules until no more terminals or λ can be added to it

1. If X is a terminal (i.e., $X \in \Sigma$), then FIRST(X)={X}
2. If $X \Rightarrow \lambda$ is a production (i.e., $X \in N$ and $X \Rightarrow \lambda$), then add λ to FIRST(X)
3. If X is a non-terminal (i.e., $X \in N$) and $X \Rightarrow Y_1 Y_2 \ldots Y_k$ is a production for some k>=1, then place α in FIRST(X) if for some i, α is in FIRST(Y_i), and λ is in all of FIRST(Y_1),…,FIRST(Y_{i-1})

   If λ is in FIRST(Y_j) for all j=1,2,…,k, then add λ to FIRST(X)

*Courtesy of Compilers: Principles, Techniques, & Tools. Alfred V. Aho and et al. Pearson/Addison Wesley, 2007 (Section 4.4.2)



FIRST(α) Function

```
function First(α) returns Set
    foreach A ∈ NonTerminals( ) do VisitedFirst(A) ← false                (9)
    ans ← InternalFirst(α)
    return (ans)
end
function InternalFirst(Xβ) returns Set
    if Xβ = ⊥                                  ——→ Rule 2                 (10)
    then return (∅)
    if X ∈ Σ                                   ——→ Rule 1                 (11)
    then return ({X})
    /⋆  X is a nonterminal.                    ——→ Rule 3          ⋆/     (12)
    ans ← ∅
    if not VisitedFirst(X)
    then
        VisitedFirst(X) ← true                                            (13)
        foreach rhs ∈ ProductionsFor(X) do
            ans ← ans ∪ InternalFirst(rhs)                                (14)
    if SymbolDerivesEmpty(X)                                              (15)
    then ans ← ans ∪ InternalFirst(β)
    return (ans)                                                          (16)
end
```

Figure 4.8: Algorithm for computing First(α).

- **SymbolDerivesEmpty(A)** indicates whether or not the nonterminal A can derive λ
- **VisitedFirst(X)** is to indicate that the productions of X is already participate in the computation of FIRST(α)



Handling Endless Recursion

$$A \Rightarrow B$$
$$\cdot \quad \cdot \quad \cdot$$
$$B \Rightarrow C$$
$$\cdot \quad \cdot \quad \cdot$$
$$C \Rightarrow A$$

```
function First(α) returns Set
    foreach A ∈ NonTerminals( ) do VisitedFirst(A) ← false        (9)
    ans ← InternalFirst(α)
    return (ans)
end
function InternalFirst(Xβ) returns Set
    if Xβ = ⊥                                                     (10)
    then return (∅)
    if X ∈ Σ                                                      (11)
    then return ({X})
    /*  X is a nonterminal.                                       */ (12)
    ans ← ∅
    if not VisitedFirst(X)
    then
        VisitedFirst(X) ← true                                    (13)
        foreach rhs ∈ ProductionsFor(X) do
            ans ← ans ∪ InternalFirst(rhs)                        (14)
    if SymbolDerivesEmpty(X)                                      (15)
    then ans ← ans ∪ InternalFirst(β)
    return (ans)                                                  (16)
end
```

Figure 4.8: Algorithm for computing First(α).

- Termination of FIRST(A) must be handled properly in grammars
  - where **the computation of FIRST(A) appears to depend on FIRST(A)**
- **VisitedFirst(X)** is to indicate that the productions of X is already participate in the computation of FIRST(α)



FIRST(Cont'd)

- More about the Rule 3 in the previous page

3. If X is a non-terminal (i.e., $X \in N$) and $X \Rightarrow Y_1 Y_2 \ldots Y_k$ is a production for some k>=1, then place α in FIRST(X) if for some i, α is in $FIRST(Y_i)$, and λ is in all of $FIRST(Y_1),\ldots,FIRST(Y_{i-1})$

- Examples:
  - Everything in $FIRST(Y_1)$ is surely in FIRST(X)
  - If Y_1 does not derive λ, then we add nothing more to FIRST(X)
  - If $Y_1 \Rightarrow^* \lambda$, then we add $FIRST(Y_2)$, and so on



Example

Given the grammar,

- $E \Rightarrow TE'$
- $E' \Rightarrow +TE' \mid \lambda$
- $T \Rightarrow FT'$
- $T' \Rightarrow *FT' \mid \lambda$
- $F \Rightarrow (E) \mid id$

where E is for expression, T is for term, and F is for factor.

Please find the FIRST set of each symbol

- FIRST sets for E, T, F, E', and T':
  - FIRST(E) = FIRST(T) = FIRST(F) = {(, id }
  - FIRST(E') = {+, λ}
  - FIRST(T') = {*, λ}



Another Example for FIRST

Given the grammar,

- *input* ⇒ *expression*
- *expression* ⇒ *term rest_expression*
- *term* ⇒ *ID* | *parenthesized_expression*
- *parenthesized_expression* ⇒ '(' *expression* ')'
- *rest_expression* ⇒ '+' *expression* | λ

FIRST sets for input, expression, term, parenthesized_expression, and rest_expression:

- FIRST (input) = FIRST(expression) =FIRST (term) ={ ID, (}
- FIRST (parenthesized_expression) = { (}
- FIRST (rest_expression) = { +, λ}



You Should …

- Refer to Section 4.5.3 to do the exercise in Fig. 4.9 and 4.10

Exercise

- Note the *ans* does not contain λ, which may cause trouble for you to derive the results

```
1  E      → Prefix ( E )
2         | v Tail
3  Prefix → f
4         | λ
5  Tail   → + E
6         | λ
```

Figure 4.1: A simple expression grammar.

```
function First(α) returns Set
    foreach A ∈ NonTerminals( ) do VisitedFirst(A) ← false        (9)
    ans ← InternalFirst(α)
    return (ans)
end
function InternalFirst(Xβ) returns Set
    if Xβ = ⊥                                                     (10)
    then return (∅)
    if X ∈ Σ                                                      (11)
    then return ({X})
    /*  X is a nonterminal.                                       */ (12)
    ans ← ∅
    if not VisitedFirst(X)
    then
        VisitedFirst(X) ← true                                    (13)
        foreach rhs ∈ ProductionsFor(X) do
            ans ← ans ∪ InternalFirst(rhs)                        (14)
    if SymbolDerivesEmpty(X)                                      (15)
    then ans ← ans ∪ InternalFirst(β)
    return (ans)                                                  (16)
end
```

Figure 4.8: Algorithm for computing First(α).

Level	First X	First β	*ans*	Marker	Done? (⋆=Yes)	Comment
			First(Tail)			
0	Tail	⊥	{ }	(12)		
1	+	E	{+}	(11)	⋆	Tail → +E
1	⊥	⊥	{ }	(10)	⋆	Tail → λ
0			{+}	(14)		After all rules for Tail
1	⊥	⊥	{ }	(10)	⋆	Since β = ⊥
0			{+}	(15)	⋆	Final answer
			First(Prefix)			
0	Prefix	⊥	{ }	(12)		
1	f	⊥	{f}	(11)	⋆	Prefix → f
1	⊥	⊥	{ }	(10)	⋆	Prefix → λ
0			{f}	(14)		After all rules for Prefix
1	⊥	⊥	{ }	(10)	⋆	Since β = ⊥
0			{f}	(15)	⋆	Final answer
			First(E)			
0	E	⊥	{ }	(12)		
1	Prefix	(E)	{ }	(12)		E → Prefix (E)
1			{f}	(16)		Computation shown above
2	(	E)	{(}	(11)	⋆	Since Prefix ⇒* λ
1			{f,(}	(15)	⋆	Results due to E → Prefix (E)
1	v	Tail	{v}	(11)	⋆	E → v Tail
1	⊥	⊥	{ }	(10)		Since β = ⊥
0			{f,(,v}	(15)	⋆	Final answer

Figure 4.9: First sets for the nonterminals of Figure 4.1.



FOLLOW

- FOLLOW(β)
  - refers to the set of terminals a that can appear immediately to the right of non-terminal β in some sentential form

  Example:
  - If S ⇒ αAaβ, then a is in the set of FOLLOW(A)



FOLLOW (Cont'd)

- *FOLLOW(B)
  - where B is non-terminal ,
  - *S* is the start symbol for the grammar, and
  - $ is the input right end-marker
  - We apply the following rules for all nonterminals B until nothing can be added

1. Place $ in FOLLOW(*S*)
2. If there is a production A ⇒ αBβ,
   then everything in **FIRST(β)** except *λ* is added to FOLLOW(B)
3. If there is a production
   (a) A ⇒ αB, or
   (b) A ⇒ αBβ, where FIRST(β) contains λ,
   then everything in **FOLLOW(A)** is added to FOLLOW(B)

*Courtesy of Compilers: Principles, Techniques, & Tools. Alfred V. Aho and et al. Pearson/Addison Wesley, 2007 (Section 4.4.2)



FOLLOW (A) Function

```
function FOLLOW(A) returns Set
    foreach A ∈ NONTERMINALS( ) do
        VisitedFollow(A) ← false                                   (17)
    ans ← INTERNALFOLLOW(A)
    return (ans)
end
function INTERNALFOLLOW(A) returns Set
    ans ← ∅
    if not VisitedFolow(A)                                         (18)
    then
        VisitedFollow(A) ← true                                    (19)
        foreach a ∈ OCCURRENCES(A) do                              (20)
            ans ← ans ∪ FIRST(TAIL(a))                             (21)   → Rule 2
            if ALLDERIVEEMPTY(TAIL(a))                             (22)
            then
                targ ← LHS(PRODUCTION(a))
                ans ← ans ∪ INTERNALFOLLOW(targ)                   (23)   → Rule 3
    return (ans)                                                   (24)
end
function ALLDERIVEEMPTY(γ) returns Boolean
    foreach X ∈ γ do
        if not SymbolDerivesEmpty(X) or X ∈ Σ
        then return (false)
    return (true)                                                          → Rule 3
end
```

(when argument γ is empty set (3a) or the following non-terminals in γ all contain λ (3b))

Figure 4.11: Algorithm for computing Follow(A).

- **SymbolDerivesEmpty(A)** indicates whether or not the nonterminal A can derive λ
- **VisitedFollow(X)** is to indicate that the productions of X is already participate in the computation of FOLLOW(A)
- What happens to `$'? It is fine. Rule 1 is not defined in the primary textbook



Example

Given the grammar,

- $E \Rightarrow TE'$
- $E' \Rightarrow +TE' \mid \lambda$
- $T \Rightarrow FT'$
- $T' \Rightarrow *FT' \mid \lambda$
- $F \Rightarrow (E) \mid id$

where E is for expression, T is for term, and F is for factor

Please find the FOLLOW set of each symbol



Example (Cont'd)

Given the grammar,

- $E \Rightarrow TE'$
- $E' \Rightarrow +TE' \mid \lambda$
- $T \Rightarrow FT'$
- $T' \Rightarrow *FT' \mid \lambda$
- $F \Rightarrow (E) \mid id$

E is start symbol

FOLLOW(B)

1. Place $ in FOLLOW(S)
2. If there is a production A ⇒ αBβ, then everything in FIRST(β) except λ is added to FOLLOW(B)
3. If there is a production A ⇒ αB, or A ⇒ αBβ, where FIRST(β) contains λ, then everything in FOLLOW(A) is added to FOLLOW(B)

- FIRST sets for E, T, F, E', and T':
  - FIRST(E) = FIRST(T) = FIRST(F) = {(, id }
  - FIRST(E') = {+, λ}
  - FIRST(T') = {*, λ}

- FOLLOW sets for E, T, F, E', and T':
  - FOLLOW(E) = {$,)}
  - FOLLOW(E') = FOLLOW(E) = {$,)}
  - FOLLOW(T) = FIRST(E')+FOLLOW(E') = {+, $,) }
  - FOLLOW(T') = FOLLOW(T)= {+, $,) }
  - FOLLOW(F) = FIRST(T')+FOLLOW(T) + FOLLOW(T')= {*, +, $,)}



Another Example for FOLLOW

Given the grammar,

- *input* ⇒ *expression*
- *expression* ⇒ *term rest_expression*
- *term* ⇒ *ID* | *parenthesized_expression*
- *parenthesized_expression* ⇒ '(' *expression* ')'
- *rest_expression* ⇒ '+' *expression* | λ

FOLLOW(B)

1. Place $ in FOLLOW(S)
2. If there is a production A ⇒ αBβ, then everything in FIRST(β) except λ is added to FOLLOW(B)
3. If there is a production (a) A ⇒ αB, or (b) A ⇒ αBβ, where FIRST(β) contains λ, then everything in FOLLOW(A) is added to FOLLOW(B)

FOLLOW sets for input, expression, term, parenthesized_expression, and rest_expression

- FOLLOW (input) = {$ } Rule 1
- FOLLOW (expression) = {$,)} Rule 3(a) got $; Rule 2 got)
- FOLLOW (term) = FOLLOW (parenthesized_expression) Rule3(a)
  = {+, $,) } Rule 2 got +; Rule 3(b) got $)
- FOLLOW (rest_expression) = { $,)} Rule 3(a)



You Should …

- Refer to Section 4.5.4 to do the exercise in Fig. 4.12 and 4.13



Just Another Example for FOLLOW

- Fig. 4.10 grammar
- Can you derive the FOLLOW sets for A, B?
- Because of different definition used in the book, S does not contain $

```
1  S → A B c
2  A → a
3     | λ
4  B → b
5     | λ
```

```
function FOLLOW(A) returns Set
    foreach A ∈ NONTERMINALS( ) do
        VisitedFollow(A) ← false                                   (17)
    ans ← INTERNALFOLLOW(A)
    return (ans)
end
function INTERNALFOLLOW(A) returns Set
    ans ← ∅
    if not VisitedFolow(A)                                         (18)
    then
        VisitedFollow(A) ← true                                    (19)
        foreach a ∈ OCCURRENCES(A) do                              (20)
            ans ← ans ∪ FIRST(TAIL(a))                             (21)
            if ALLDERIVEEMPTY(TAIL(a))                             (22)
            then
                targ ← LHS(PRODUCTION(a))
                ans ← ans ∪ INTERNALFOLLOW(targ)                   (23)
    return (ans)                                                   (24)
end
function ALLDERIVEEMPTY(γ) returns Boolean
    foreach X ∈ γ do
        if not SymbolDerivesEmpty(X) or X ∈ Σ
        then return (false)
    return (true)
end
```

Figure 4.11: Algorithm for computing Follow(A).

Level	Rule	Marker	Result	Comment
			FOLLOW(B)	
0			FOLLOW(B)	
0	S→A B c	(21)	{c}	
0		(24)	{c}	Returns
			FOLLOW(A)	
0			FOLLOW(A)	
0	S→ A B c	(21)	{b,c}	
0		(24)	{b,c}	Returns
			FOLLOW(S)	
0			FOLLOW(S)	
0		(24)	{ }	Returns

Figure 4.12: Follow sets for the grammar in Figure 4.10. Note that Follow(S) = { } because S does not appear on the RHS of any production.



QUESTIONS?