

# Backtracking

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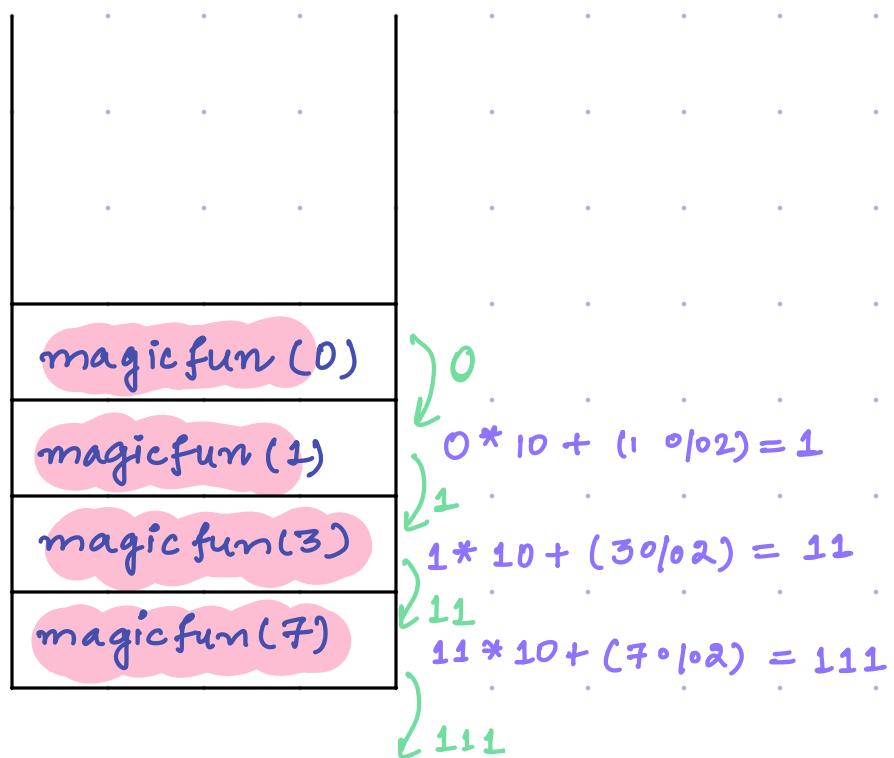
Class starts at 7:05 IST



# QUIZ

What is the output of below code for N = 7 ?

```
int magicfun( int N) {  
    if ( N == 0)  
        return 0;  
    else  
        return magicfun(N/2) * 10 + (N % 2);  
}
```



$N \rightarrow N/2 \rightarrow N/4 \dots \rightarrow 0$

ans: 111

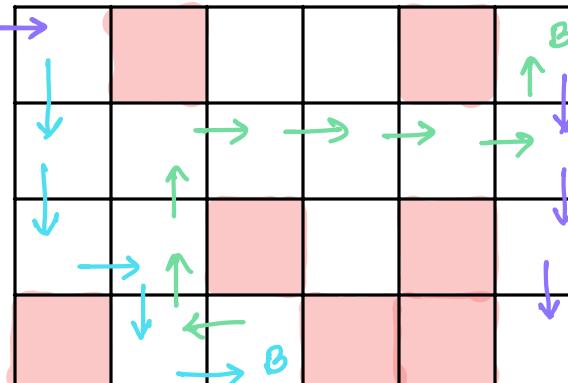
T.C :  $O(\log N)$



# Backtracking



entry →



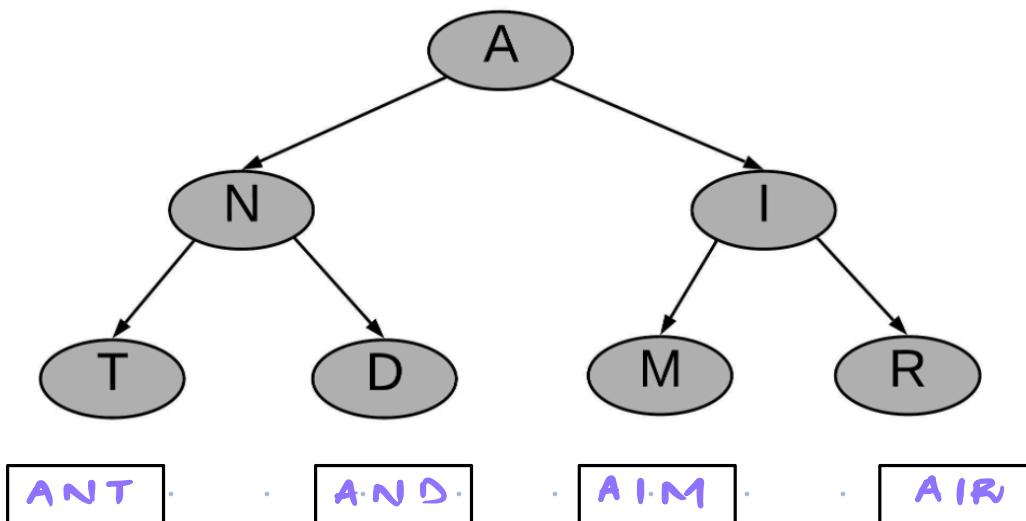
Blocked



Restart



Given a set of words represented in the form of a tree, the tree is formed such that every branch ends in a word. Our task is to find out if a given word is present in the tree or not.



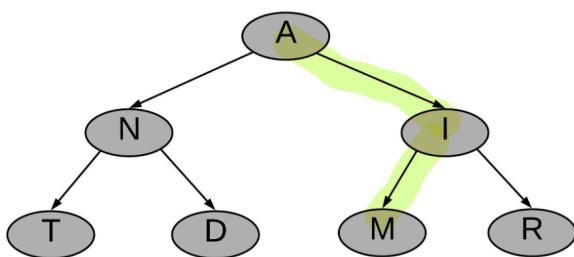
Word to be Searched : AIM

Approach 1 :



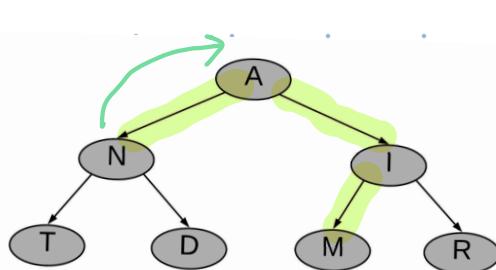


Checking all branches  
till we get our desired  
word.



$AIM == AIM$

### Backtracking approach :



$AIM$

$AN \neq AIM$

### What is backtracking?

Algorithmic Technique of exploring all possibilities using Recursion.

In Backtracking, we begin by choosing an option & explore all possibility until we reach a state where we can't go any further so now be backtrack & go to next option.

We repeat this step until we get the desired result.



PRUNING : Reducing the search space by discarding the branches that are not worth exploring.

General pattern for Backtracking code :-

Solve() {

    Base Case

    Option 1

        Recur

    or { Do Recur Undo }

    Option 2

        Recur

    or { Do Recur Undo }

        |  
        |  
        |

    y



# Valid Parentheses

Question - Given an integer  $A$ , write a function to generate all valid parentheses of length  $2 \times A$ .

( )

Traversing  $L \rightarrow R$   
 $\#C \leq \#O$

Overall

$\#C == \#O == A$

$A = 1$  ( )

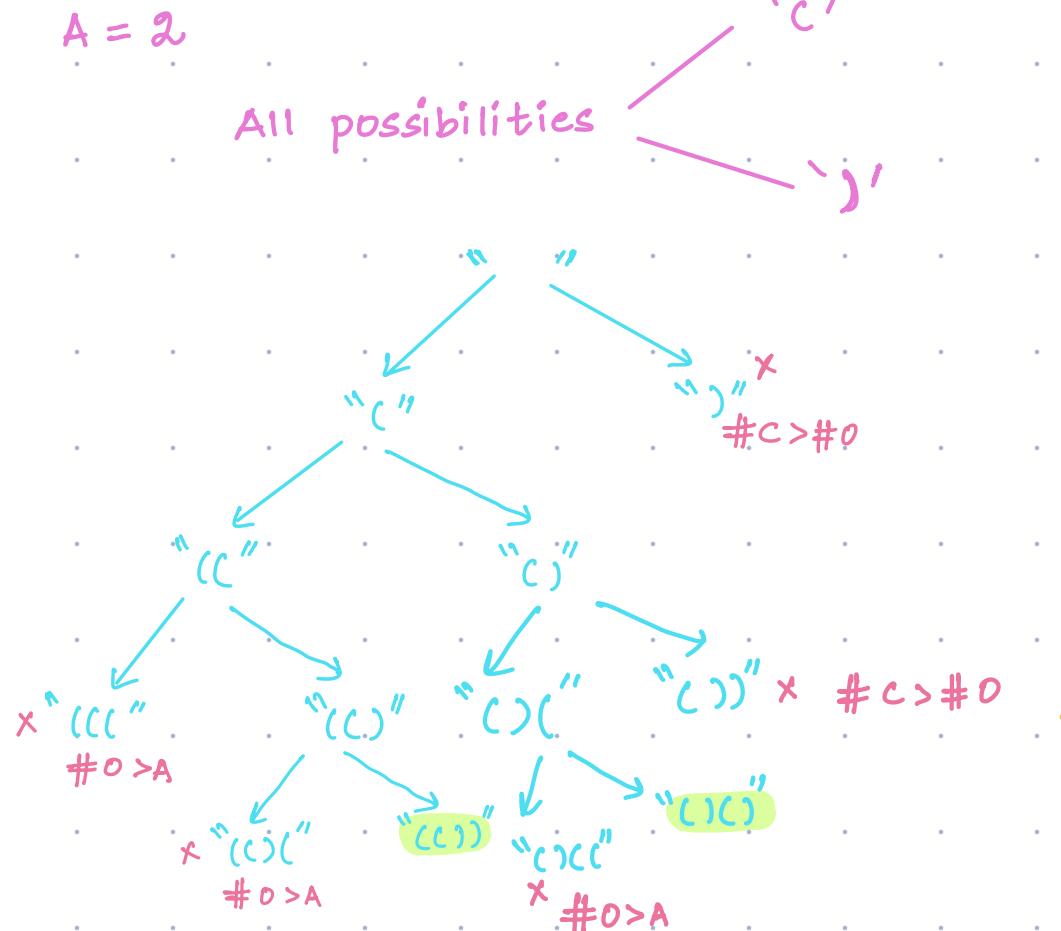
$A = 2$  (( )) ✓

( ) ( ) ✓

) ( ) ( ✗

) ) ( ( ✗

Approach :





We need to keep track of #O & #C.

solve (A, open-count, close-count)

Conditions for adding "(" and ")"

When  $\text{open\_count} < A$

Add ')' & recur.

When  $\text{close\_count} < \text{open\_count}$

Add '(' & recur.

Base case

str. length == 2A



## CODE :

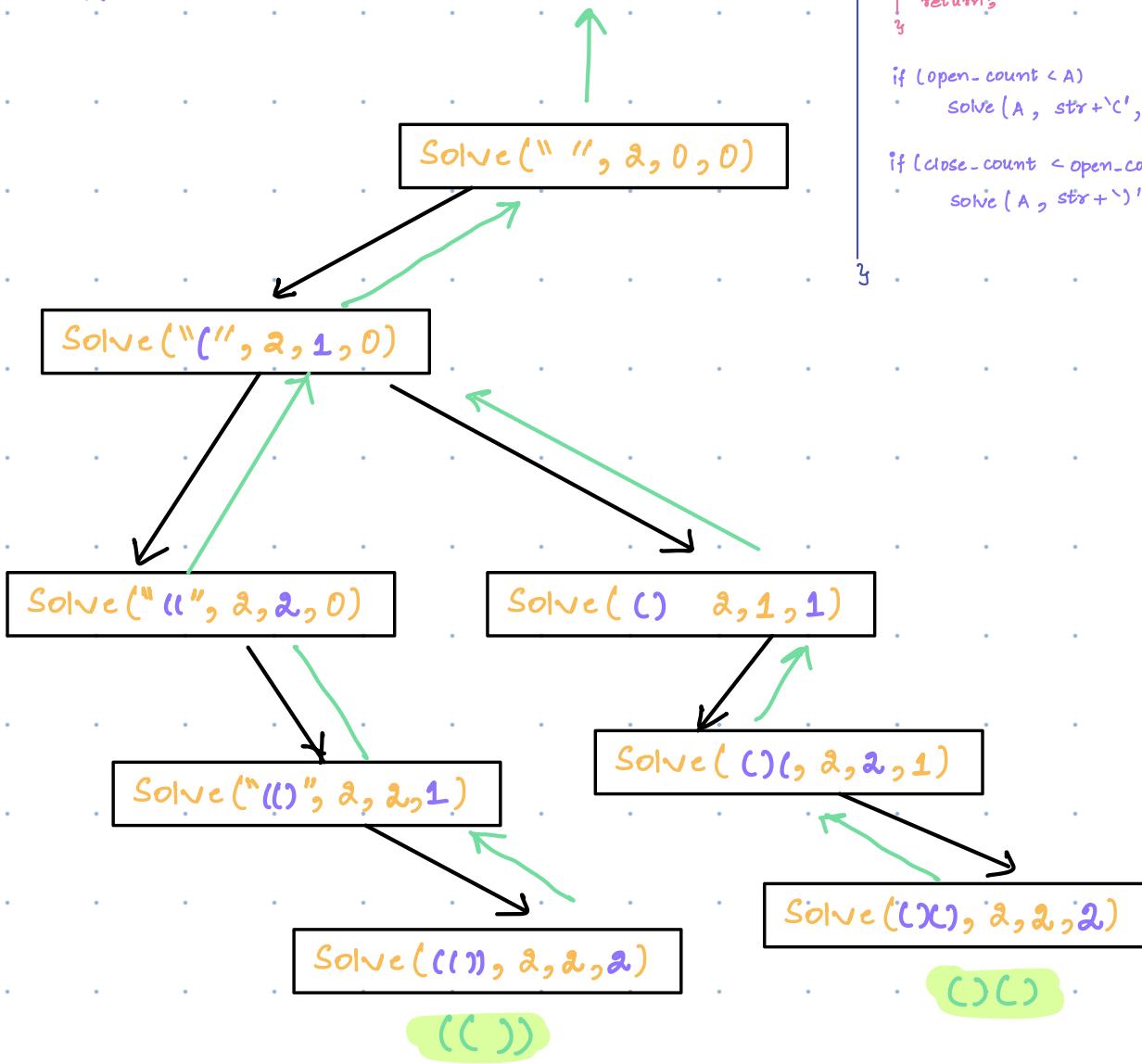
```
solve( A , str , open-count , close-count ) {  
    // Base Case  
    if (str.length == 2A) {  
        print(str);  
        return;  
    }  
  
    if (open-count < A)  
        solve( A , str + '(', open-count + 1 , close-count );  
  
    if (close-count < open-count)  
        solve( A , str + ')' + open-count , close-count + 1 );  
}
```

$$T.C \leq O(2^A)$$

$$S.C = O(2A) = O(A)$$



## Dry run

 $A = 2$ 

```
solve(str, A, open_count, close_count) {  
    // Base case  
    if (str.length == 2A) {  
        print(str);  
        return;  
    }  
  
    if (open_count < A)  
        solve(A, str + '(', open_count + 1, close_count);  
  
    if (close_count < open_count)  
        solve(A, str + ')', open_count, close_count + 1);  
}
```

Break till 8:30



## Subarray, Subsequence, Subsets

Subarray : Continuous part of an array.

Subsequence      ↴ Non-continuous part of array.  
Subsets

Subsequence	Subsets
<ul style="list-style-type: none"><li>Relative order of elements is maintained.</li></ul>	<ul style="list-style-type: none"><li>Order not maintained.</li></ul>

[5 1]      ↴ These are not  
[1 5]      considered different subsets



1	2	3	4	5
---	---	---	---	---

	Subarray ?	Subsequence ?	Subset ?
[1 3 4]	✗	✓	✓
[1 5 3]	✗	✗	✓
[1 2 3]	✓	✓	✓
[ ]	✗	✓	✓
[1 2 3 4 5]	✓	✓	✓
[2 5]	✗	✓	✓
[4]	✓	✓	✓
[5 1]	✗	✗	✓
[4 3 2]	✗	✗	✓

- Subarray is a subsequence but not vice-versa.
- Subsequence is a subset but not vice-versa.
- Subarray is a subset but not vice-versa.



## Subsets

Question - Given an array with distinct integers. Print all the subsets using recursion.

a	b	c
---	---	---

{ } { }

{ a }

{ b }

{ c }

{ a b }

{ a c }

{ b c }

{ a b c }

### Observations:

Each element has 2 options

Include in set

Exclude in set

- - -

a - -

- b -

- - c

a b -

a - c

- b c

a b c

$$2 \times 2 \times 2 = 8$$

n elements

$$2 \times 2 \times 2 \cdots 2 = 2^N$$

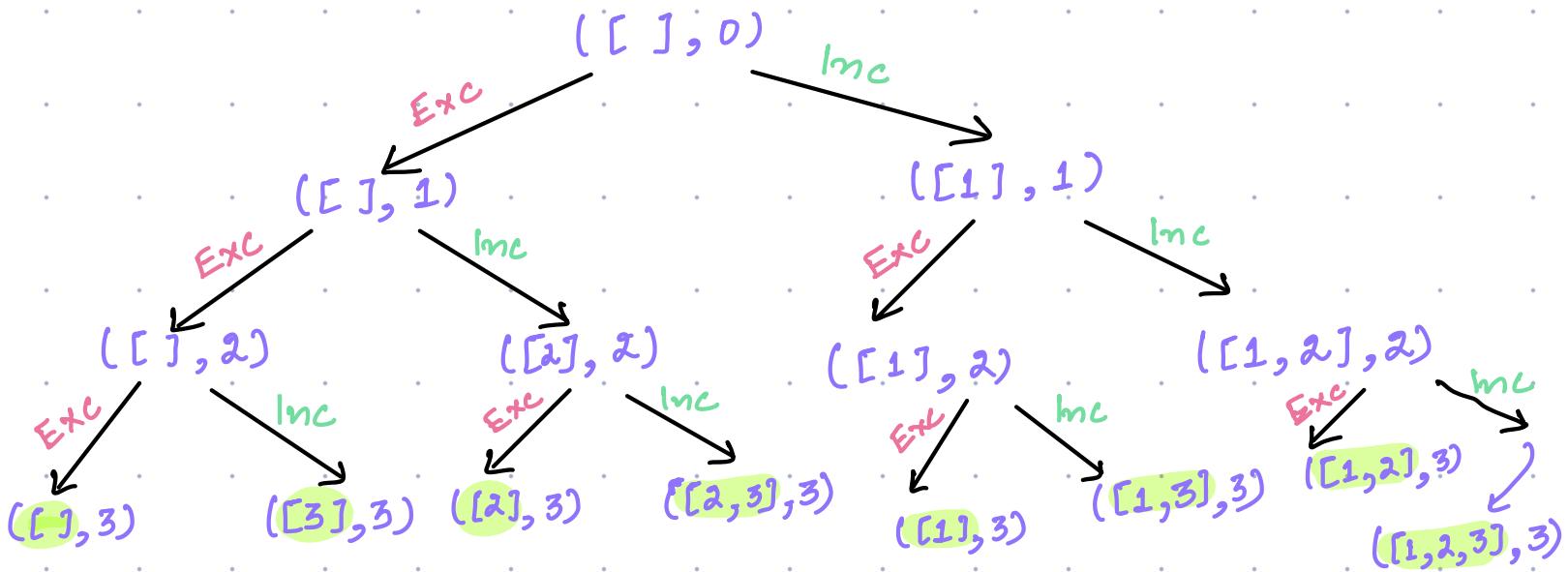
Total Subsets:  $2^N$



0	1	2
1	2	3

(current\_set, index)

Approach:



Base condition

$$\text{index} = N$$



## CODE

```
Subsets( A[], curr[], index) {
```

// Base Case

```
if (index == A.size()) {  
    print(curr[]);  
    return;  
}
```

// Option 1 : Include element in current set.

```
curr.add(A[index]);    } Do  
subset(A[], curr[], index + 1); } Recur  
curr.remove(curr.length() - 1); } Undo
```

// Option 2 : Exclude element

```
subset(A[], curr[], index + 1);
```



Dry run :

Subset (A, curr[], index) {

Base Case

Option 1: Include

- Do
- Recur
- Undo

Option 2: Exclude

Recur

Subsets (A[], curr[], index) {

// Base Case

```
if (index == A.size()) {
    print(curr[]);
    return;
}
```

// Option 1 : Include element in current set.

```
curr.add(A[index]);
subset(A[], curr[], index+1); // Recur
curr.remove(curr.length()-1); // Undo
```

// Option 2 : Exclude element

subset(A[], curr[], index+1);

 $A[] = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}$ 

curr = [ ]

Subset (A, curr, 0)

subset (A, curr, 1)

subset (A, curr, 1)

Subset (A, curr, 2)

subset (A, curr, 2)

subset (A, curr, 2)

ab

a

b

[ ]

T.C :  $O(2^N)$ S.C :  $O(N)$



- A popular Fitness app FitBit, is looking to make workouts more exciting for its users. The app has noticed that people get bored when the same exercises are shown in the same order every time they work out. To mix things up, FitBit wants to show all the different ways the exercises can be arranged so that each workout feels new.
- Your challenge is to write a program for FitBit that takes a string A as input, where each character in the string represents a different exercise. Your program should then find and display all possible arrangements of these exercises.

Example:

A = Push-ups

B = Squats

C = Burpees

D = Planks

Then different ways of doing the exercise includes -

ABCD

ACBD

ADBC

ADCB

Permutations

etc...



$$N = 3$$

No of ways to arrange 3 balls :

$$\frac{3}{\cancel{3}} \times \frac{2}{\cancel{2}} \times \frac{1}{\cancel{1}} = 3! = 6$$

$$N(N-1)(N-2) \cdots 1 = N! \rightarrow \text{Total no of permutations}$$



## Permutations

Question - Given a string with distinct elements. Print all permutations of it without modifying it.

[a b c]

N = 3

3! = 6

abc

acb

bac

bca

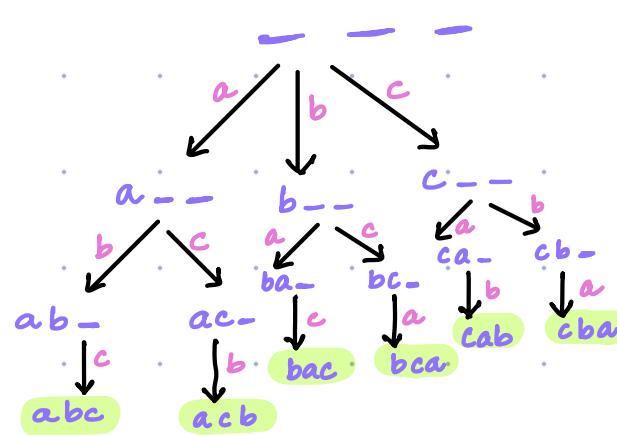
cab

cba

Approach :

N = 3

[a b c]



Base case :

all N spots filled.



## CODE

```
void permutation (str, index, string ans, boolean [ ] visited) {  
    // Base case  
    if (index == str.length) {  
        print (ans);  
        return;  
    }  
  
    for (i=0 ; i<str.length() ; i++) { // All Possibility  
        if (visited[i] == false) { // Valid Possibility  
            visited[i] = true; // Do  
            permutations (str, index+1, ans+str[i], visited); // Recur  
            visited[i] = false; // Undo  
        }  
    }  
}
```



## Dry Run :

```

void permutation (Str, index, string ans, boolean [] visited) {
    // Base case
    if (index == str.length) {
        print (ans);
        return;
    }

    for (i=0 ; i<str.length(); i++) { // All Possibility
        if (visited[i] == false) { // Valid Possibility
            visited[i] = true; // Do
            permutations (str, index+1, ans+str[i], visited); // Recur
            visited[i] = false; // Undo
        }
    }
}

```

permutation (---)

## Base Case

Option 1 :  $i = 0$ 

- Do
- Recur
- Undo

Option 2 :  $i = 1$ 

- Do
- Recur
- Undo

 $i = N-1$ 