

Given:

$$\begin{aligned}
\hat{y} &= O = R_4(P_4) \\
P_4 &= H_3 w_4 \\
H_3 &= R_3(P_3) \\
P_3 &= H_2 w_3 \\
H_2 &= R_2(P_2) \\
P_2 &= H_1 w_2 \\
H_1 &= R_1(P_1) \\
P_1 &= x w_1 \\
C &= \frac{1}{2}(\hat{y} - y)^2
\end{aligned}$$

1 Find $\frac{d}{dw_4}[C]$:

$$\begin{aligned}
&= (\hat{y} - y) \frac{d}{dw_4} \hat{y} \\
&= (\hat{y} - y) \frac{d}{dw_4} [R_4(P_4)] \\
&= (\hat{y} - y) R_4'(P_4) \frac{d}{dw_4} [H_3 w_4] \\
&= (\hat{y} - y) R_4'(P_4) H_3
\end{aligned}$$

2 Find $\frac{d}{dw_3}[C]$:

$$\begin{aligned}
&= (\hat{y} - y) R_4'(P_4) \frac{d}{dw_3} [H_3 w_4] \\
&= (\hat{y} - y) R_4'(P_4) w_4 \frac{d}{dw_3} [H_3] \\
&= (\hat{y} - y) R_4'(P_4) w_4 \frac{d}{dw_3} [R_3(P_3)] = (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) \frac{d}{dw_3} [P_3] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) \frac{d}{dw_3} [H_2 w_3] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) H_2
\end{aligned}$$

3 Find $\frac{d}{dw_1}[C]$:

$$\begin{aligned}
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) \frac{d}{dw_1} [H_2 w_3] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 \frac{d}{dw_1} [H_2] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 \frac{d}{dw_1} [R_2(P_2)] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 R_2'(P_2) \frac{d}{dw_1} [P_2] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 R_2'(P_2) \frac{d}{dw_1} [H_1 w_2] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 R_2'(P_2) w_2 \frac{d}{dw_1} [R_1(P_1)] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 R_2'(P_2) w_2 R_1'(P_1) \frac{d}{dw_1} [P_1] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 R_2'(P_2) w_2 R_1'(P_1) \frac{d}{dw_1} [x w_1] \\
&= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 R_2'(P_2) w_2 R_1'(P_1) x
\end{aligned}$$

4 Output layer error

$$\hat{y} - y$$

5 Find $\frac{d}{dP_2}[C]$:

$$\begin{aligned} &= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 \frac{d}{dP_2} [R_2(P_2)] \\ &= (\hat{y} - y) R_4'(P_4) w_4 R_3'(P_3) w_3 R_2'(P_2) \end{aligned}$$