STA6856 (Summer, 2025) **Mid-term Examination**

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UWF Honor Code applies to this unit examination. By writing your name above, you agree with the following: *you have neither given nor received unauthorized aidfor this examination.*

This exam contains 6 pages (including this cover page) and 7 questions. The total number of possible points is 100 points.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear order will receive very little credit. If extra space is needed, use the back of the previous page. Indicate the location of the work clearly and be neat.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations may still receive partial credit.
- **Provide exact answers** unless otherwise instructed.
- **If question directed, simplify all answers as much as possible.** This means that you need to need to combine like terms, reduce fractions, rationalize denominators, etc.
- **Be sure to state units for applied problems.**
- **Clearly identify your answer for each problem.**
- **For Question 7, please provide R codes and the outputs.**

1. (15 points) Suppose $E(X) = 2$, $\text{Var}(X) = 9$, $E(Y) = 0$, $\text{Var}(Y) = 4$, and $\text{Corr}(X, Y) = 0.25$. Find

   (a) $\text{Var}(X + Y)$

   (b) $\text{Cov}(X, X + Y)$

   (c) $\text{Corr}(X + Y, X - Y)$

2. (10 points) Consider the following model

$$Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.5e_{t-1}$$

   (a) Characterize this model as models in the ARMA(p,q) family, that is identify p and q.

   (b) Is this model causal? Is this a stationary model?

3. (20 points) For each of the following, find the mean function and the autocovariance function, then state if it is a stationary process. Here $\{e_t\}$ is an i.i.d. $N(0,1)$ and $E(t) = t$

   (a) $Y_t = t + e_3$

   (b) $Y_t = e_t e_{t-2}$

4. (10 points) Identify the following as specific ARMA models. That is, what are p, q, and what are the values of the parameters (the ϕ's and θ's)

$$Y_t = 0.5Y_{t-1} - 0.5Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$$

5. (20 points) Consider the following AR process

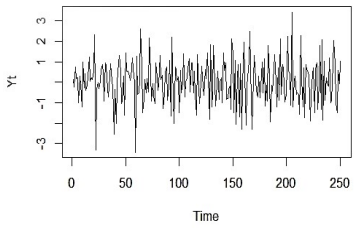
$$Y_t = 1.1Y_{t-1} - 0.18Y_{t-2} + e_t, \qquad e_t \sim WN(0, \sigma^2)$$

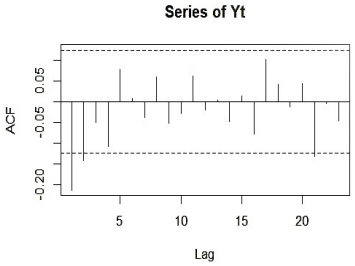
(a) Show that this series is stationary

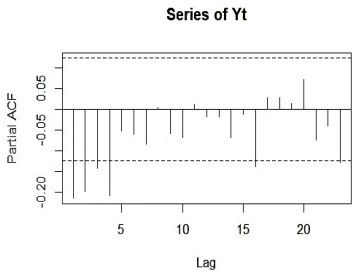
(b) Derive the autocovariance functions.

6. (10 points) Foe each case, identify an ARMA(p,q) model for the time series variable Y_t, given its ACF, PACF and EACF. Explain your reasoning for getting credits.

(a)







EACF

AR/MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	o	o	o	o	o	o	o	o	o	o	o	o
1	x	o	o	o	o	o	o	o	o	o	o	o	o	o
2	x	x	o	x	o	o	o	o	o	o	o	o	o	o
3	x	x	o	x	o	o	o	o	o	o	o	o	o	o
4	x	x	x	o	o	o	o	o	o	o	o	o	o	o
5	x	o	x	o	o	o	o	o	o	o	o	o	o	o
6	x	o	x	x	x	o	o	o	o	o	o	o	o	o
7	o	x	x	x	x	o	o	o	o	o	o	o	o	o

Mean is around zero, variance isn't changing which suggests a stationary process.

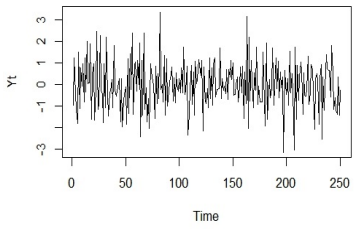
ACF tapers off instead of cutting off which suggests an AR components instead of pure MA.

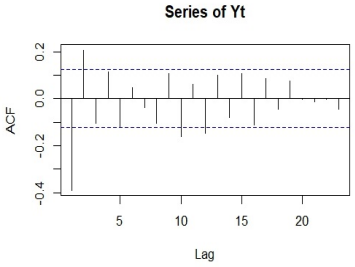
PACF also tapers off meaning it is likely not a pure AR process.

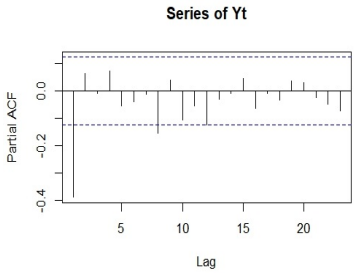
Given the above, the smallest clean triangle's vertex would be row 1, column 1. There are some X's but that could be noise.

Therefore a possible model would be ARMA(1, 1).

(b)







EACF

AR/MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	o	o	o	o	o	o	o	x	o	x	o	o
1	x	o	o	o	o	o	o	o	o	x	o	o	o	o
2	o	o	o	o	o	o	o	o	o	o	o	o	o	o
3	o	x	o	o	o	o	o	o	o	o	o	o	o	o
4	x	x	x	o	o	o	o	o	o	o	o	o	o	o
5	x	o	x	x	o	o	o	o	o	o	o	o	o	o
6	x	o	x	x	o	o	o	o	o	o	o	o	o	o
7	o	x	x	x	o	o	o	o	o	o	o	o	o	o

Mean is around zero, variance isn't changing which suggests a stationary process.

ACF tapers off instead of cutting off which suggests an AR components instead of pure MA.

PACF also tapers off meaning it is likely not a pure AR process.

Given the above, the smallest clean triangle's vertex would be row 1, column 1.

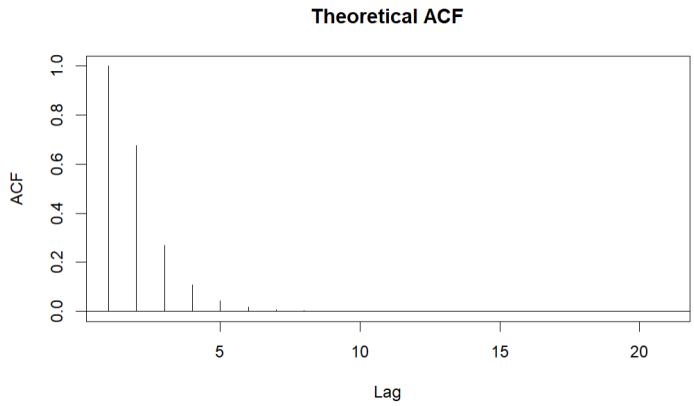
Therefore a possible model would be ARMA(1, 1).

7. (15 points) Simulate a mixed ARMA(1,1) model of length $n = 60$ with $\phi = 0.4$ and $\theta = 0.6$. Provide R codes and the outputs to achieve the following tasks.

   (a) Plot the theoretical autocorrelation function for this model. Plot sufficient lags until the correlations are negligible.

```
phi <- c(0.4)
theta <- c(0.6)
n <- 70

theoretical_acf <- ARMAacf(ar = phi,
                           ma = theta,
                           lag.max = 20)
plot(
  theoretical_acf,
  type = "h",
  main = "Theoretical ACF",
  xlab = "Lag",
  ylab = "ACF"
)
abline(h = 0)
```

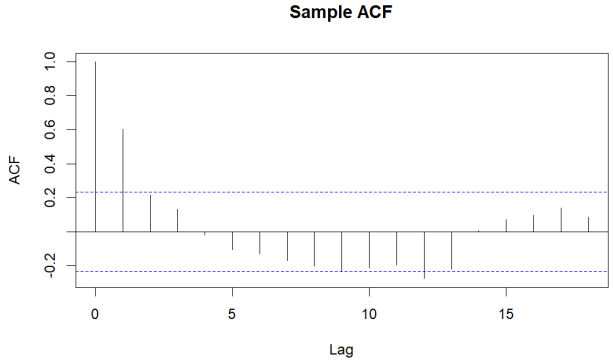


   (b) Plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)?

```
set.seed(123)

series <- arima.sim(
  n = n,
  model = list(ar = phi, ma = theta))

acf(series, main = "Sample ACF")
```



The sample ACF closely resembles the theoretical ACF in shape. The overall pattern of slow decay after significant values in lags 1 and 2 shows up in both plots.

   (c) Plot and interpret the sample EACF for this series. Does the EACF help you specify the correct orders for the model?

```
library(TSA)
eacf(series)
```

```
AR/MA
  0 1 2 3 4 5 6 7 8 9 10 11 12 13
0 x o o o o o o o o o o  x  o  o
1 x o o o o o o o o o o  o  o  o
2 x o x o o o o o o o o  o  o  o
3 x x x o o o o o o o o  o  o  o
4 x x x o o o o o o o o  o  o  o
5 x x o x o o o o o o o  o  o  o
6 o x o x o o o o o o o  o  o  o
7 o x x x o o o o o o o  o  o  o
```

Yes, there is a clear triangle starting at (1,1) => ARMA(1,1). There are a few X's which is probably due to the small sample size.