STA6856 **HW #4** **Instructor: Tharindu De Alwis**

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Student Name (print): ____________________________

Question:	1	2	3	4	Total
Points:	10	10	20	20	60
Score:					

1. (10 points) Consider the following time series model.

$$Y_t = 0.25y_{t-1} - 0.25Y_{t-12} + 0.0625Y_{t-13} + e_t - 0.1e_{t-1} + 0.1e_{t-12} \text{ - } 0.01e_{t-13}$$

   (a) Recognize the model as $ARIMA(p, d, q) \times (P, D, Q)$ model. That is, what are the values for p, d, q, P, D, and Q.

   (b) Write down all the coefficient values using the standard notation. That is, ϕ's, θ's, Φ's, and Θ's.

2. (10 points) An AR model has AR characteristic polynomial

$$(1 - 1.6x + 0.7x^2)(1 - 0.8x^{12})$$

   (a) Is th model stationary?

   (b) Identify the model as a certain seasonal ARIMA model.

3. (20 points) Consider *electricity* dataset in the *TSA* R package that contains the monthly electricity generated in the United States.

   (a) Construct the sample ACF of the data, is the data stationary?

   (b) Calculate the sample ACF of the first difference of the logged transformed series. Is the seasonality visible in this display? If so, what is your seasonal component?

   (c) Plot the time series of seasonal difference, use $s = 12$ or the s value you have suggested in part (b), and first difference of the logged series. Does a stationary model seem appropriate now?

   (d) Display the sample ACF of the series after a seasonal difference and a first difference have been taken of the logged series in part (d). What model(s) might you consider for the electricity series?

4. (20 points) Consider the air passenger miles time series in *TSA* R package. The file is named *airmiles*.

   (a) Create the sample ACF and PACF for the *airmiles* data. Suggest the candidate values for p, d, q, P, D, Q, and s.

   (b) Fit $ARIMA(0, 1, 1) \times (0, 1, 0)_{12}$ and assess its adequacy.

   (c) Use auto.arima{forecast} to select the parameters p, d, q, P, D, Q, and s.

   (d) Refit the selected model from part(c) using arima{stats} function and assess its adequacy.

① $Y_T = 0.25Y_{t-1} - 0.25Y_{t-12} + 0.0625Y_{t-13}$
$+e_t - 0.1e_{t-1} + 0.1e_{t-12} - 0.01e_{t-13}$

$Y_{t-1} \Rightarrow \phi_1 = 0.25 \rightarrow$ non-seasonal AR(1) $\Rightarrow p = 1$

$Y_{t-12} \Rightarrow \Phi_1 = -0.25 \rightarrow$ Seasonal AR(1) $\Rightarrow P = 1$

$e_{t-1} \Rightarrow \theta_1 = -0.1 \rightarrow$ nonseasonal MA(1) $\Rightarrow q = 1$

$e_{t-12} \Rightarrow \Theta_1 = 0.1 \rightarrow$ seasonal MA(1) $\Rightarrow Q = 1$

No differencing applied $\Rightarrow d = D = 0$

2A $1 - 1.6x + 0.7x^2 \Rightarrow$ get the roots $\Rightarrow$ $x = \frac{1.6\sqrt{(-1.6)^2 - 4(0.7)}}{2(0.7)} = \frac{1.6 \pm \sqrt{2.56 - 2.8}}{1.4}$

$= \frac{1.6 \pm \sqrt{-0.24}}{1.4} = \frac{1.6 \pm i\sqrt{0.24}}{1.4}$

$|x| = \frac{\sqrt{1.6^2 + 0.24}}{1.4} = \frac{\sqrt{2.56 + 0.24}}{1.4} \approx \frac{1.673}{1.4} \Rightarrow$ greater than 1 so stationary

$1 - 0.8x^{12} = 0 \Rightarrow \frac{-0.8x^{12}}{-0.8} = \frac{-1}{-0.8} \Rightarrow x^{12} = \frac{1}{0.8} \Rightarrow x = \left(\frac{1}{0.8}\right)^{\frac{1}{12}} \approx 1.019$

$\Rightarrow$ greater than 1 so stationary

Since all roots of polynomial lie outside of unit circle, model is stationary

2B $\phi_1 = 1.6 \quad \phi_2 = -0.7 \Rightarrow p = 2$

$\Phi_1 = 0.8 \Rightarrow P = 1$

no MA terms; no differencing $\Rightarrow d = D = 0$

$\Rightarrow$ ARIMA $(2,0,0) \times (1,0,0)_{12}$