STA6856 **HW #2** **Instructor: Tharindu De Alwis**

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Due on Tuesday, February 4th at 11:59 PM

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

1. (10 points) If X and Y are dependent but $\mathrm{Var}(X) = \mathrm{Var}(Y)$, find $\mathrm{Cov}(X+Y, X-Y)$

2. (10 points) Let $Y_1, Y_2, \cdots, Y_n$ be iid with $E(Y_i) = \mu$ and $\mathrm{Var}(Y_i) = \sigma^2$. Recall that $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Find $E[(\overline{Y} - E(\overline{Y}))^2]$

3. (10 points) Suppose $Y_t = X$ for all t where $E(X) = \mu$ and $\mathrm{Var}(X) = \sigma^2$.
   (a) Show that $\{Y_t\}$ is stationary.
   (b) Find the autocovariance function γ_k for $\{Y_t\}$.

4. (10 points) Recall: To show that a process $\{Y_t\}$ is not stationary, try to show that $E(Y_t)$ depends on t. If this fails, try to show that $\mathrm{Var}(Y_t)$ depends on t. If this fails, show that $\gamma_k = \mathrm{Cov}(Y_t, Y_{t-k})$ depends on t.
   (a) Let $Y_t = \sum_{i=1}^{t} e_t$ where the e_t are iid with $E(e_t) = \mu > 0$ and $\mathrm{Var}(e_t) = \sigma^2$. Show that $\{Y_t\}$ is not stationary.
   (b) Let $Y_t = \sum_{i=1}^{t} e_t$ where the e_t are iid with $E(e_t) = 0$ and $\mathrm{Var}(e_t) = \sigma^2$. Show that $\{Y_t\}$ is not stationary.

5. (10 points) Consider the $MA(3)$ model with $\theta_1 = 0.8$, $\theta_2 = 0.6$, and $\theta_3 = 0.4$.
   (a) Find the theoretical ACF.
   (b) Generate $n = 150$ observations of this $MA(3)$ time series. Then, plot the sample autocorrelation function (sample ACF).

(1) $Cov(X+Y, X-Y) = Cov(X,X) - Cov(X,Y) + Cov(Y,X) - Cov(Y,Y)$

Given $\boxed{\begin{array}{l} Cov(X,X) = Var(X) \\ Cov(Y,Y) = Var(Y) \end{array}}$

$= Var(X) - \cancel{Cov(X,Y)} + \cancel{Cov(X,Y)} - Var(Y)$

$= Var(X) - Var(Y)$

Given $\boxed{Var(X) = Var(Y)}$

therefore

$\boxed{Cov(X+Y, X-Y) = 0}$

(2) $E[(\bar{Y} - E(\bar{Y}))^2] = Var(\bar{Y})$

Given $\boxed{Y_1, Y_2 \ldots Y_n \text{ is iid with variance } \sigma^2}$

therefore $Var(\bar{Y}) = \dfrac{\sigma^2}{n}$

therefore

$\boxed{E[(\bar{Y} - E(\bar{Y}))^2] = \dfrac{\sigma^2}{n}}$

(3A) Given $\boxed{\begin{array}{l} Y_T = X \text{ for all } t \\ E(X) = \mu \\ Var(X) = \sigma^2 \end{array}}$

therefore $E(X) = E(Y_t) = \mu$ ← constant mean

$Var(X) = Var(Y_t) = \sigma^2$ ← constant variance

$Cov(X,X) = Cov(Y, Y_{t-k}) = \sigma^2$ ← constant covariance for all t, k

all conditions for stationarity are satisfied

(3B) $\gamma_K = Cov(Y_t, Y_{t-k})$

Given $\boxed{Y_T = X \text{ for all } t}$

$\boxed{\text{then } \gamma_K = Cov(X,X) = \sigma^2}$

4A $E[Y_t] = E\left[\sum_{i=1}^{t} e_t\right] = \sum_{i=1}^{t} E[e_t] = \mu t$ depends on t so not stationary

4B $E[Y_t] = E\left[\sum_{i=1}^{t} e_t\right] = \sum_{i=1}^{t} E[e_t] = 0$ constant so check variance

$\text{Var}(Y_t) = \text{Var}\left(\sum_{i=1}^{t} e_t\right) = \sum_{i=1}^{t} \text{Var}(e_t) = \sigma^2 t$ depends on t so not stationary

5A $Y_t = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3}$

e_t is iid with mean=0 and variance $= \sigma^2$

$\rho_k = \dfrac{\gamma_k}{\gamma_0}$ $\gamma_0 = \text{Var}(Y_t)$

$\gamma_0 = \sigma^2(1 + 0.8^2 + 0.6^2 + 0.4^2) = 2.16\sigma^2$

$\gamma_1 = \sigma^2(\theta_1 + \theta_1\theta_2 + \theta_2\theta_3) = \sigma^2(0.8 + (0.8 \cdot 0.6) + (0.6 \cdot 0.4))$
$= 1.52\sigma^2$

$\gamma_2 = \sigma^2(\theta_2 + \theta_1\theta_3) = \sigma^2(0.6 + (0.8 \cdot 0.4)) = 0.92\sigma^2$

$\gamma_3 = \sigma^2\theta_3 = 0.4\sigma^2$

$\rho_0 = 1$

$\rho_1 = \dfrac{1.52\sigma^2}{2.16\sigma^2} \approx .704\sigma^2$

$\rho_2 = \dfrac{0.92\sigma^2}{2.16\sigma^2} = .426\sigma^2$

$\rho_3 = \dfrac{0.4\sigma^2}{2.16\sigma^2} \approx .185\sigma^2$

$\rho_k = 0$ for all $k > 3$

HW 2

Farooq Mahmud

Problem 5b

```
set.seed(42)
theta <- c(0.8, 0.6, 0.4)

ma3_series <- arima.sim(n = 150, model = list(ma = theta))
acf(ma3_series)
```