**STA6856** **HW #3** **Instructor: Tharindu De Alwis**

**Student Name (print):** Farooq Mahmud

**Due on Tuesday, March 4th at 11:59 PM**

| Question: | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Points: | 10 | 10 | 10 | 10 | 20 | 60 |
| Score: | | | | | | |

1. (10 points) Identify the following as specific ARMA models. That is, what are $p$, $q$, and what are the values of the parameters (the $\phi$'s and $\theta$'s)
   (a) $Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$
   (b) $Y_t = 0.5Y_{t-1} - 0.5Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

2. (10 points) Consider the following time series model
$$Y_t = 0.64Y_{t-2} + e_t - 0.8e_{t-1}$$
   (a) Characterize this model as models in the ARMA(p,q) family, that is identify $p$ and $q$.
   (b) Is this model causal? Is this model a stationary model?

3. (10 points) Consider the following time series model
$$(1 - 0.8B)(1 - 1.2B)(1 - B)Y_t = e_t$$
   (a) Is this model can be characterized as models in the ARMA(p,q) family? if so, then identify p and q.
   (b) Is this series stationary?

4. (10 points) Suppose that $Y_t$ follows the ARMA(1,1) model, $(1-\phi B)Y_t = (1-\theta B)e_t$, where $e_t$ is a white noise. Let $X_t = (1 - \gamma B)Y_t$. What is the model for $X_t$, that is find $p$ and $q$.

5. (20 points) Simulate an AR(2) time series of length $n = 72$ with $\phi_1 = 0.7$ and $\phi_2 = -0.4$.
   (a) Calculate and plot the theoretical autocorrelation function for this model. Plot sufficient lags until the correlations are negligible. Hint: use ARMAacf() function.
   (b) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF of part (a)?
   (c) What are the theoretical partial autocorrelations for this model? Hint: use ARMAacf(...,pacf = TRUE) function.
   (d) Calculate and plot the sample PACF for your simulated series. How well do the values and patterns match the theoretical ACF of part (c)?

1A $Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$

$p=2 \quad q=1 \quad \phi_1 = 1 \quad \phi_2 = 0.25$

$\theta_1 = -0.1$

1B $Y_t = 0.5Y_{t-1} - 0.5Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

$p=2 \quad q=2 \quad \phi_1 = -0.5 \quad \phi_2 = 0.5$

$\theta_1 = -0.5 \quad \theta_2 = 0.25$

2A $Y_t = 0.64Y_{t-2} + e_t - 0.8e_{t-1}$

ARMA(2,1)

2B $\phi(B) = 1 - 0.64B^2$

$1 - 0.64B^2 = 0$

$1 = 0.64B^2$

$\frac{1}{0.64} = B^2$

$\sqrt{\frac{1}{0.64}} = B$

$\boxed{\pm 1.25 = B}$ ⇒ causal because roots are $> 1$
also stationary for same reason

(3A) $(1-0.8B)(1-1.2B)(1-B)Y_t = e_t$

$\phi(B)Y_t = \theta(B)e_t$

$\phi(B) = (1-0.8B)(1-1.2B)(1-B)$

$\theta(B) = 1 \Rightarrow$ $\boxed{\text{Not an ARMA model because } (1-B) \text{ is unit root}}$

(3B) Not stationary because $(1-B)$ is a unit root

(4) $(1-\phi B)Y_t = (1-\theta B)e_t \qquad X_t = (1-\gamma B)Y_t$

$Y_T = \dfrac{1-\theta B}{1-\phi B} e_t$

$$\boxed{\begin{aligned} X_t &= (1-\gamma B)\left(\frac{1-\theta B}{1-\phi B}\right)e_t \\ p &= 1 \\ q &= 2 \end{aligned}}$$

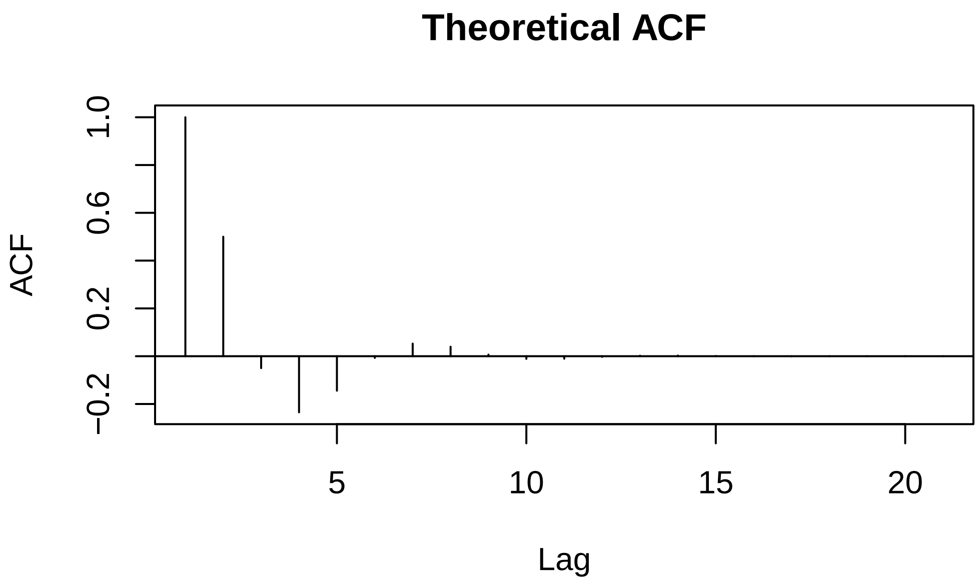
# HW3

Farooq Mahmud

## Problem 5a

```
phi <- c(0.7, -0.4)

theoretical_acf <- ARMAacf(ar = phi,
                           ma = numeric(0),
                           lag.max = 20)
plot(
  theoretical_acf,
  type = "h",
  main = "Theoretical ACF",
  xlab = "Lag",
  ylab = "ACF"
)
abline(h = 0)
```

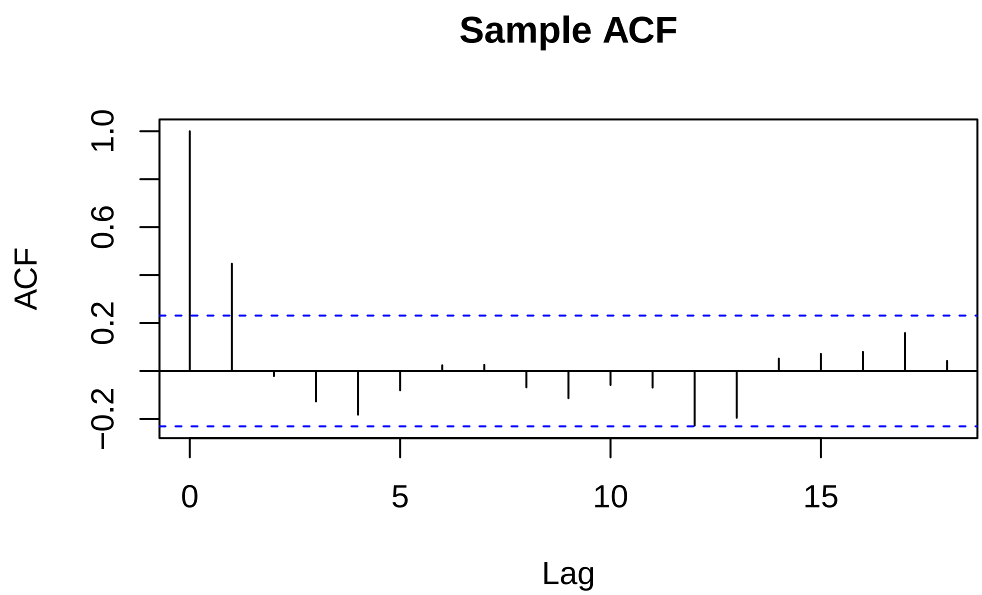



**Problem 5b**

```
set.seed(123)

phi <- c(0.7, -0.4)
n <- 72
ar2_series <- arima.sim(n = n, model = list(ar = phi))

acf(ar2_series, main = "Sample ACF")
```

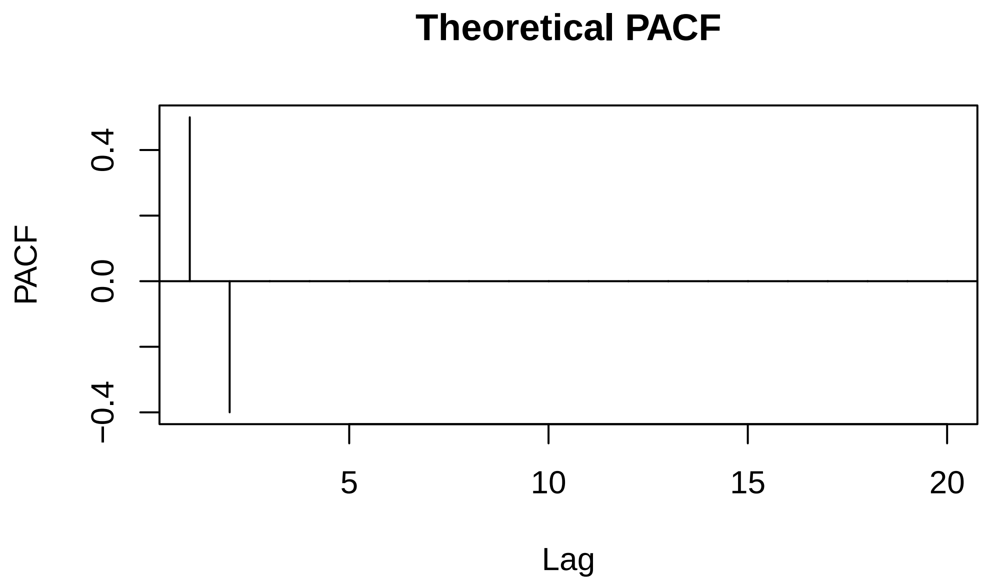



The sample ACF closely resembles the theoretical ACF in shape. The overall pattern of slow decay after significant values in lags 1 and 2 shows up in both plots which is congruent with this being an AR(2) time series.

### Problem 5c

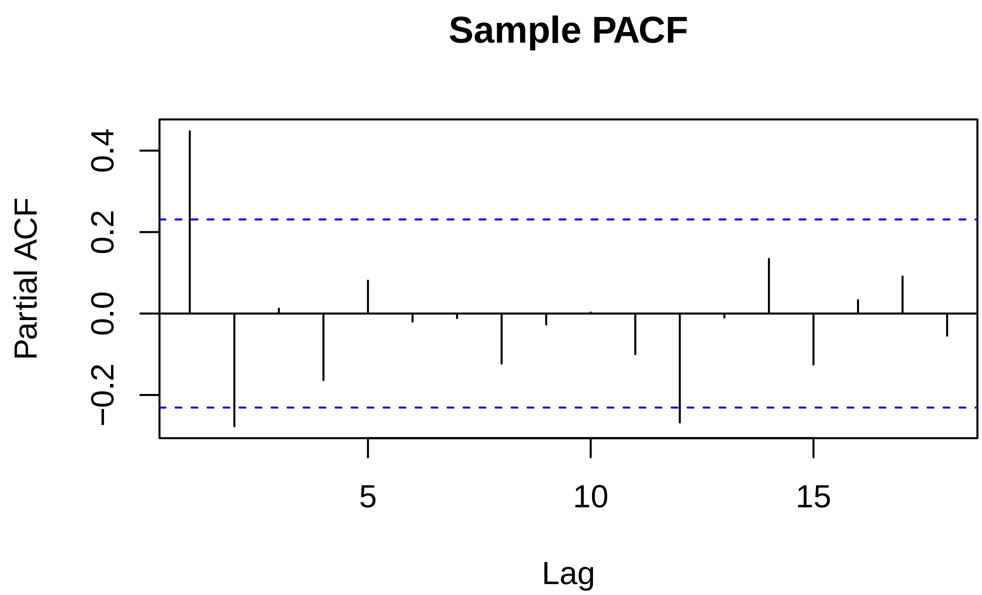
```
theoretical_pacf <- ARMAacf(
  ar = phi,
  ma = numeric(0),
  lag.max = 20,
  pacf = TRUE
)

plot(
  theoretical_pacf,
  type = "h",
  main = "Theoretical PACF",
  xlab = "Lag",
  ylab = "PACF"
)
abline(h = 0)
```




**Problem 5d**

```
pacf(ar2_series, main = "Sample PACF")
```




The sample PACF matches well with the theoretical ACF. There are significant spikes at lags 1 and 2 and insignificant values after. This is congruent with this being an AR(2) time series.