# Operations Research - Assignment 1 Report

## **Team Members:**

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## 1. Introduction

This report presents the implementation of a Linear Programming (LP) solver using the Simplex Method and its variations. The solver is designed to handle different types of LP problems efficiently and provides a step-by-step solution process.

# 2. Objective

The objective of this assignment is to develop a software tool capable of solving LP problems using:

- **Standard Simplex Method** for standard form LP problems (≤ constraints, non-negative variables).
- BIG-M Method to handle "greater-than-or-equal-to" (≥) and equality
  (=) constraints.
- Two-Phase Method as an alternative to the BIG-M method for artificial variables.
- **Preemptive Method for Goal Programming** for multi-objective optimization.

Additionally, support for unrestricted variables has been included.

# 3. Implementation Details

# 3.1 Programming Language

The solver was implemented using **Python**.

# 3.2 Input Format

The program accepts LP problems in the following standard format:

- Objective function coefficients
- Constraint coefficients
- · Right-hand side values
- Constraint types  $(\leq, \geq, =)$
- Variable restrictions (non-negative, unrestricted)
- Chosen method (BIG-M or Two-Phase, if applicable)
- Goal values and priority levels for goal programming

# 3.3 Output Format

The program provides:

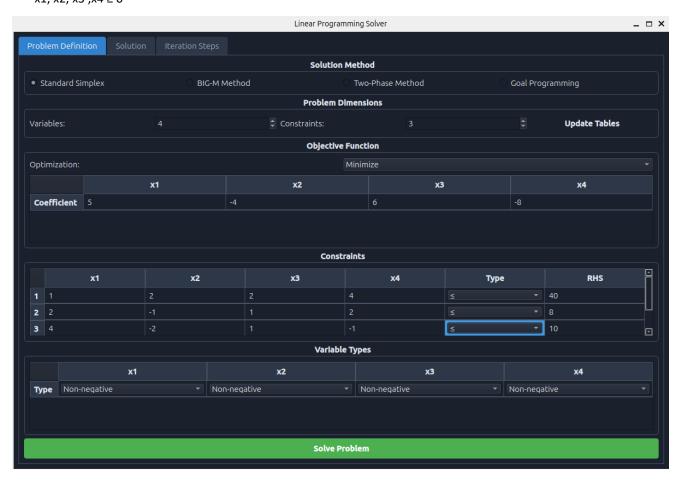
- Optimal solution (values of decision variables)
- Optimal objective function value
- Problem status (optimal, infeasible, unbounded)
- Goal satisfaction (for goal programming)
- Step-by-step solution tables

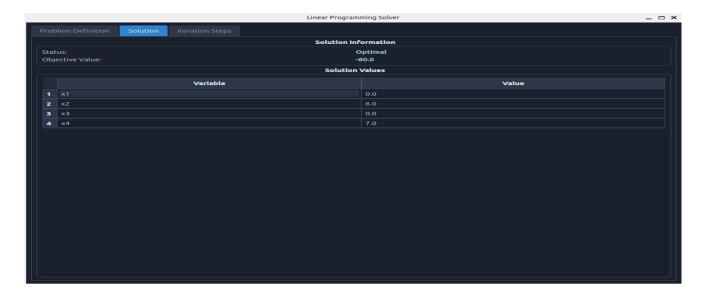
# 4. Sample Runs

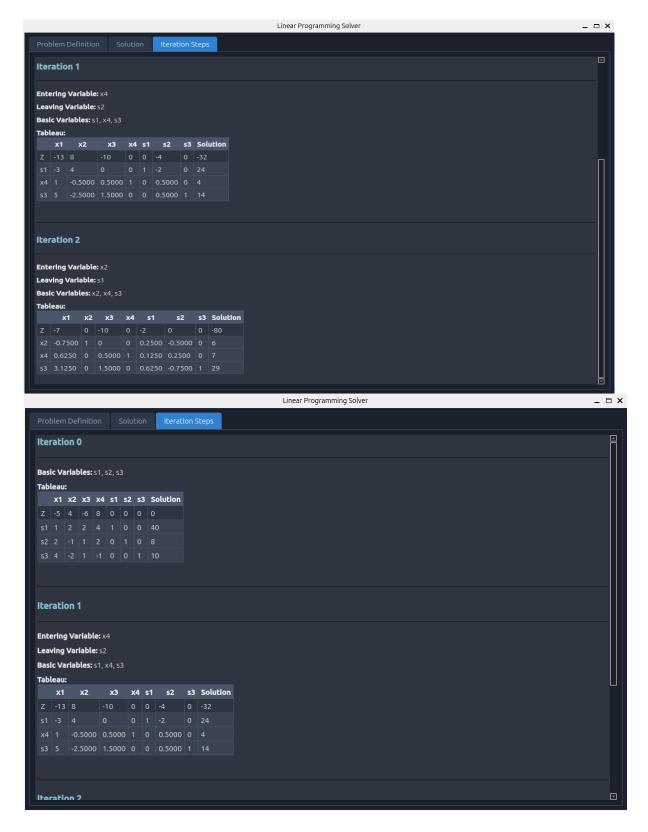
Below is example cases solved using our program:

# 4.1 Example 1 - Simplex Method

Minimize Z = 5x1 - 4x2 + 6x3 - 8x4  $x1 + 2x2 + 2x3 + 4x4 \le 40$   $2x1 - x2 + x3 + 2x4 \le 8$   $4x1 - 2x2 + x3 - x4 \le 8$   $2x1 - x2 + x3 + 2x4 \le 10$  $x1, x2, x3, x4 \ge 0$ 



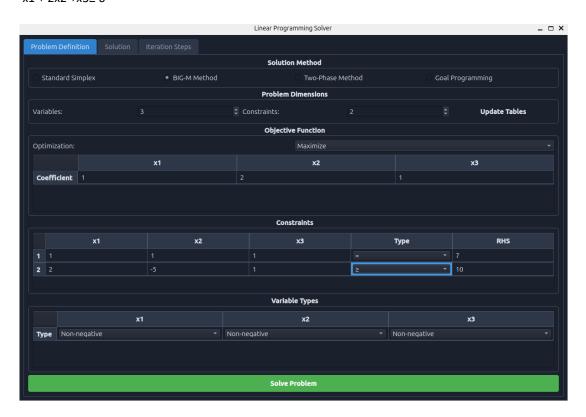


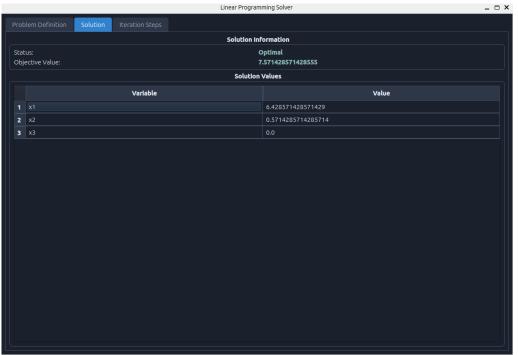


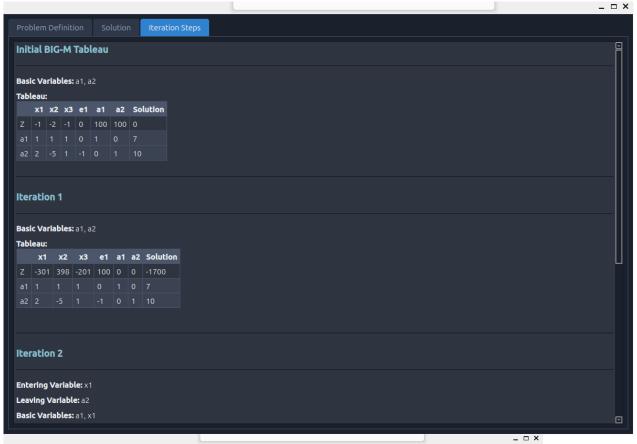
- Optimal Solution: x1 = 0, x2 = 6, x3 = 0, x4 = 7
- Optimal Objective Value: Z = -80

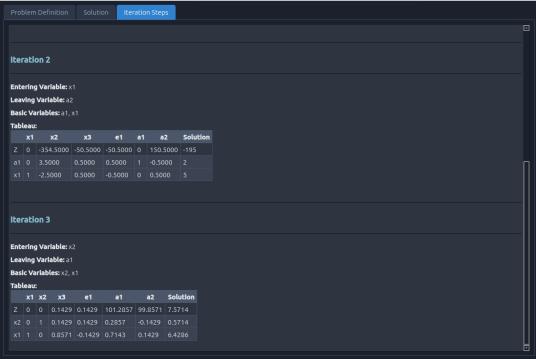
## 4.2 Example 2 - Big-M Method

Maximize Z = x1 + 2x2 + x3 x1 + x2 + x3 = 7 2x1 + 5x2 + x3 = 10 $x1 + 2x2 + x3 \ge 0$ 





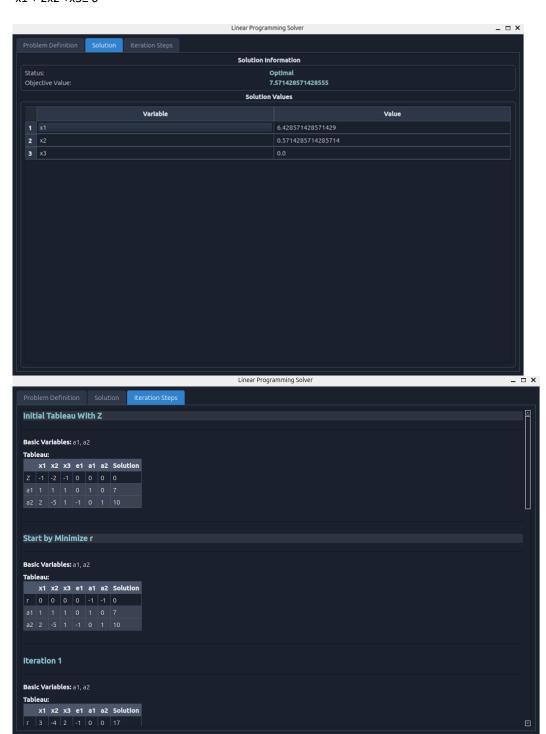


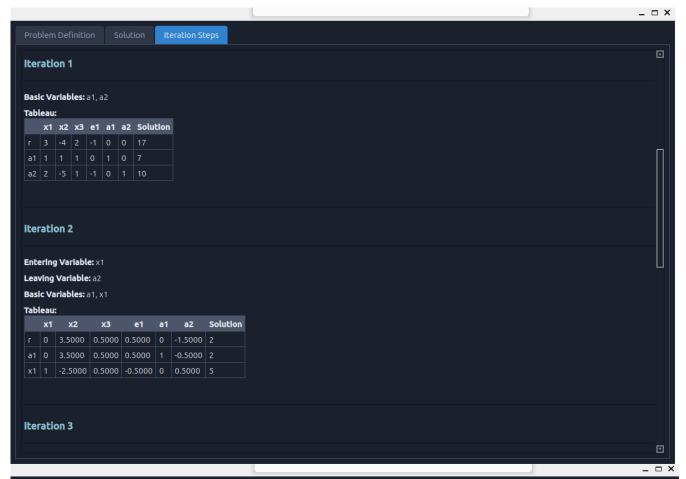


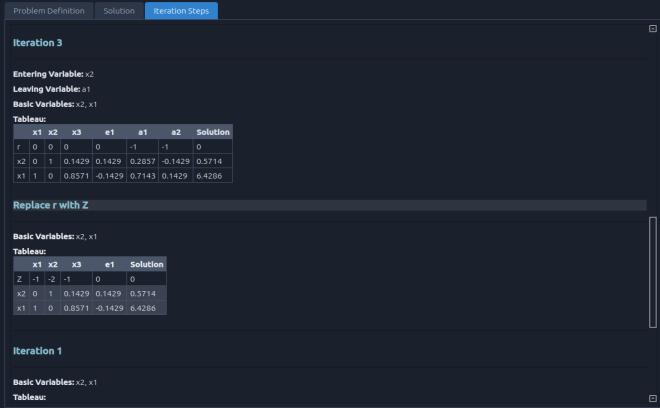
- Optimal Solution: x1 = 45/7, x2 = 4/7
- Optimal Objective Value: Z = 53/7

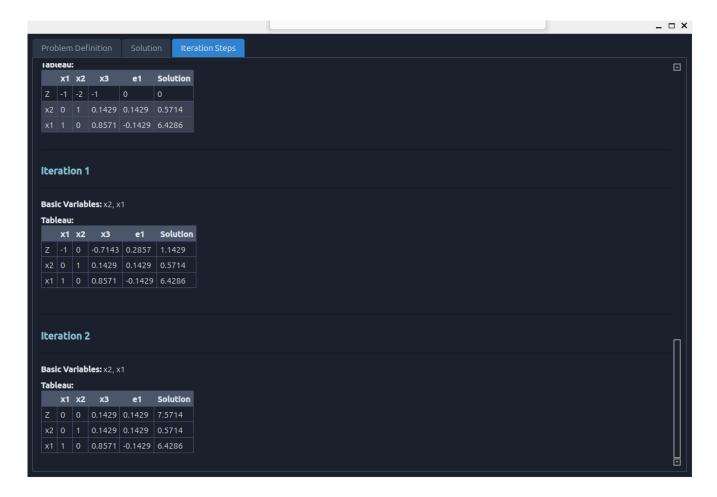
### 4.3 Example 3 - Two-Phase Method

Maximize Z = x1 + 2x2 + x3 x1 + x2 + x3 = 7 2x1 + 5x2 + x3 = 10 $x1 + 2x2 + x3 \ge 0$ 









• Optimal Solution: x1 = 45/7, x2 = 4/7

• Optimal Objective Value: Z = 53/7

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# 4.4 Example 4 - Goal-programming Method

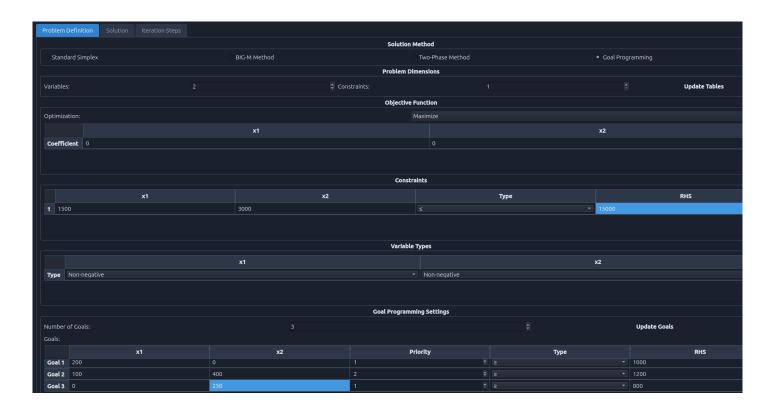
Goals: 200x1>=1000 priority:1

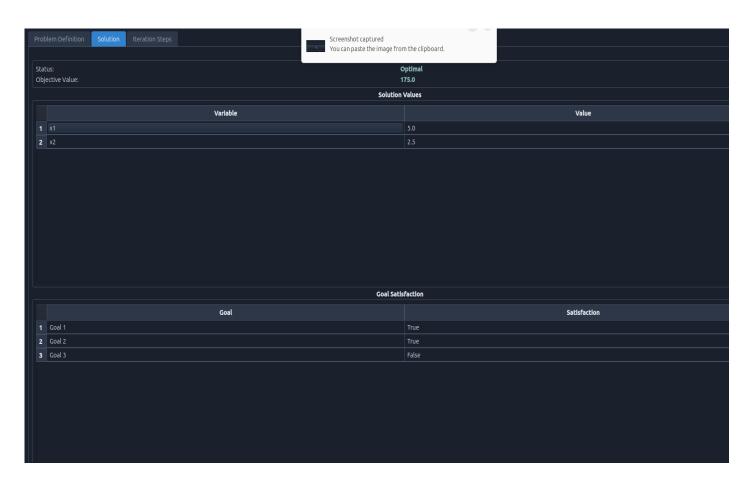
100x1+ 400x2 >=1200 priority:2

250x2 >= 800 priority:1

Constraints:

1500x1+3000x2<=15000





3 a a . b . a 3 . a . . . . . . . . . . . .

Basic Variables: d1-, d2-, d3-, s1

Tableau:

	<b>x1</b>	x2	d1-	d1+	d2-	d2+	d3-	d3+	<b>s1</b>	Solution
Z1	0			0	0	0	0		0	0
Z2			0	0	-2	0	0			0
Z3			0	0	0	0				0
d1-	200									1000
d2-	100	400								1200
d3-		250								800
	1500	3000								15000

#### Iteration 1

Basic Variables: d1-, d2-, d3-, s1

Tableau:

	<b>x1</b>	x2	d1-	d1+	d2-	d2+	d3-	d3+	<b>s1</b>	Solution
Z1	200		0		0					1000
Z2	200	800	0	0	0	-2	0			2400
Z3		250	0	0	0		0		0	800
d1-	200				0					1000
d2-	100	400	0	0	1		0			1200
d3-		250	0	0	0		1			800
s1	1500	3000	0	0	0		0		1	15000

#### Iteration 2

Entering Variable: x2

Leaving Variable: d2-

Basic Variables: d1-, x2, d3-, s1

Tableau:

	x1 x2 d1-		d1-	d1+	d2-	d2+	d3-	d3+	<b>s1</b>	Solution
Z1	200	0	0	-1	0 0		0		0	1000
Z2	0		0	0	-2	0	0		0	0
Z3	-62.5000		0	0	-0.6250	-0.6250 0.6250			0	50
d1-	200	0	1	-1	0	0	0		0	1000
x2	0.2500		0	0	0.0025	-0.0025	0		0	
d3-	-62.5000		0	0	-0.6250	0.6250			0	50
s1	750	0	0	0	-7.5000	7.5000	0		1	6000

Problem Definition

Solution

Iteration Steps

Tableau:

	x1	X2	d1-	d1+	d2-	d2+	d3-	d3+	<b>s1</b>	Solution
Z1	200						0	0		1000
Z2					-2		0			
Z3	-62.5000				-0.6250	0.6250	0			50
d1-	200						0			1000
x2	0.2500				0.0025	-0.0025	0			
d3-	-62.5000				-0.6250	0.6250				50
s1	750		0		-7.5000	7.5000	0	0		6000

#### Iteration 3

Entering Variable: x1

Leaving Variable: d1-

Basic Variables: x1, x2, d3-, s1

Tableau:

	х1	X2	d1-	d1+	d2-	d2+	d3-	d3+	<b>s1</b>	Solution
Z1				0		0	0	0	0	
Z2				0	-2	0	0	0	0	
Z3			0.3125	-0.3125	-0.6250	0.6250	0		0	362.5000
x1			0.0050	-0.0050	0	0	0	0	0	
x2			-0.0013	0.0013	0.0025	-0.0025	0	0	0	1.7500
d3-			0.3125	-0.3125	-0.6250	0.6250	1	-1	0	362.5000
s1			-3.7500	3.7500	-7.5000	7.5000	0	0	1	2250

### Iteration 4

Entering Variable: d2+

Leaving Variable: s1

Basic Variables: x1, x2, d3-, d2+

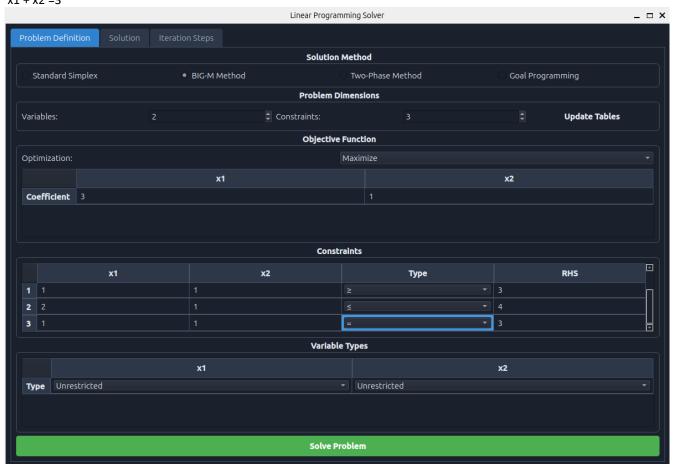
Tableau:

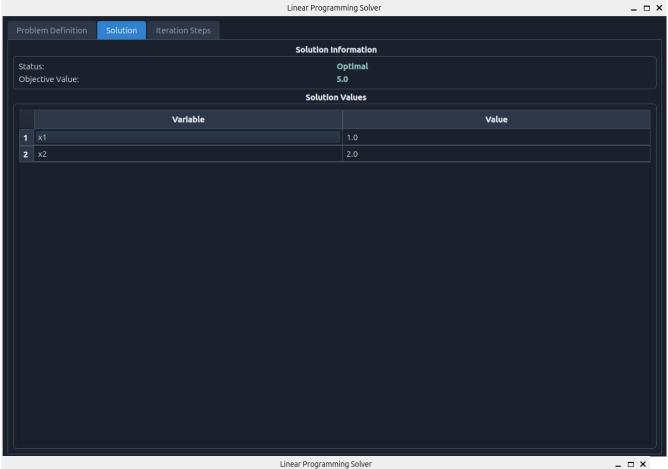
	x1	X2	d1-	d1+	d2-	d2+	d3-	d3+	<b>s1</b>	Solution
Z1				0	0	0				
Z2			0	0	-2	0				
Z3			0.6250	-0.6250	0	0			-0.0833	175
x1			0.0050	-0.0050	0	0	0		0	5
x2			-0.0025	0.0025	0	0	0		0.0003	2.5000
d3-			0.6250	-0.6250	0	0			-0.0833	175
d2+		0	-0.5000	0.5000	-1	1	0		0.1333	300

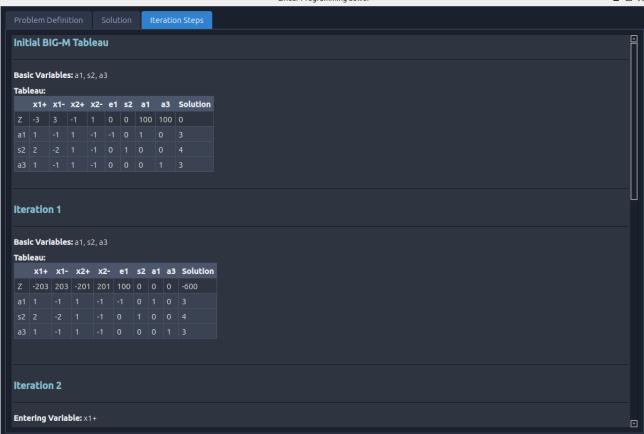
• Optimal Solution: x1 = 5, x2 = 2.5

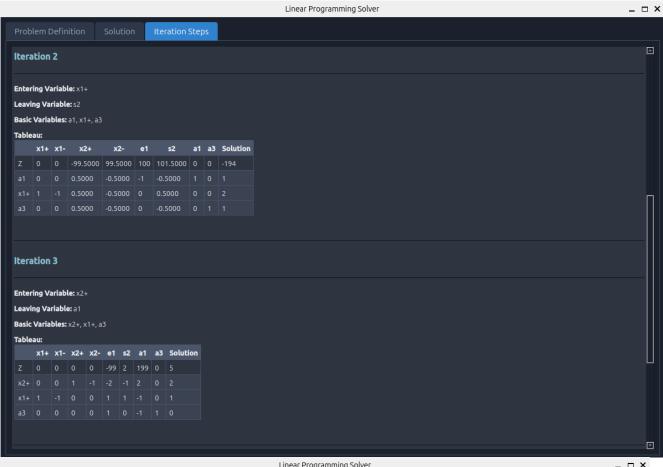
## 4.4 Example 4 – Unrestricted variables

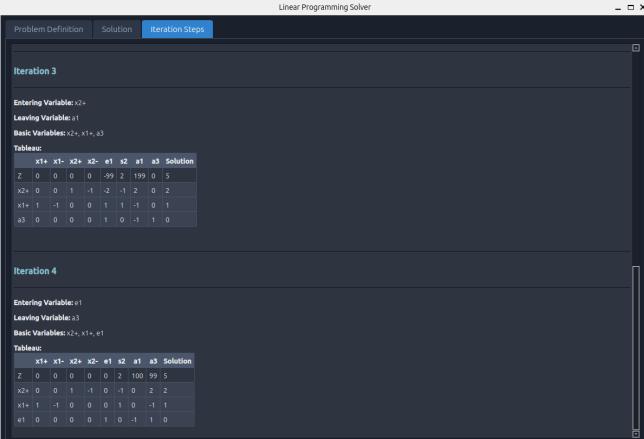
Maximize  $Z = 3 \times 1 + x2$   $x1 + x2 \ge 7$   $2x1 + x2 \le 4$ x1 + x2 = 3







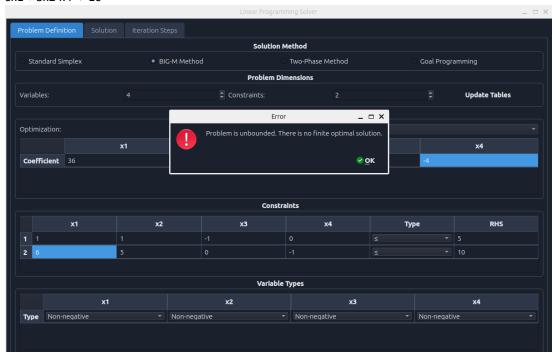


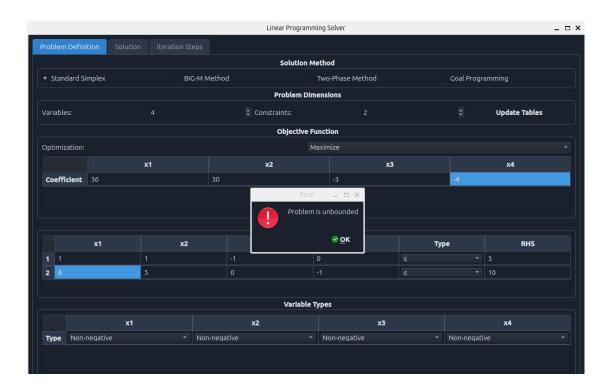


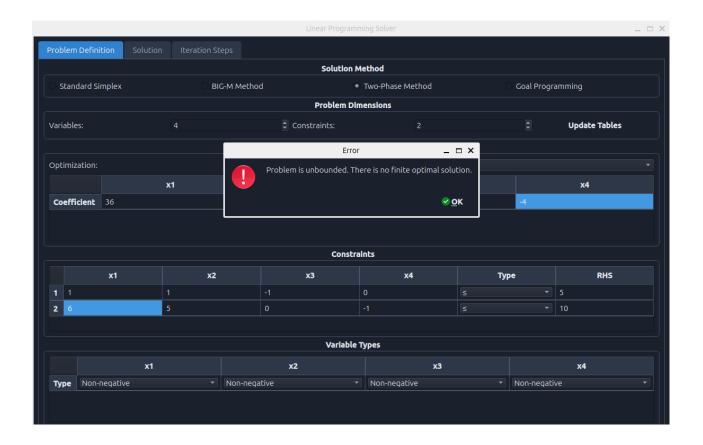
## 4.5 Example 5 – unbounded solution

Maximize  $Z = 36 \times 1 + 30 \times 2 - 3 \times 3 - 4 \times 4 \times 1 + x^2 - x^3 < 5$ 

6x1 + 5x2-x4 <=10







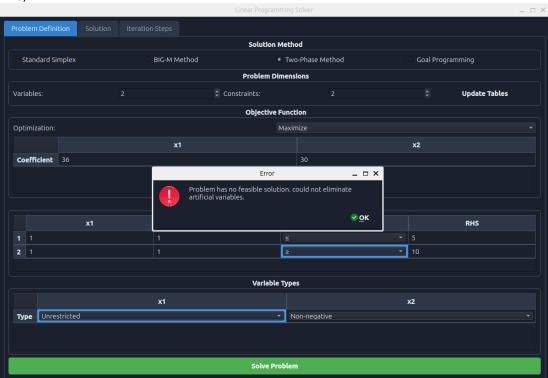
## 4.6 Example 6 – infeasible solution

Maximize  $Z = 36 \times 1 + 30 \times 2$ 

x1 + x2 ≥10

x1 + x2 <= 5

x2>0, x1



### 5. Bonus Feature

We developed a **user-friendly interface** by pyQt5 in python that allows users to input LP problems easily and view the solution process interactively.

### 6. Conclusion

This project provided hands-on experience in solving LP problems using different methods. The solver successfully handles various constraints and outputs detailed step-by-step solutions. Future improvements include extending support for additional optimization techniques and graphical visualization of results.