

Operations Research - Assignment 1 Report

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1. Introduction

This report presents the implementation of a Linear Programming (LP) solver using the Simplex Method and its variations. The solver is designed to handle different types of LP problems efficiently and provides a step-by-step solution process.

2. Objective

The objective of this assignment is to develop a software tool capable of solving LP problems using:

- **Standard Simplex Method** for standard form LP problems (\leq constraints, non-negative variables).
- **BIG-M Method** to handle "greater-than-or-equal-to" (\geq) and equality ($=$) constraints.
- **Two-Phase Method** as an alternative to the BIG-M method for artificial variables.
- **Preemptive Method for Goal Programming** for multi-objective optimization.

Additionally, support for unrestricted variables has been included.

3. Implementation Details

3.1 Programming Language

The solver was implemented using **Python**.

3.2 Input Format

The program accepts LP problems in the following standard format:

- Objective function coefficients
- Constraint coefficients
- Right-hand side values
- Constraint types (\leq , \geq , $=$)
- Variable restrictions (non-negative, unrestricted)
- Chosen method (BIG-M or Two-Phase, if applicable)
- Goal values and priority levels for goal programming

3.3 Output Format

The program provides:

- **Optimal solution** (values of decision variables)
- **Optimal objective function value**
- **Problem status** (optimal, infeasible, unbounded)
- **Goal satisfaction (for goal programming)**
- **Step-by-step solution tables**

4. Sample Runs

Below is example cases solved using our program:

4.1 Example 1 - Simplex Method

Minimize $Z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

$x_1 + 2x_2 + 2x_3 + 4x_4 \leq 40$

$2x_1 - x_2 + x_3 + 2x_4 \leq 8$

$4x_1 - 2x_2 + x_3 - x_4 \leq 8$

$2x_1 - x_2 + x_3 + 2x_4 \leq 10$

$x_1, x_2, x_3, x_4 \geq 0$

Linear Programming Solver

Problem Definition Solution Iteration Steps

Solution Method

☒ Standard Simplex ☐ BIG-M Method ☐ Two-Phase Method ☐ Goal Programming

Problem Dimensions

Variables: 4 Constraints: 3 **Update Tables**

Objective Function

Optimization: Minimize

	x1	x2	x3	x4
Coefficient	5	-4	6	-8

Constraints

	x1	x2	x3	x4	Type	RHS
1	1	2	2	4	\leq	40
2	2	-1	1	2	\leq	8
3	4	-2	1	-1	\leq	10

Variable Types

	x1	x2	x3	x4
Type	Non-negative	Non-negative	Non-negative	Non-negative

Solve Problem

Linear Programming Solver

Problem Definition **Solution** Iteration Steps

Solution Information

Status: Optimal
Objective Value: -80.0

Solution Values

	Variable	Value
1	x1	0.0
2	x2	6.0
3	x3	0.0
4	x4	7.0

Linear Programming Solver

Problem Definition Solution **Iteration Steps**

Iteration 1

Entering Variable: x4
Leaving Variable: s2
Basic Variables: s1, x4, s3

Tableau:

	x1	x2	x3	x4	s1	s2	s3	Solution
Z	-13	8	-10	0	0	-4	0	-32
s1	-3	4	0	0	1	-2	0	24
x4	1	-0.5000	0.5000	1	0	0.5000	0	4
s3	5	-2.5000	1.5000	0	0	0.5000	1	14

Iteration 2

Entering Variable: x2
Leaving Variable: s1
Basic Variables: x2, x4, s3

Tableau:

	x1	x2	x3	x4	s1	s2	s3	Solution
Z	-7	0	-10	0	-2	0	0	-80
x2	-0.7500	1	0	0	0.2500	-0.5000	0	6
x4	0.6250	0	0.5000	1	0.1250	0.2500	0	7
s3	3.1250	0	1.5000	0	0.6250	-0.7500	1	29

Linear Programming Solver

Problem Definition Solution **Iteration Steps**

Iteration 0

Basic Variables: s1, s2, s3

Tableau:

	x1	x2	x3	x4	s1	s2	s3	Solution
Z	-5	4	-6	8	0	0	0	0
s1	1	2	2	4	1	0	0	40
s2	2	-1	1	2	0	1	0	8
s3	4	-2	1	-1	0	0	1	10

Iteration 1

Entering Variable: x4
Leaving Variable: s2
Basic Variables: s1, x4, s3

Tableau:

	x1	x2	x3	x4	s1	s2	s3	Solution
Z	-13	8	-10	0	0	-4	0	-32
s1	-3	4	0	0	1	-2	0	24
x4	1	-0.5000	0.5000	1	0	0.5000	0	4
s3	5	-2.5000	1.5000	0	0	0.5000	1	14

Iteration 2

Output:

- Optimal Solution: $x_1 = 0$, $x_2 = 6$, $x_3 = 0$, $x_4 = 7$
- Optimal Objective Value: $Z = -80$

4.2 Example 2 - Big-M Method

Maximize $Z = x_1 + 2x_2 + x_3$

$x_1 + x_2 + x_3 = 7$

$2x_1 + 5x_2 + x_3 = 10$

$x_1 + 2x_2 + x_3 \geq 0$

Linear Programming Solver

Problem Definition Solution Iteration Steps

Solution Method

☐ Standard Simplex ☒ BIG-M Method ☐ Two-Phase Method ☐ Goal Programming

Problem Dimensions

Variables: 3 Constraints: 2 [Update Tables](#)

Objective Function

Optimization: Maximize

	x1	x2	x3
Coefficient	1	2	1

Constraints

	x1	x2	x3	Type	RHS
1	1	1	1	=	7
2	2	-5	1	≥	10

Variable Types

	x1	x2	x3
Type	Non-negative	Non-negative	Non-negative

[Solve Problem](#)

Linear Programming Solver

Problem Definition **Solution** Iteration Steps

Solution Information

Status: Optimal
Objective Value: 7.571428571428555

Solution Values

	Variable	Value
1	x1	6.428571428571429
2	x2	0.5714285714285714
3	x3	0.0

Problem Definition Solution **Iteration Steps**

Initial BIG-M Tableau

Basic Variables: a1, a2

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
Z	-1	-2	-1	0	100	100	0
a1	1	1	1	0	1	0	7
a2	2	-5	1	-1	0	1	10

Iteration 1

Basic Variables: a1, a2

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
Z	-301	398	-201	100	0	0	-1700
a1	1	1	1	0	1	0	7
a2	2	-5	1	-1	0	1	10

Iteration 2

Entering Variable: x1
Leaving Variable: a2
Basic Variables: a1, x1

Problem Definition Solution **Iteration Steps**

Iteration 2

Entering Variable: x1
Leaving Variable: a2
Basic Variables: a1, x1

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
Z	0	-354.5000	-50.5000	-50.5000	0	150.5000	-195
a1	0	3.5000	0.5000	0.5000	1	-0.5000	2
x1	1	-2.5000	0.5000	-0.5000	0	0.5000	5

Iteration 3

Entering Variable: x2
Leaving Variable: a1
Basic Variables: x2, x1

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
Z	0	0	0.1429	0.1429	101.2857	99.8571	7.5714
x2	0	1	0.1429	0.1429	0.2857	-0.1429	0.5714
x1	1	0	0.8571	-0.1429	0.7143	0.1429	6.4286

Output:

- Optimal Solution: $x_1 = 45/7$, $x_2 = 4/7$
- Optimal Objective Value: $Z = 53/7$

4.3 Example 3 - Two-Phase Method

Maximize $Z = x_1 + 2x_2 + x_3$

$x_1 + x_2 + x_3 = 7$

$2x_1 + 5x_2 + x_3 = 10$

$x_1 + 2x_2 + x_3 \geq 0$

Linear Programming Solver

Problem DefinitionSolutionIteration Steps

Solution Information

Status:Optimal

Objective Value:7.571428571428555

Solution Values

	Variable	Value
1	x1	6.428571428571429
2	x2	0.5714285714285714
3	x3	0.0

Linear Programming Solver

Problem DefinitionSolutionIteration Steps

Initial Tableau With Z

Basic Variables: a1, a2

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
Z	-1	-2	-1	0	0	0	0
a1	1	1	1	0	1	0	7
a2	2	-5	1	-1	0	1	10

Start by Minimize r

Basic Variables: a1, a2

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
r	0	0	0	0	-1	-1	0
a1	1	1	1	0	1	0	7
a2	2	-5	1	-1	0	1	10

Iteration 1

Basic Variables: a1, a2

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
r	3	-4	2	-1	0	0	17

Problem Definition

Solution

Iteration Steps

Iteration 1

Basic Variables: a1, a2

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
r	3	-4	2	-1	0	0	17
a1	1	1	1	0	1	0	7
a2	2	-5	1	-1	0	1	10

Iteration 2

Entering Variable: x1

Leaving Variable: a2

Basic Variables: a1, x1

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
r	0	3.5000	0.5000	0.5000	0	-1.5000	2
a1	0	3.5000	0.5000	0.5000	1	-0.5000	2
x1	1	-2.5000	0.5000	-0.5000	0	0.5000	5

Iteration 3

Problem Definition

Solution

Iteration Steps

Iteration 3

Entering Variable: x2

Leaving Variable: a1

Basic Variables: x2, x1

Tableau:

	x1	x2	x3	e1	a1	a2	Solution
r	0	0	0	0	-1	-1	0
x2	0	1	0.1429	0.1429	0.2857	-0.1429	0.5714
x1	1	0	0.8571	-0.1429	0.7143	0.1429	6.4286

Replace r with Z

Basic Variables: x2, x1

Tableau:

	x1	x2	x3	e1	Solution
Z	-1	-2	-1	0	0
x2	0	1	0.1429	0.1429	0.5714
x1	1	0	0.8571	-0.1429	6.4286

Iteration 1

Basic Variables: x2, x1

Tableau:

Problem Definition						Solution						Iteration Steps					
Tableau:																	
	x1	x2	x3	e1	Solution												
Z	-1	-2	-1	0	0												
x2	0	1	0.1429	0.1429	0.5714												
x1	1	0	0.8571	-0.1429	6.4286												
Iteration 1																	
Basic Variables: x2, x1																	
Tableau:																	
	x1	x2	x3	e1	Solution												
Z	-1	0	-0.7143	0.2857	1.1429												
x2	0	1	0.1429	0.1429	0.5714												
x1	1	0	0.8571	-0.1429	6.4286												
Iteration 2																	
Basic Variables: x2, x1																	
Tableau:																	
	x1	x2	x3	e1	Solution												
Z	0	0	0.1429	0.1429	7.5714												
x2	0	1	0.1429	0.1429	0.5714												
x1	1	0	0.8571	-0.1429	6.4286												

Output:

- Optimal Solution: $x_1 = 45/7$, $x_2 = 4/7$
- Optimal Objective Value: $Z = 53/7$
-

4.4 Example 4 - Goal-programming Method

Goals: $200x_1 \geq 1000$ priority:1

$100x_1 + 400x_2 \geq 1200$ priority:2

$250x_2 \geq 800$ priority:1

Constraints:

$1500x_1 + 3000x_2 \leq 15000$

Problem Definition

Solution

Iteration Steps

Solution Method

Standard Simplex

BIG-M Method

Two-Phase Method

• Goal Programming

Problem Dimensions

Variables: 2

Constraints: 1

Update Tables

Objective Function

Optimization: Maximize

	x1	x2
Coefficient	0	0

Constraints

	x1	x2	Type	RHS
1	1500	3000	≤	15000

Variable Types

	x1	x2
Type	Non-negative	Non-negative

Goal Programming Settings

Number of Goals: 3

Update Goals

	x1	x2	Priority	Type	RHS
Goal 1	200	0	1	≥	1000
Goal 2	100	400	2	≥	1200
Goal 3	0	250	1	≥	800

Problem Definition

Solution

Iteration Steps

Status: Optimal

Objective Value: 175.0

Solution Values

	Variable	Value
1	x1	5.0
2	x2	2.5

Goal Satisfaction

	Goal	Satisfaction
1	Goal 1	True
2	Goal 2	True
3	Goal 3	False

Iteration 0

Basic Variables: d1-, d2-, d3-, s1

Tableau:

	x1	x2	d1-	d1+	d2-	d2+	d3-	d3+	s1	Solution
Z1	0	0	-1	0	0	0	0	0	0	0
Z2	0	0	0	0	-2	0	0	0	0	0
Z3	0	0	0	0	0	0	-1	0	0	0
d1-	200	0	1	-1	0	0	0	0	0	1000
d2-	100	400	0	0	1	-1	0	0	0	1200
d3-	0	250	0	0	0	0	1	-1	0	800
s1	1500	3000	0	0	0	0	0	0	1	15000

Iteration 1

Basic Variables: d1-, d2-, d3-, s1

Tableau:

	x1	x2	d1-	d1+	d2-	d2+	d3-	d3+	s1	Solution
Z1	200	0	0	-1	0	0	0	0	0	1000
Z2	200	800	0	0	0	-2	0	0	0	2400
Z3	0	250	0	0	0	0	0	-1	0	800
d1-	200	0	1	-1	0	0	0	0	0	1000
d2-	100	400	0	0	1	-1	0	0	0	1200
d3-	0	250	0	0	0	0	1	-1	0	800
s1	1500	3000	0	0	0	0	0	0	1	15000

Iteration 2

Entering Variable: x2

Leaving Variable: d2-

Basic Variables: d1-, x2, d3-, s1

Tableau:

	x1	x2	d1-	d1+	d2-	d2+	d3-	d3+	s1	Solution
Z1	200	0	0	-1	0	0	0	0	0	1000
Z2	0	0	0	0	-2	0	0	0	0	0
Z3	-62.5000	0	0	0	-0.6250	0.6250	0	-1	0	50
d1-	200	0	1	-1	0	0	0	0	0	1000
x2	0.2500	1	0	0	0.0025	-0.0025	0	0	0	3
d3-	-62.5000	0	0	0	-0.6250	0.6250	1	-1	0	50
s1	750	0	0	0	-7.5000	7.5000	0	0	1	6000

Tableau:

	x1	x2	d1-	d1+	d2-	d2+	d3-	d3+	s1	Solution
Z1	200	0	0	-1	0	0	0	0	0	1000
Z2	0	0	0	0	-2	0	0	0	0	0
Z3	-62.5000	0	0	0	-0.6250	0.6250	0	-1	0	50
d1-	200	0	1	-1	0	0	0	0	0	1000
x2	0.2500	1	0	0	0.0025	-0.0025	0	0	0	3
d3-	-62.5000	0	0	0	-0.6250	0.6250	1	-1	0	50
s1	750	0	0	0	-7.5000	7.5000	0	0	1	6000

Iteration 3

Entering Variable: x1

Leaving Variable: d1-

Basic Variables: x1, x2, d3-, s1

Tableau:

	x1	x2	d1-	d1+	d2-	d2+	d3-	d3+	s1	Solution
Z1	0	0	-1	0	0	0	0	0	0	0
Z2	0	0	0	0	-2	0	0	0	0	0
Z3	0	0	0.3125	-0.3125	-0.6250	0.6250	0	-1	0	362.5000
x1	1	0	0.0050	-0.0050	0	0	0	0	0	5
x2	0	1	-0.0013	0.0013	0.0025	-0.0025	0	0	0	1.7500
d3-	0	0	0.3125	-0.3125	-0.6250	0.6250	1	-1	0	362.5000
s1	0	0	-3.7500	3.7500	-7.5000	7.5000	0	0	1	2250

Iteration 4

Entering Variable: d2+

Leaving Variable: s1

Basic Variables: x1, x2, d3-, d2+

Tableau:

	x1	x2	d1-	d1+	d2-	d2+	d3-	d3+	s1	Solution
Z1	0	0	-1	0	0	0	0	0	0	0
Z2	0	0	0	0	-2	0	0	0	0	0
Z3	0	0	0.6250	-0.6250	0	0	0	-1	-0.0833	175
x1	1	0	0.0050	-0.0050	0	0	0	0	0	5
x2	0	1	-0.0025	0.0025	0	0	0	0	0.0003	2.5000
d3-	0	0	0.6250	-0.6250	0	0	1	-1	-0.0833	175
d2+	0	0	-0.5000	0.5000	-1	1	0	0	0.1333	300

Output:

- Optimal Solution: $x_1 = 5$, $x_2 = 2.5$

4.4 Example 4 – Unrestricted variables

Maximize $Z = 3x_1 + x_2$

$x_1 + x_2 \geq 7$

$2x_1 + x_2 \leq 4$

$x_1 + x_2 = 3$

Linear Programming Solver

Problem Definition

Solution

Iteration Steps

Solution Method

☐ Standard Simplex

☒ BIG-M Method

☐ Two-Phase Method

☐ Goal Programming

Problem Dimensions

Variables: Constraints:

Update Tables

Objective Function

Optimization:

Maximize

	x_1	x_2
Coefficient	3	1

Constraints

	x_1	x_2	Type	RHS
1	1	1	\geq	3
2	2	1	\leq	4
3	1	1	$=$	3

Variable Types

	x_1	x_2
Type	Unrestricted	Unrestricted

Solve Problem

Linear Programming Solver

Problem DefinitionSolutionIteration Steps

Solution Information

Status:Optimal
Objective Value:5.0

Solution Values

	Variable	Value
1	x1	1.0
2	x2	2.0

Linear Programming Solver

Problem DefinitionSolutionIteration Steps

Initial BIG-M Tableau

Basic Variables: a1, s2, a3

Tableau:

	x1+	x1-	x2+	x2-	e1	s2	a1	a3	Solution
Z	-3	3	-1	1	0	0	100	100	0
a1	1	-1	1	-1	-1	0	1	0	3
s2	2	-2	1	-1	0	1	0	0	4
a3	1	-1	1	-1	0	0	0	1	3

Iteration 1

Basic Variables: a1, s2, a3

Tableau:

	x1+	x1-	x2+	x2-	e1	s2	a1	a3	Solution
Z	-203	203	-201	201	100	0	0	0	-600
a1	1	-1	1	-1	-1	0	1	0	3
s2	2	-2	1	-1	0	1	0	0	4
a3	1	-1	1	-1	0	0	0	1	3

Iteration 2

Entering Variable: x1+

Iteration 2Entering Variable: $x1+$ Leaving Variable: $s2$ Basic Variables: $a1, x1+, a3$

Tableau:

	$x1+$	$x1-$	$x2+$	$x2-$	$e1$	$s2$	$a1$	$a3$	Solution
Z	0	0	-99.5000	99.5000	100	101.5000	0	0	-194
$a1$	0	0	0.5000	-0.5000	-1	-0.5000	1	0	1
$x1+$	1	-1	0.5000	-0.5000	0	0.5000	0	0	2
$a3$	0	0	0.5000	-0.5000	0	-0.5000	0	1	1

Iteration 3Entering Variable: $x2+$ Leaving Variable: $a1$ Basic Variables: $x2+, x1+, a3$

Tableau:

	$x1+$	$x1-$	$x2+$	$x2-$	$e1$	$s2$	$a1$	$a3$	Solution
Z	0	0	0	0	-99	2	199	0	5
$x2+$	0	0	1	-1	-2	-1	2	0	2
$x1+$	1	-1	0	0	1	1	-1	0	1
$a3$	0	0	0	0	1	0	-1	1	0

Iteration 3Entering Variable: $x2+$ Leaving Variable: $a1$ Basic Variables: $x2+, x1+, a3$

Tableau:

	$x1+$	$x1-$	$x2+$	$x2-$	$e1$	$s2$	$a1$	$a3$	Solution
Z	0	0	0	0	-99	2	199	0	5
$x2+$	0	0	1	-1	-2	-1	2	0	2
$x1+$	1	-1	0	0	1	1	-1	0	1
$a3$	0	0	0	0	1	0	-1	1	0

Iteration 4Entering Variable: $e1$ Leaving Variable: $a3$ Basic Variables: $x2+, x1+, e1$

Tableau:

	$x1+$	$x1-$	$x2+$	$x2-$	$e1$	$s2$	$a1$	$a3$	Solution
Z	0	0	0	0	0	2	100	99	5
$x2+$	0	0	1	-1	0	-1	0	2	2
$x1+$	1	-1	0	0	0	1	0	-1	1
$e1$	0	0	0	0	1	0	-1	1	0

4.5 Example 5 – unbounded solution

Maximize $Z = 36x_1 + 30x_2 - 3x_3 - 4x_4$

$x_1 + x_2 - x_3 \leq 5$

$6x_1 + 5x_2 - x_4 \leq 10$

Linear Programming Solver

Problem Definition Solution Iteration Steps

Solution Method

☐ Standard Simplex ☒ BIG-M Method ☐ Two-Phase Method ☐ Goal Programming

Problem Dimensions

Variables: 4 Constraints: 2 **Update Tables**

Optimization:

	x1	x2	x3	x4
Coefficient	36			-4

Constraints

	x1	x2	x3	x4	Type	RHS
1	1	1	-1	0	\leq	5
2	6	5	0	-1	\leq	10

Variable Types

	x1	x2	x3	x4
Type	Non-negative	Non-negative	Non-negative	Non-negative

Error

Problem is unbounded. There is no finite optimal solution.

OK

Linear Programming Solver

Problem Definition Solution Iteration Steps

Solution Method

☒ Standard Simplex ☐ BIG-M Method ☐ Two-Phase Method ☐ Goal Programming

Problem Dimensions

Variables: 4 Constraints: 2 **Update Tables**

Objective Function

Optimization: Maximize

	x1	x2	x3	x4
Coefficient	36	30	-3	-4

Constraints

	x1	x2	x3	x4	Type	RHS
1	1	1	-1	0	\leq	5
2	6	5	0	-1	\leq	10

Variable Types

	x1	x2	x3	x4
Type	Non-negative	Non-negative	Non-negative	Non-negative

Error

Problem is unbounded

OK

Linear Programming Solver

Problem Definition | Solution | Iteration Steps

Solution Method

☐ Standard Simplex ☐ BIG-M Method ☒ Two-Phase Method ☐ Goal Programming

Problem Dimensions

Variables: 4 Constraints: 2 **Update Tables**

Optimization:

	x1	x2	x3	x4
Coefficient	36			-4

Constraints

	x1	x2	x3	x4	Type	RHS
1	1	1	-1	0	≤	5
2	6	5	0	-1	≤	10

Variable Types

	x1	x2	x3	x4
Type	Non-negative	Non-negative	Non-negative	Non-negative

Error

Problem is unbounded. There is no finite optimal solution.

OK

4.6 Example 6 – infeasible solution

Maximize $Z = 36x_1 + 30x_2$

$x_1 + x_2 \geq 10$

$x_1 + x_2 \leq 5$

$x_2 > 0, x_1$

Linear Programming Solver

Problem Definition | Solution | Iteration Steps

Solution Method

☐ Standard Simplex ☐ BIG-M Method ☒ Two-Phase Method ☐ Goal Programming

Problem Dimensions

Variables: 2 Constraints: 2 **Update Tables**

Objective Function

Optimization: Maximize

	x1	x2
Coefficient	36	30

Constraints

	x1	x2	Type	RHS
1	1	1	≤	5
2	1	1	≥	10

Variable Types

	x1	x2
Type	Unrestricted	Non-negative

Error

Problem has no feasible solution. could not eliminate artificial variables.

OK

Solve Problem

5. Bonus Feature

We developed a **user-friendly interface** by PyQt5 in python that allows users to input LP problems easily and view the solution process interactively.

6. Conclusion

This project provided hands-on experience in solving LP problems using different methods. The solver successfully handles various constraints and outputs detailed step-by-step solutions. Future improvements include extending support for additional optimization techniques and graphical visualization of results.