Neutrino Oscillations

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Chapter 1

Introduction

Neutrinos are elementary particles, electrically neutral fermions with spin-$\frac{1}{2}$ and very tiny mass $\mathcal{O}(eV)$. In Standard Model of Particle Physics, Neutrinos are Leptons (see Figure 1). Also, they come in three different flavors: electron neutrino ν_e, muon neutrino ν_μ, and tau neutrino ν_τ; and they interact only via weak interactions and gravity. Further, Neutrinos exist only as left-handed particles, i.e., they can only acted upon by $SU(2)_L$ in the Electro-weak gauge theory[4].

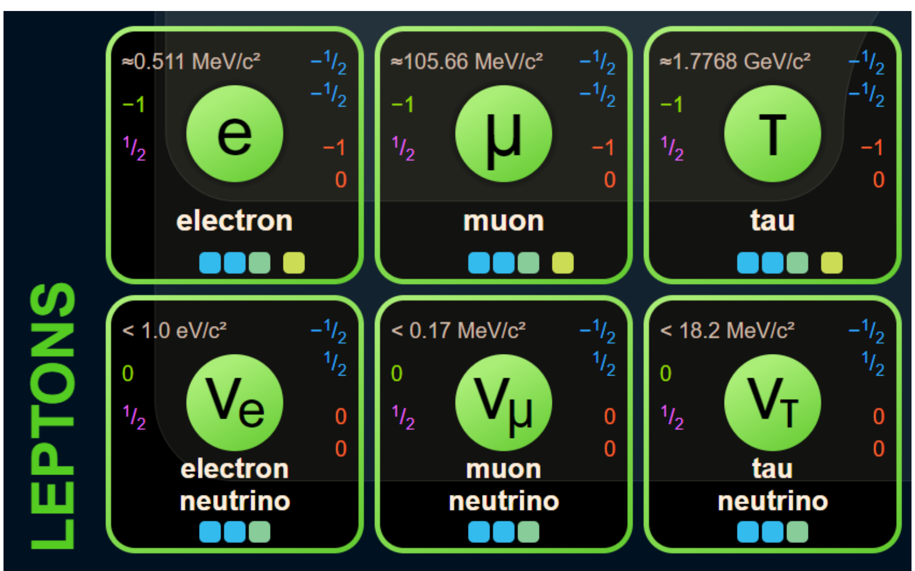


Figure 1

However, Parity was assumed to be a good symmetry of nature, week interactions have proved to maximally violate Parity! Since, left-handed fermions feel weak interactions. This was found experimentally by Wu in 1963 (see Figure 2).

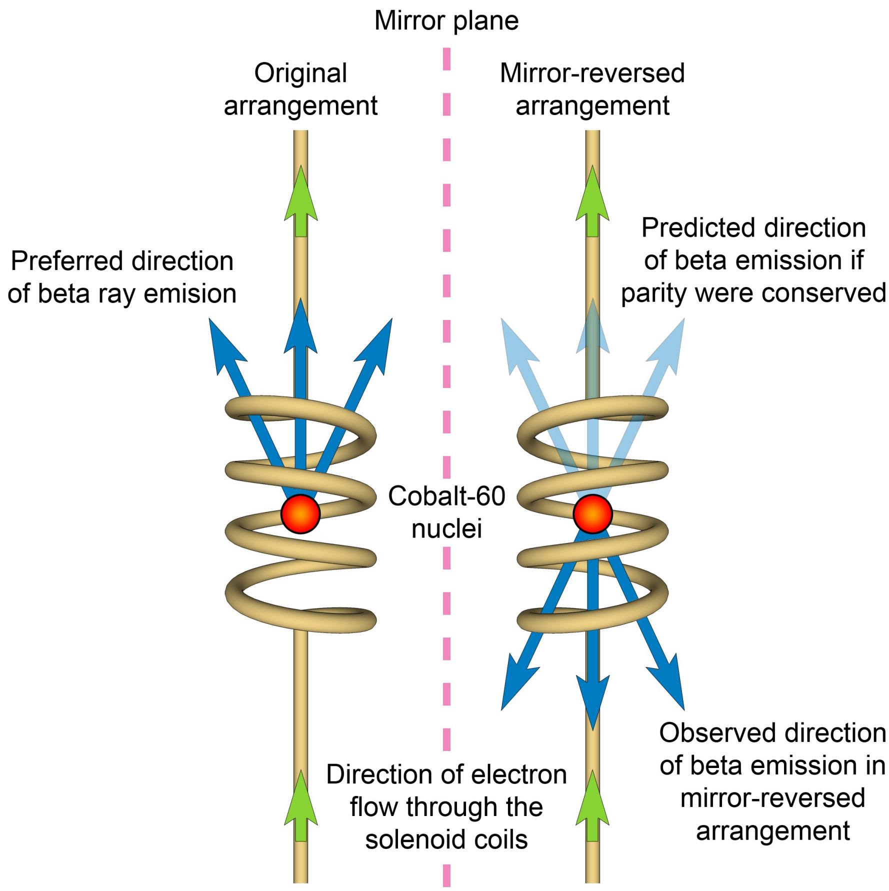


Figure 2

Chapter 2

Neutrino Masses and Mixing

2.1 The Solar Neutrino Problem

At the core of our Sun enormous amount of nuclear reaction is happening all the time which leads to the creation of large amount of energy and gigantic number of neutrinos. One example is the fusion of two protons under very high temperature and pressure which leads to the creation of a deuterium nuclei, a positron, and an electron neutrino: $p + p \longrightarrow {}^2_1H + e^+ + \nu_e$. Notice that since the deuterium nuclei comes with one proton and one neutron, this means a beta decay must have happened to convert a proton to a neutron. Therefore, this leads to the creation of a positron, and an electron neutrino[2].

This is part of a broader cycle called *proton-proton cycle* which is one of the dominant cycles of nuclear fusion reactions occurring at the core of the Sun. Therefore, a vast majority of the neutrinos emitted from the Sun are essentially electron neutrinos. However, a quite interesting puzzle appeared upon measuring the flux of the e-neutrinos reaching the surface of Earth. It turns out that the theoretically predicted value of this flux and the experimentally measured flux were of ratio two-thirds to one-third, respectively. This is known as the *solar neutrino problem*. This problem took long time to be resolved from the 1960s to the 2000s.

2.2 Neutrino Mixing & Neutrino Mass Hierarchy

We have mentioned earlier that there are three different flavors: electron neutrino ν_e, muon neutrino ν_μ, and tau neutrino ν_τ. They are different because they come from different interactions. For example[3],

$$\begin{aligned} p &\longrightarrow n + e^+ + \nu_e \\ \pi^+ &\longrightarrow \mu^+ + \nu_\mu \\ \tau^- &\longrightarrow \pi^- + \nu_\tau \end{aligned}$$

Neutrinos were assumed to be massless in the Standard Model of particle physics. However, there now exist many experimentally established measurements of the neutrinos solar flux and the phenomenon of neutrino oscillations which we will be discussed in the next chapter. These observations requires neutrinos to be massive. In the 1950s, Bruno Pontecorvo conceived the idea of massive neutrinos. In fact, Pontecorvo suggested that the neutrino flavors are mixture of certain quantum states and as these flavors travel through space-time their identities transform.

In other words, one of the possible phenomena resulting if neutrinos have masses is *neutrino mixing*. That is, neutrinos (flavor eigenstates) do not have definite masses but they are linear combinations of three other states namely, mass eigenstates ν_1, ν_2, ν_3. So, equation (2.1) defines the PMNS mixing matrix of neutrinos for Pontecorvo, Maki, Nakagawa and Sakata who were the pioneers of the field.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{2.1}$$

with

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 0.82 \pm 0.01 & 0.54 \pm 0.02 & -0.15 \pm 0.03 \\ -0.35 \pm 0.06 & 0.70 \pm 0.06 & 0.62 \pm 0.06 \\ 0.44 \pm 0.06 & -0.45 \pm 0.06 & 0.77 \pm 0.06 \end{pmatrix} \tag{2.2}$$

Equation (2.2) tells approximately all elements are in the same order of magnitude. Also, we have assumed that the flavor eigenstates that couple with the gauge bosons $W^{\pm}$ by weak interactions, are coherent superpositions of three mass eigenstates.

Assigning masses to neutrinos can be done in two ways; first, by the Dirac mechanism which would require an $SU(2)$ singlet framework. Or, second, mass can be generated by the Majorana mechanism which would demand the neutrino and anti-neutrino to be the same particle. Notice that, the Dirac mechanism assumes that neutrinos have the Yukawa interactions with the neutral component of the Higgs doublet, but otherwise would have no interactions with Standard Model particles, so is called a *sterile* neutrino.

Thereafter, the PMNS mixing matrix can be parametrized in terms of the three angles θ_{12}, θ_{23}, and θ_{13} and the CP phase δ_{CP}. Thus, it can be understood as follows[3]:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13}\, e^{-i\delta^{CP}} \\ 0 & 1 & 0 \\ -S_{13}\, e^{i\delta^{CP}} & 0 & C_{13} \end{pmatrix} \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.3}$$

where $C_{ij} = cos(\theta_{ij})$ and $S_{ij} = sin(\theta_{ij})$ are the mixing angles. Also, α and β (both unknown at the present) are so-called *Majorana phases* that are decoupled from the phenomenon of neutrino oscillations. Notwithstanding, we currently cannot determine whether the ν_3 mass eigenstate is heavier or lighter than the mass eigenstates of

ν_1 the and ν_2. There are two scenarios; the first scenario suggests that ν_3 is heavier and is called *normal mass hierarchy* (NH: $m_3 > m_2 > m_1$), the second suggests the opposite and is called *inverted mass hierarchy* (IH: $m_2 > m_1 > m_3$), (see Figure 3[3]).

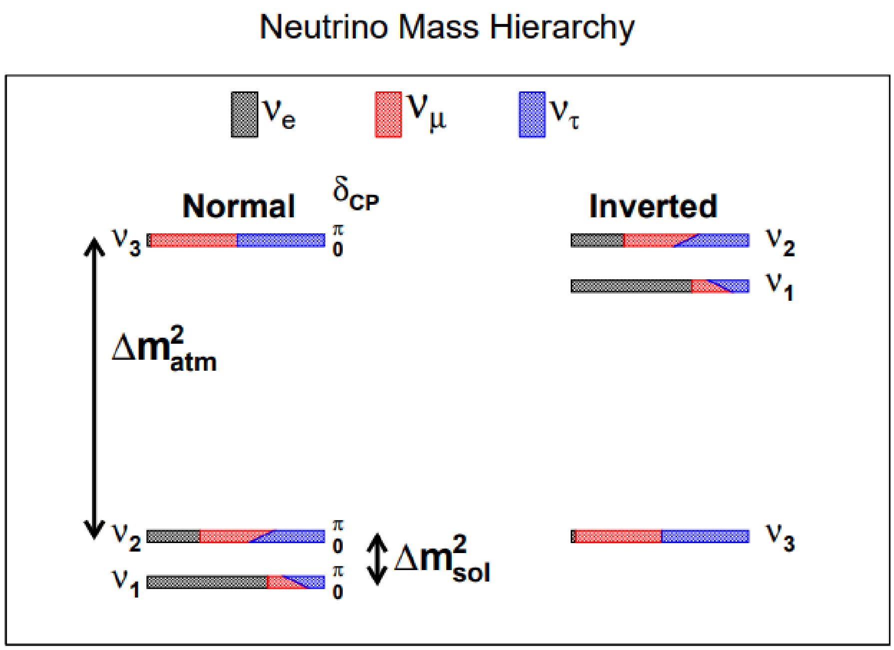


Figure 3

A natural question now is: Why is it important to determine the MH? Actually, the goal of all experiments and theories focused on neutrinos is to formulate a rigours standard model of particle physics that explains the observed masses of neutrinos and its mixing patterns, and relates them to the well known charged lepton masses (and possibly to the quark masses and mixings). Evidently, most of the current theoretical models of neutrino mass assume that neutrinos are massive Majorana fermions. Indeed, the determination of MH would strengthen, or role out, roughly half of the proposed models.

Chapter 3

Neutrino Oscillations

3.1 Classical Oscillations

For a particle of mass m in one dimension, the classical Lagrangian for forced oscillations by a force $F(t)$ and a frequency ω is:

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 + xF(t) \tag{3.1}$$

From Euler-Lagrange equations, one can get the equation of motion:

$$\ddot{x} + \omega^2 x = \frac{F(t)}{m} \tag{3.2}$$

Equation (3.2) admits the following free solution ($F(t) = 0$):

$$x(t) = A\, cos(\omega t + \delta) \qquad \text{with amplitude } A \text{ and phase } \delta \tag{3.3}$$

However, if we consider the source to be $F(t) = f\, cos(\gamma t + \beta)$, then near the resonance: $\gamma = \omega$ (i.e., $\gamma = \omega + \epsilon$ and $\epsilon << 1$) the motion can be though of as small oscillations with variable amplitude:

$$x(t) = C(t)\, e^{i\omega t} \tag{3.4}$$

where $C(t) = a^2 + b^2 + 2\, ab\, cos(\epsilon t + (\beta - \alpha))$ with constant a and b. Also, since $C(t)$ depends on the perturbation frequency parameter ϵ, the amplitude varies very slowly between limits, i.e., $|a - b| \leq C \leq |a + b|$. This phenomena is called beats and the beat frequency is ϵ.

3.2 Two flavor Mixing Approximation

We consider a mixing between only two flavors ν_α and ν_β:

$$\begin{aligned} \nu_\alpha &= \quad \nu_i\, cos(\theta_{ij}) + \nu_j\, sin(\theta_{ij}) \\ \nu_\beta &= -\nu_i\, sin(\theta_{ij}) + \nu_j\, cos(\theta_{ij}) \end{aligned} \tag{3.5}$$

where ν_i and ν_j are the mass eigenstates involved in such mixing. Further, when a neutrino ν_α is created at time $t = 0$ with momentum $\vec{p}$, the mass eigenstates will have slightly different energies E_i, E_j because of their slightly different masses.

3.2.1 Classical Beats as Quantum Oscillations

In Quantum Mechanics, due to this difference in masses the frequencies will therefore, be difference which gives rise to the beats phenomenon. This analogous to the classical case in section (3.1). As a result to this difference in frequencies, the original beam of ν_α particles develops a ν_β component whose intensity oscillates during traveling through space-time. This phenomenon is called Neutrino Oscillations.

3.2.2 Propagation in Space-Time

For the neutrino ν_α created at time $t = 0$ with momentum $\vec{p}$, its initial state is:

$$|\nu_\alpha,\, \vec{p}\rangle = |\nu_i,\, \vec{p}\rangle\; cos(\theta_{ij}) + |\nu_j,\, \vec{p}\rangle\; sin(\theta_{ij}) \tag{3.6}$$

After time t, the state evolution is

$$|\nu_\alpha,\, \vec{p},\, t\rangle = e^{-iE_i t}\; |\nu_i,\, \vec{p}\rangle\;\; cos(\theta_{ij}) + e^{-iE_j t}\; |\nu_j,\, \vec{p}\rangle\;\; sin(\theta_{ij}) \tag{3.7}$$

Similarly, if we have a neutrino ν_β created at time $t = 0$ with momentum $\vec{p}$, from (3.5) its state evolution is

$$|\nu_\beta,\, \vec{p},\, t\rangle = -e^{-iE_i t}\; |\nu_i,\, \vec{p}\rangle\;\; sin(\theta_{ij}) + e^{-iE_j t}\; |\nu_j,\, \vec{p}\rangle\;\; cos(\theta_{ij}) \tag{3.8}$$

3.2.3 Oscillations Probability

One can invert equations (3.5) to get the mass eigenstates in terms of the flavors:

$$\begin{aligned} \nu_i &= \nu_\alpha\, cos(\theta_{ij}) - \nu_\beta\, sin(\theta_{ij}) \\ \nu_j &= \nu_\alpha\, cos(\theta_{ij}) + \nu_\beta\, sin(\theta_{ij}) \end{aligned} \tag{3.9}$$

Substituting in equation (3.6), we get:

$$|\nu_\alpha(t),\, \vec{p}\rangle = A(t)\;\; |\nu_\alpha(0),\, \vec{p}\rangle + B(t)\;\; |\nu_\beta(0),\, \vec{p}\rangle \tag{3.10}$$

where

$$\begin{aligned} A(t) &= e^{-iE_i t}\, cos^2(\theta_{ij}) + e^{-iE_j t}\, sin^2(\theta_{ij}) \\ B(t) &= \left(e^{-iE_j t} - e^{-iE_i t}\right)\; sin(\theta_{ij})\, cos(\theta_{ij}) \end{aligned} \tag{3.11}$$

Therefore, the probabilities of finding ν_β and ν_α are,

$$\begin{aligned} P(\nu_\alpha \longrightarrow \nu_\beta) &= |\, \langle \nu_\beta(0),\, \vec{p}|\, |\nu_\alpha(t),\, \vec{p}\rangle\, | = |B(t)|^2 = sin^2(2\theta_{ij})\, sin^2(\frac{1}{2}(E_j - E_i)t) \\ P(\nu_\alpha \longrightarrow \nu_\alpha) &= |\, \langle \nu_\alpha(0),\, \vec{p}|\, |\nu_\alpha(t),\, \vec{p}\rangle\, | = |A(t)|^2 = 1 - |B(t)|^2 \end{aligned} \tag{3.12}$$

Now, if we apply the NH scenario to $P(\nu_e \longrightarrow \nu_\mu)$, we would have the following oscillation probabilities:

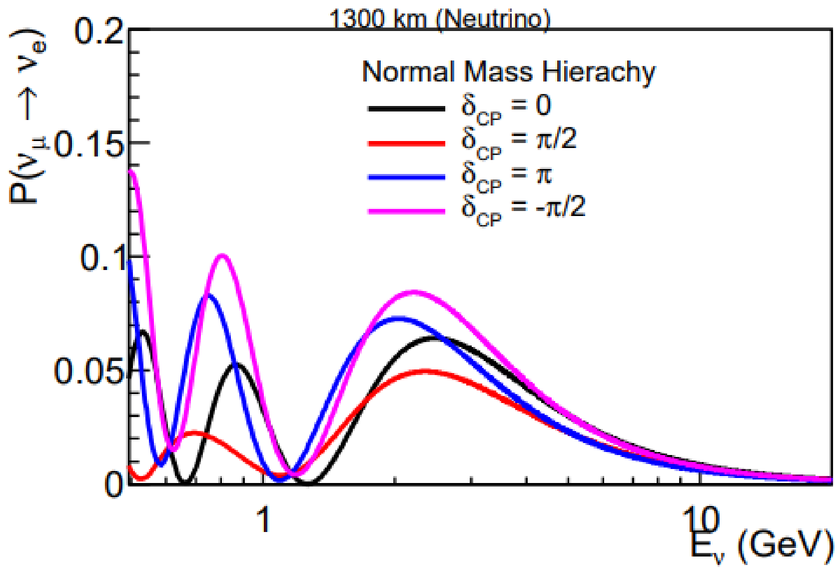


Figure 4

If we apply the IH scenario to $P(\nu_e \longrightarrow \nu_\mu)$, we would have the following oscillation probabilities:

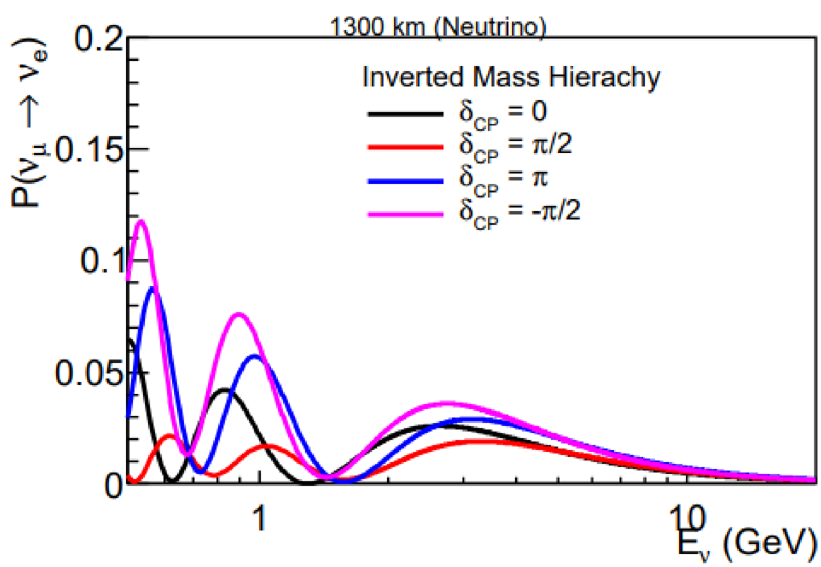


Figure 5

3.2.4 Remarks

We point out some remarks from the previous results. First, these formulas assume the vacuum is the background of neutrinos propagation. Actually, this is a very good approximation due to the large mean free paths for neutrinos to interact with matter. Also, this manifests that neutrino oscillations can be enhanced when neutrinos traverse very long distances in matter. This is known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect and is drastically confirmed in the 'Sudbury Neutrino Observatory (SNO)'. This resolves the solar neutrino problem. However, the MH is not resolved yet.

Indeed, there are no oscillations if the mixing angle vanishes, i.e., zero probabilities. Further, for large energy difference (large difference in frequency), the oscillations may be within the uncertainty time. Yet, for small mixing angles and large energy difference, the oscillation is negligible.

Chapter 4

Experimental Work and Explorative Directions

The time t taken by a traveling neutrino is determined by L the distance from the neutrinos source to the neutrino detector. However, the momenta are usually much larger than their possible masses and they travel approximately at the speed of light[4]. Therefore, $t = L$ and hence,

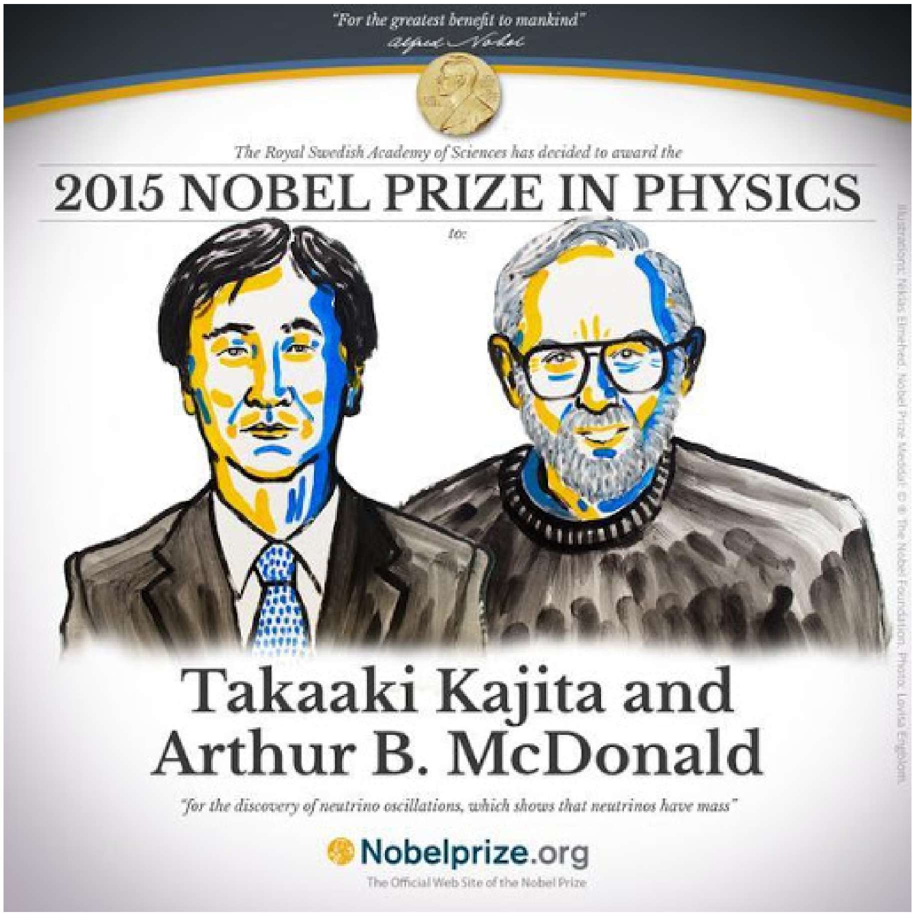
$$E_j - E_i = \sqrt{m_j^2 + p^2} - \sqrt{m_i^2 + p^2} \approx \frac{m_j^2 - m_i^2}{2p} = \frac{\Delta m_{ji}^2}{2p} \tag{4.1}$$

Consequently, the probabilities in (3.12) are, respectively:

$$\begin{aligned} P(\nu_\alpha \longrightarrow \nu_\beta) &= sin^2(2\theta_{ij})\, sin^2(\frac{L}{L_0}) \\ P(\nu_\alpha \longrightarrow \nu_\alpha) &= 1 - P(\nu_\alpha \longrightarrow \nu_\beta) = \end{aligned} \tag{4.2}$$

with the oscillation length $L_0 = \frac{4E}{m_j^2 - m_i^2}$.

Evidently, these oscillation lengths are of order 100 km or more which can be neglected under normal laboratory conditions. However, in 2015 the Nobel Prize in physics was warded by Takaaki Kajita and Arthur B. McDonald for the discovery of neutrino oscillations.



4.1 Super-Kamiokande

The Super-Kamiokande detector is a very large tank of water, located underground (approximately 1 km). The water in the tank acts as both the target for neutrinos, and the detecting medium for the by-products of neutrino interactions[6]. Figure 7 shows a model of the detector.

Figure 7

The inside surface of the tank is lined with thousands of 50-cm diameter light collectors called *photo-multiplier tubes.* Additionally, there is a layer of water called the outer detector and is instrumented light sensors to detect any charged particles entering the central volume, and to shield it by absorbing any neutrons produced in the nearby rock. In addition to that, a forest of electronics, computers, calibration devices, and water purification equipment is installed in or near the detector cavity.

Super-Kamiokande experiment[6] carries out a dedicated analysis that used only events with $\frac{L}{E_\nu}$ value could be determined with good precision. Using only the high $\frac{L}{E_\nu}$ resolution events, Super-Kamiokande showed that the measured ν_μ survival probability has a dip corresponding to the first minimum of the theoretical survival probability near $\frac{L}{E_\nu} = 500\, km/GeV$. This was the first evidence that the ν_μ survival probability obeys the sinusoidal function predicted by neutrino oscillations.

4.2 Open Questions

In this section we present a number of questions that are currently left open:

• Is the mass hierarchy normal ($\Delta m^2_{23} > 0$) or inverted ($\Delta m^2_{23} < 0$)?

• Are the neutrinos and anti-neutrinos identical (Majorana particles)? This cannot be answered by an oscillation experiment.

• What is the value of δ_{CP}?

• What is the absolute neutrino mass? This cannot be measured by an oscillation experiments.

• Why is the MNSP matrix so different in form to the CKM matrix? This will require a deeper understanding of flavour and mass generation mechanisms.

The discovery of neutrino oscillations opens a window to study physics beyond the Standard Model of particle physics, i.e., GUT scale. Also, there maybe still many observations to be made about neutrinos themselves. Moreover, further studies of neutrinos might give us information about the origin of the matter in the Universe.

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