Neutrino Oscillations

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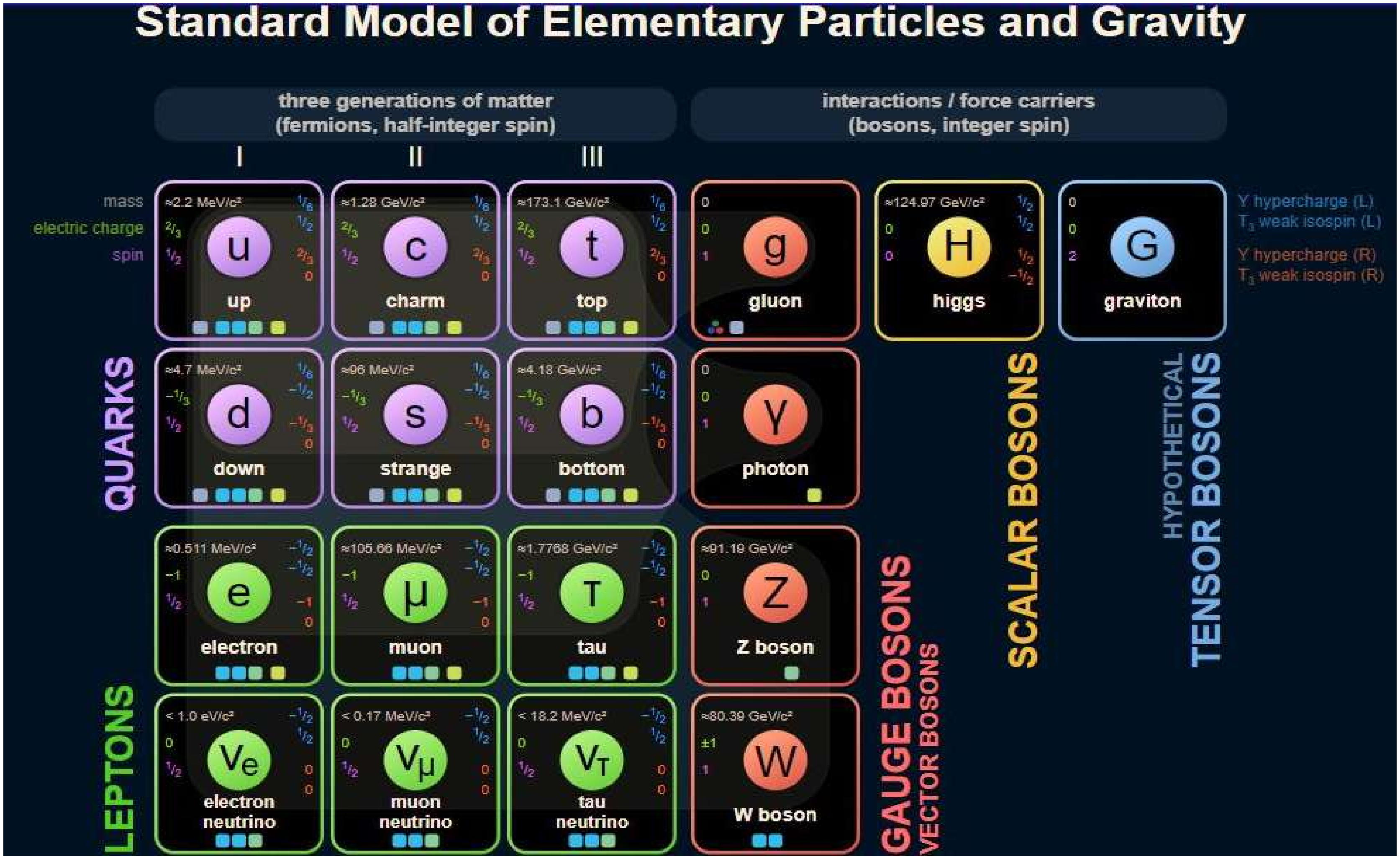
❖ Introduction

Neutrinos are elementary particles, electrically neutral fermions with spin-1/2 and very tiny mass (order of few eVs). In Standard Model of Particle Physics, Neutrinos are Leptons. Also, they come in three different flavors: electron neutrino, muon neutrino, and tau neutrino; and they interact only via weak interactions and gravity. Further, Neutrinos exist only as left-handed particles, i.e., they can only acted upon by $SU(2)_L$ in the Electro-Weak gauge theory (see Figure 1).

However, Parity was assumed to be a good symmetry of nature, week interactions have proved to maximally violate Parity! Since, left-handed fermions feel weak interactions. This was found experimentally by Wu in 1963.


Standard Model of Elementary Particles and Gravity
three generations of matter
(fermions, half-integer spin)
interactions / force carriers
(bosons, integer spin)
I
II
III
mass
electric charge
spin
≈2.2 MeV/c²
u
up
≈1.28 GeV/c²
c
charm
≈173.1 GeV/c²
t
top
0
0
1
g
gluon
≈124.97 GeV/c²
0
0
H
higgs
0
0
2
G
graviton
Y hypercharge (L)
T3 weak isospin (L)
Y hypercharge (R)
T3 weak isospin (R)
QUARKS
≈4.7 MeV/c²
d
down
≈96 MeV/c²
s
strange
≈4.18 GeV/c²
b
bottom
0
0
1
γ
photon
SCALAR BOSONS
HYPOTHETICAL
TENSOR BOSONS
≈0.511 MeV/c²
e
electron
≈105.66 MeV/c²
μ
muon
≈1.7768 GeV/c²
τ
tau
≈91.19 GeV/c²
0
1
Z
Z boson
LEPTONS
< 1.0 eV/c²
νe
electron neutrino
< 0.17 MeV/c²
νμ
muon neutrino
< 18.2 MeV/c²
ντ
tau neutrino
≈80.39 GeV/c²
±1
1
W
W boson
GAUGE BOSONS
VECTOR BOSONS

❖ Neutrino Masses & Mixing

❑ The Solar Neutrino Problem

At the core of our Sun enormous amount of nuclear reaction is happening all the time which leads to the creation of large amount of energy and gigantic number of neutrinos. One example is the fusion of two protons under very high temperature and pressure which leads to the creation of a deuterium nuclei, a positron, and an electron neutrino: $p + p \longrightarrow {}^2_1H + e^+ + \nu_e$.

Notice that since the deuterium nuclei comes with one proton and one neutron, this means a beta decay must have happened to convert a proton to a neutron. Therefore, this leads to the creation of a positron, and an electron neutrino.

❑ The Solar Neutrino Problem

This is part of a broader cycle called 'proton-proton cycle' which is one of the dominant cycles of nuclear fusion reactions occurring at the core of the Sun. Therefore, a vast majority of the neutrinos emitted from the Sun are essentially electron neutrinos. However, a quite interesting puzzle appeared upon measuring the flux of the electron-neutrinos reaching the surface of Earth. It turns out that the theoretically predicted value of this flux and the experimentally measured flux were of ratio two-thirds to one-third, respectively. This is known as the 'solar neutrino problem'. This problem took long time to be resolved from the 1960s to the 2000s.

❑ Neutrino Mixing & Mass Hierarchy

We have mentioned earlier that there are three different flavors: electron neutrino, muon neutrino, and tau neutrino. They are different because they come from different interactions.

Neutrinos were assumed to be massless in the Standard Model of particle physics. However, there now exist many experimentally established measurements of the neutrinos solar flux and the phenomenon of neutrino oscillations which we will be discussed next. These observations requires neutrinos to be massive. In the 1950s, Bruno Pontecorvo conceived the idea of massive neutrinos. In fact, Pontecorvo suggested that the neutrino flavors are mixture of certain quantum states and as these flavors travel through space-time their identities transform.

❑ Neutrino Mixing & Mass Hierarchy

In other words, one of the possible phenomena resulting if neutrinos have masses is 'neutrino mixing'. That is, neutrinos (flavor eigenstates) do not have definite masses but they are linear combinations of three other states namely, mass eigenstates. So, equation (2.1) defines the PMNS mixing matrix of neutrinos for Pontecorvo, Maki, Nakagawa and Sakata who were the pioneers of the field.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{2.1}$$

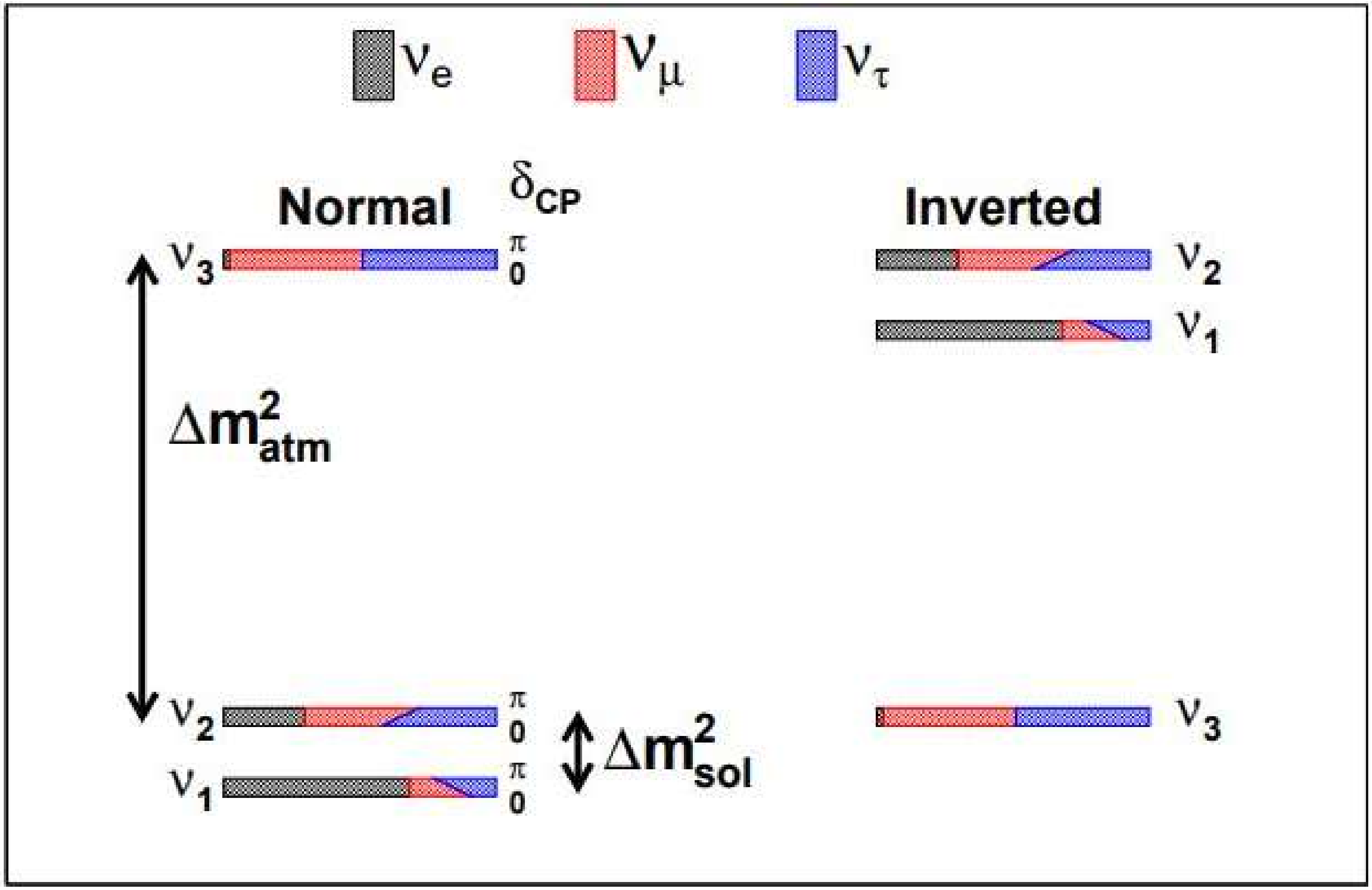
❑ Neutrino Mixing & Mass Hierarchy

Equation (2.2) tells approximately all elements are in the same order of magnitude. Also, we have assumed that the flavor eigenstates that couple with the gauge bosons W± by weak interactions, are coherent superpositions of three mass eigenstates.

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 0.82 \pm 0.01 & 0.54 \pm 0.02 & -0.15 \pm 0.03 \\ -0.35 \pm 0.06 & 0.70 \pm 0.06 & 0.62 \pm 0.06 \\ 0.44 \pm 0.06 & -0.45 \pm 0.06 & 0.77 \pm 0.06 \end{pmatrix} \tag{2.2}$$

Notwithstanding, we currently cannot determine whether the 3rd mass eigenstate is heavier or lighter than the other two mass eigenstates. There are two scenarios; the first scenario suggests that the 3rd is heavier and is called 'normal mass hierarchy' (NH: $m_3 > m_2 > m_1$), the second suggests the opposite and is called 'inverted mass hierarchy' (IH: $m_2 > m_1 > m_3$), (see Figure 2).

Neutrino Mass Hierarchy



❖ Neutrino Oscillations

❑ Classical Oscillations

For a particle of mass *m* in one dimension, the classical Lagrangian for forced oscillations is:

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 + xF(t)$$

The E.O.M is: $\ddot{x} + \omega^2 x = \frac{F(t)}{m}$ and the free solution is: $x(t) = A\,cos(\omega t + \delta)$

However, if we consider a source force $F(t) = f\,cos(\gamma t + \beta)$, then, near the resonance frequency the motion can be thought of as a small oscillation with variable amplitude: $x(t) = C(t)\,e^{i\omega t}$

Now, we carry an analogous analysis but in the quantum mechanical sense.

Two Flavour Mixing Approximation

We consider a mixing between only two flavors ν_α and ν_β:

$$\nu_\alpha = \nu_i \, cos(\theta_{ij}) + \nu_j \, sin(\theta_{ij})$$

$$\nu_\beta = -\nu_i \, sin(\theta_{ij}) + \nu_j \, cos(\theta_{ij})$$

where ν_i and ν_j are the mass eigenstates involved in such mixing. Further, when a neutrino ν_α is created at time $t = 0$ with momentum $\vec{p}$, the mass eigenstates will have slightly different energies E_i, E_j because of their slightly different masses.

In Quantum Mechanics, due to this difference in masses the frequencies will therefore, be difference which gives rise to the beats phenomenon. This analogous to the classical case. As a result to this difference in frequencies, the original 1st beam of particles develops a component along the 2nd beam whose intensity oscillates during traveling through space-time. This phenomenon is called 'Neutrino Oscillations'.

❑ <u>Two Flavour Mixing Approximation:</u> Space-Time Propagation

For the neutrino ν_α created at time $t = 0$ with momentum $\vec{p}$, its initial state is:

$$|\nu_\alpha, \vec{p}\rangle = |\nu_i, \vec{p}\rangle \; cos(\theta_{ij}) + |\nu_j, \vec{p}\rangle \; sin(\theta_{ij})$$

After time t, the state evolution is

$$|\nu_\alpha, \vec{p}, t\rangle = e^{-iE_i t} \; |\nu_i, \vec{p}\rangle \; cos(\theta_{ij}) + e^{-iE_j t} \; |\nu_j, \vec{p}\rangle \; sin(\theta_{ij})$$

Similarly, if we have a neutrino ν_β created at time $t = 0$ with momentum $\vec{p}$, its state evolution is

$$|\nu_\beta, \vec{p}, t\rangle = -e^{-iE_i t} \; |\nu_i, \vec{p}\rangle \; sin(\theta_{ij}) + e^{-iE_j t} \; |\nu_j, \vec{p}\rangle \; cos(\theta_{ij})$$

❑ <u>Two Flavour Mixing Approximation:</u> Oscillation Probability

One can invert the equations to get the mass eigenstates in terms of the flavors:

$$\nu_i = \nu_\alpha \, cos(\theta_{ij}) - \nu_\beta \, sin(\theta_{ij})$$
$$\nu_j = \nu_\alpha \, cos(\theta_{ij}) + \nu_\beta \, sin(\theta_{ij})$$

Substituting in the equations of state evolution , we get:

$$|\nu_\alpha(t), \vec{p}\rangle = A(t) \; |\nu_\alpha(0), \vec{p}\rangle + B(t) \; |\nu_\beta(0), \vec{p}\rangle$$

where

$$A(t) = e^{-iE_i t} \, cos^2(\theta_{ij}) + e^{-iE_j t} \, sin^2(\theta_{ij})$$
$$B(t) = \left(e^{-iE_j t} - e^{-iE_i t}\right) \; sin(\theta_{ij}) \, cos(\theta_{ij})$$

❑ <u>Two Flavour Mixing Approximation:</u> Oscillation Probability

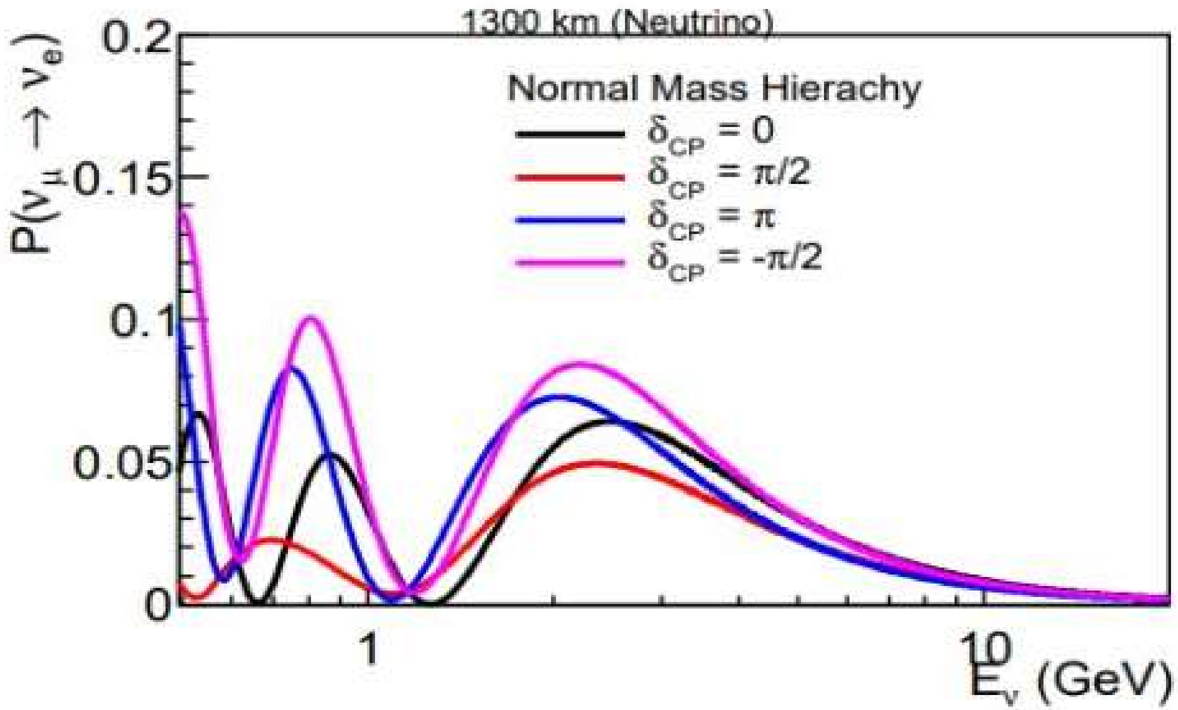
Therefore, the probabilities of finding ν_β and ν_α are,

$$P(\nu_\alpha \longrightarrow \nu_\beta) = |\langle \nu_\beta(0), \vec{p}| |\nu_\alpha(t), \vec{p}\rangle| = |B(t)|^2 = sin^2(2\theta_{ij})\, sin^2(\frac{1}{2}(E_j - E_i)t)$$

$$P(\nu_\alpha \longrightarrow \nu_\alpha) = |\langle \nu_\alpha(0), \vec{p}| |\nu_\alpha(t), \vec{p}\rangle| = |A(t)|^2 = 1 - |B(t)|^2$$

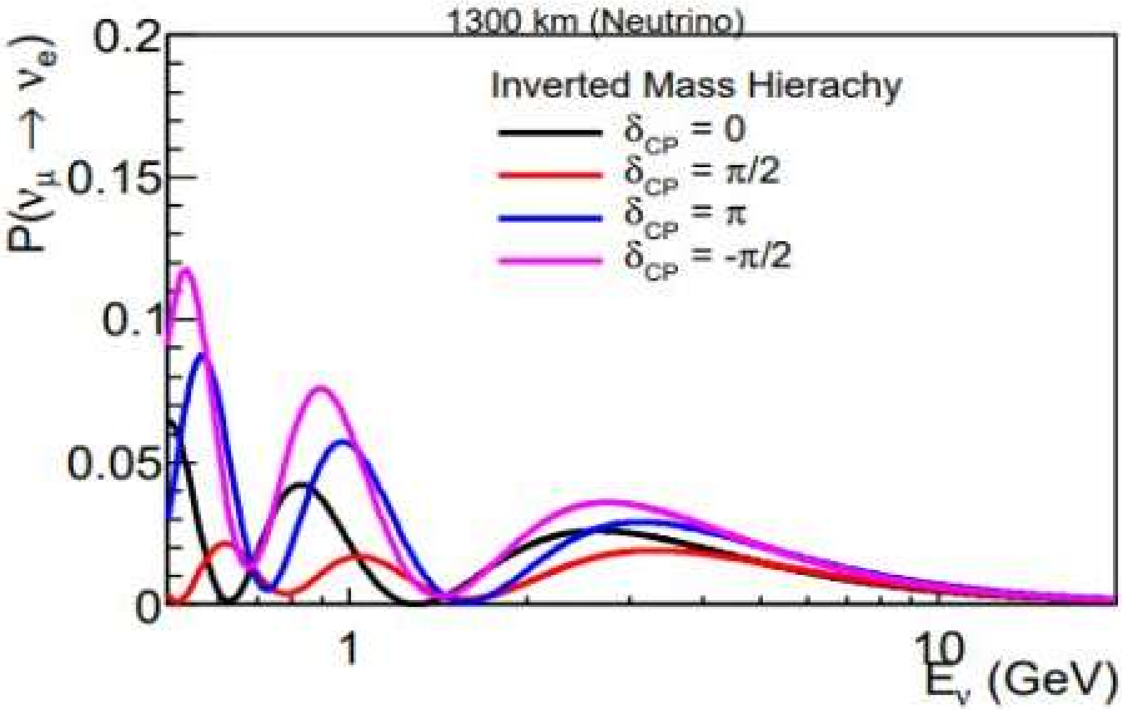
- <u>Two Flavour Mixing Approximation:</u> Oscillation Probability

Now, if we apply the NH scenario to $P(\nu_e \longrightarrow \nu_\mu)$, we would have the following oscillation probabilities:



❑ <u>Two Flavour Mixing Approximation:</u> Oscillation Probability

If we apply the IH scenario to $P(\nu_e \longrightarrow \nu_\mu)$, we would have the following oscillation probabilities:

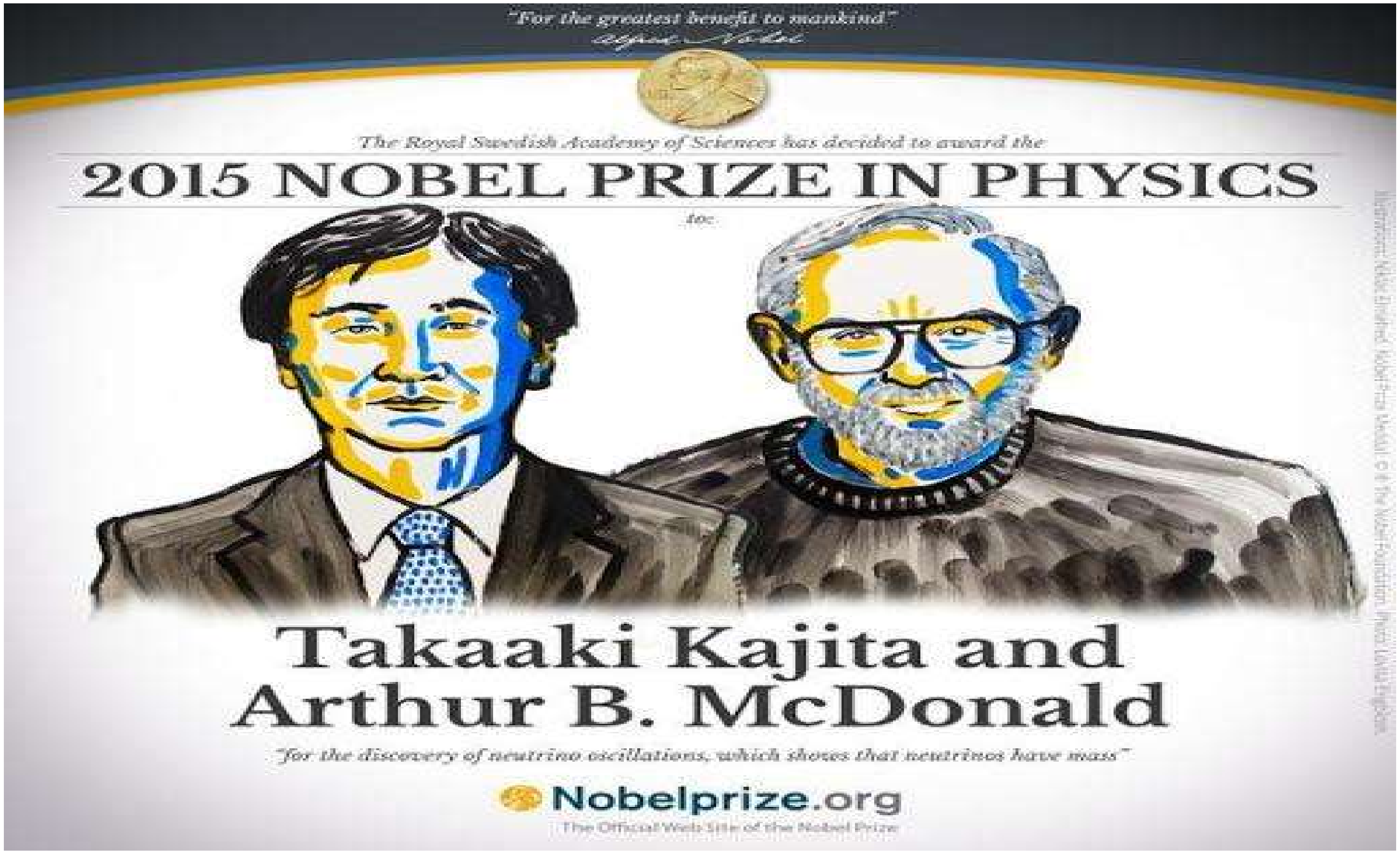


❑ Remarks

We point out some remarks from the previous results. First, these formulas assume the vacuum is the background of neutrinos propagation. Actually, this is a very good approximation due to the large mean free paths for neutrinos to interact with matter. Also, this manifests that neutrino oscillations can be enhanced when neutrinos traverse very long distances in matter. This is known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect and is drastically confirmed in the 'Sudbury Neutrino Observatory (SNO)'. This resolves the solar neutrino problem. However, the MH is not resolved yet.

Indeed, there are no oscillations if the mixing angle vanishes, i.e., zero probabilities. Further, for large energy difference (large difference in frequency), the oscillations may be within the uncertainty time. Yet, for small mixing angles and large energy difference, the oscillation is negligible.

❖ Experimental Work & Explorative Directions



The momenta of neutrino is much bigger than its mass. Hence, its speed is approaching the speed of light.

$$E_j - E_i = \sqrt{m_j^2 + p^2} - \sqrt{m_i^2 + p^2} \approx \frac{m_j^2 - m_i^2}{2p} = \frac{\Delta m_{ji}^2}{2p}$$

The neutrino move a distance L = ct from the detector.

$$P(\nu_\alpha \longrightarrow \nu_\beta) = sin^2(2\theta_{ij})\, sin^2(\frac{L}{L_0})$$

$$P(\nu_\alpha \longrightarrow \nu_\alpha) = 1 - P(\nu_\alpha \longrightarrow \nu_\beta) =$$

with the oscillation length $L_0 = \frac{4E}{m_j^2 - m_i^2}$.

The oscillation length of order 100Km which can be neglected under normal lab conditions.

❑ Super-Kamiokande

Figure 7

Open Questions

- Is the mass hierarchy normal ($\Delta m23 > 0$) or inverted ($\Delta m23 < 0$)?
- Are the neutrinos and anti-neutrinos identical (Majorana particles)? This cannot be answered by an oscillation experiment.
- What is the value of δCP?
- What is the absolute neutrino mass? This cannot be measured by an oscillation experiments.
- Why is the MNSP matrix so different in form to the CKM matrix? This will require a deeper understanding of flavour and mass generation mechanisms.

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