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When and How Can we Fit Residuals?

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ABSTRACT

I discuss the effect of fixing the *residuals* from a global fit in LISA-like data when fitting broadband signals like the inspiral of a high-redshift seed BBH.

1. INTRODUCTION

The joint likelihood for a single BBH merger waveform h and some number of white dwarf monochromatic or near-monochromatic waveforms in LISA data d is

$$\log \mathcal{L} \propto -\Delta f \sum_i \frac{\left| d_i - h_i - \sum_\alpha g_{\alpha,i} \right|^2}{2 S_i}, \tag{1}$$

where the sum runs over frequency-domain components and S_i is the noise PSD at frequency i. Expanding to quadratic order about $h_i = h_{i,0}$ in terms of the parameters θ that control the waveform, we have

$$\begin{aligned} \log \mathcal{L} \sim -\Delta f \sum_i \frac{1}{2 S_i} \Bigg(& \left| d_i - h_{i,0} - G_i \right|^2 - 2 \Re \left(d_i - h_{i,0} - G_i \right)^* \frac{\partial h_i}{\partial \theta^a} \left(\theta^a - \theta^a_0 \right) \\ & + \left(\theta^a - \theta_0^a \right) \left(\frac{\partial h_i^*}{\partial \theta^a} \frac{\partial h_i}{\partial \theta^b} + \Re \left(d_i - h_{i,0} - G_i \right)^* \frac{\partial^2 h_i}{\partial \theta^a \partial \theta^b} \right) \left(\theta^b - \theta^b_0 \right) \Bigg), \end{aligned} \tag{2}$$

where

$$G_i \equiv \sum_\alpha g_{\alpha,i}. \tag{3}$$

Collecting terms, we see that we have a Gaussian likelihood for the parameters θ, with

$$\log \mathcal{L} \sim - \left(A + B_a \Delta \theta^a + \frac{1}{2} \Delta \theta^a C_{ab} \Delta \theta^b \right), \tag{4}$$

with

$$B_a = \Delta f \sum_i \frac{1}{S_i} \left(- \Re \left(d_i - h_{i,0} - G_i \right)^* \frac{\partial h_i}{\partial \theta^a} \right) \tag{5}$$

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and

$$C_{ab} = \Delta f \sum_i \frac{1}{S_i} \left(\frac{\partial h_i^*}{\partial \theta^a} \frac{\partial h_i}{\partial \theta^b} + \Re \left(d_i - h_{i,0} - G_i \right)^* \frac{\partial^2 h_i}{\partial \theta^a \partial \theta^b} \right). \tag{6}$$

Now we make the assumption that the sources making up G are *narrowband*, that is that their S/N accumulates over a small number of bins around $i = i_0$, such that

$$\frac{\left|g_{\alpha,i}\right|^2}{S_i} \sim \begin{cases} \mathcal{O}\left(\rho_\alpha^2\right) & |i - i_0| \sim \mathcal{O}\left(1\right) \\ 0 & \text{otherwise} \end{cases}. \tag{7}$$

We further assume that h and therefore h_0 are *broadband*, so

$$\frac{\left|h_i\right|^2}{S_i} \sim \frac{\mathcal{O}\left(\rho_h^2\right)}{N} \tag{8}$$

where $N \gg 1$ is the number of bins over which the S/N of h accumulates.

Let us suppose that the data are composed of signals like h and G plus colored noise n with PSD S:

$$d_i = \bar{h}_i + \bar{G}_i + n_i \tag{9}$$