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Memory in the Population of GW Signals Detected by LVK

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ABSTRACT

Waveform inception.

1. METHODOLOGY

Gravitational wave memory is an effect predicted by general relativity, in which the passage of a gravitational wave causes a permanent displacement in the positions of test masses. This effect is expected to be present in the signals detected by the LIGO-Virgo-KAGRA (LVK) collaboration, and its detection would provide a new test of general relativity and insights into the nature of gravity. Something, something, something IR triangle. It would be wicked cool to detect, and we may have already observed it and just not know.

Gravitational memory is a non-linear effect in the wave zone. Suppose a gravitational wave with strain h is passing through a detector. The memory effect causes additional strain, h_m, to occur, which is roughly

$$h_m(t) \sim \frac{1}{2} \int_{-\infty}^{t} \mathrm{d}s \, \dot{h}^2(s). \tag{1}$$

Most waveform models do not incorporate the memory effect, but it can be calculated straightforwardly from the strain output by any given waveform model.

Since existing waveform models do not include the memory effect, the catalogs of events detected by the LVK collaboration do not include it in their estimation of each observed event's parameters, either. Suppose we have a set of parameter samples, taken from the posterior distribution of parameters, θ, for a given event, $\{\theta_i \mid i = 1, \ldots, N\}$ (here the parameters included in θ are the usual source parameters—masses, spins, distances, etc—as well as any other parameters needed to describe the observation, such as detector calibration parameters, etc). The log-likelihood used to produce these samples takes the form

$$\begin{aligned}\log \mathcal{L} &= -\frac{1}{2} \left\langle d - R(\theta)h(\theta) \mid d - R(\theta)h(\theta) \right\rangle + C \\ &= -\frac{1}{2} \left\langle d - \tilde{h}(\theta) \mid d - \tilde{h}(\theta) \right\rangle + C, \end{aligned} \tag{2}$$

where d is the data, $h(\theta)$ is the waveform model evaluated at parameters θ, $R(\theta)$ is the detector response (including calibration effects), $\tilde{h} = Rh$ is the waveform projected into the data using the detector response, $\langle \cdot \mid \cdot \rangle$ is the usual noise-weighted inner product, and C is an (unimportant) parameter-independent constant. Let $r(\theta)$ denote the *residuals* obtained by evaluating the waveform model at parameters θ and subtracting it from the data, i.e. $r(\theta) = d - R(\theta)h(\theta) = d - \tilde{h}(\theta)$; then the likelihood is

$$\log \mathcal{L} = -\frac{1}{2} \left\langle r(\theta) \mid r(\theta) \right\rangle + C. \tag{3}$$

If we now wish to add in the effects of memory, we can calculate the memory contribution to the strain, $h_m(\theta)$, and add it to the waveform model, so that the likelihood becomes

$$\log \mathcal{L} = -\frac{1}{2} \left\langle r(\theta) - \tilde{h}_m(\theta) \mid r(\theta) - \tilde{h}_m(\theta) \right\rangle + C. \tag{4}$$

If we wish to *detect* the memory effect, then we can add a "memory amplitude" parameter, A_m, which multiplies the memory contribution to the waveform, so that the likelihood becomes

$$\begin{aligned}\log \mathcal{L} &= \\ &- \frac{1}{2} \left\langle r(\theta) - A_m \tilde{h}_m(\theta) \mid r(\theta) - A_m \tilde{h}_m(\theta) \right\rangle \\ &+ C. \end{aligned} \tag{5}$$

The expected memory effect in GR is recovered when $A_m = 1$; computing the posterior on A_m allows us to test the consistency of the observed memory effect with the

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predictions of GR, and to determine whether the data prefer a non-zero memory effect at all. Conveniently, Eq. (5) takes the form of a Gaussian on A_m, peaking at

$$\hat{A}_m = \frac{\Re \left\langle \tilde{h}_m(\theta) \mid r(\theta) \right\rangle}{\left\langle \tilde{h}_m(\theta) \mid \tilde{h}_m(\theta) \right\rangle}, \tag{6}$$

with standard deviation of

$$\sigma^2_{A_m} = \frac{1}{\left\langle \tilde{h}_m(\theta) \mid \tilde{h}_m(\theta) \right\rangle}. \tag{7}$$

Marginalizing A_m out of the likelihood with an (improper) flat prior, we obtain the marginal likelihood $\bar{\mathcal{L}}$:

$$\begin{aligned} \log \bar{\mathcal{L}} = \log \int \mathrm{d}A_m \exp\left(\log \mathcal{L}\right) = \\ -\frac{1}{2}\left(\langle r(\theta) \mid r(\theta)\rangle - \hat{A}_m \Re \left\langle \tilde{h}_m(\theta) \mid r(\theta) \right\rangle\right) \\ -\frac{1}{2}\log\left(2\pi \left\langle \tilde{h}_m \mid \tilde{h}_m \right\rangle\right) + C. \end{aligned} \tag{8}$$

If we are given a set of samples as above, drawn from a posterior based on the original, no-memory likelihood $\mathcal{L}$, then the ratio of the marginal likelihood to the original likelihood serves as an importance weight that can be applied to the samples to obtain a new set of samples drawn from the posterior based on the marginal likelihood $\bar{\mathcal{L}}$:

$$\begin{aligned} \log w_i = \log \bar{\mathcal{L}}(\theta_i) - \log \mathcal{L}(\theta_i) = \\ \frac{1}{2}\hat{A}_m \Re \left\langle \tilde{h}_m(\theta_i) \mid r(\theta_i) \right\rangle \\ -\frac{1}{2}\log\left(2\pi \left\langle \tilde{h}_m \mid \tilde{h}_m \right\rangle\right). \end{aligned} \tag{9}$$

If the memory effect is not very strong in any individual event, then the importance weights will be close to 1, and the re-weighted samples will be very similar to the original samples. Each sample can also be augmented by a value of A_m drawn from the Gaussian distribution described above, to obtain a new set of samples drawn from the posterior on A_m as well.

We can see that the key parts of the calculation of the memory posterior over a catalog of events are

1. the calculation of the memory contribution to the waveform, $h_m(\theta)$, for each event;
2. the computation of the detector response, $R(\theta)$;
3. and the calculation of the inner products $\left\langle r(\theta_i) \mid \tilde{h}_m(\theta_i) \right\rangle$ and $\left\langle \tilde{h}_m(\theta_i) \mid \tilde{h}_m(\theta_i) \right\rangle$ using the noise-weighted inner product.

The latter two steps should be straightforward to extract from Bilby using the configuration settings for each original posterior estimation from the catalog. Keefe can supply the first component. Note that the above computation *does not require the log-likelihood from the original posterior estimation*, but only the samples drawn from the original posterior, and the ability to compute the detector response and the inner products. This means that we can apply this method to any existing posterior samples, regardless of how they were generated, as long as we can compute the detector response and the inner products for each sample. It may be easiest to simply compute the above using codes of our own (effectively ignoring calibration parameters, etc) rather than trying to duplicate the Bilby setup?

Once posteriors on A_m are obtained for each event, we can combine them in the usual hierarchical way (known to Max and Will) to obtain a posterior on the population distribution of A_m across the catalog, and to determine whether the data prefer a non-zero memory effect in the population as a whole.

1.1. *Hierarchical analysis*

We simultaneously model the memory and all the usual astrophysical parameters following the hierarchical framework of (Payne et al. 2023). We assume we start from regular PE samples θ_i that have been enhanced with $\hat{A}_m$ and σ_{A_m}, henceforth μ_i and σ_i for each sample, as described above. This defines the conditional distribution on A given θ_i, that is

$$p(A \mid \theta_i) = \mathcal{N}(A \mid \mu_i, \sigma_i). \tag{10}$$

We can leverage this within our hierarchical integral to marginalize over A analytically without the usual Monte Carlo sum.

The hierarchical likelihood for population parameters Λ requires computing an integral of the form

$$I = \int \frac{p(A,\theta)}{W(\theta)}\, p(A,\theta \mid \Lambda)\, \mathrm{d}A \mathrm{d}\theta\,. \tag{11}$$

Where $p(A,\theta)$ is the posterior and $W(\theta)$ is an importance weight that encodes the sampling prior and any necessary Jacobians; we will assume that the prior on A is flat, so that W is independent of A. Since $p(A,\theta) = p(A \mid \theta)\, p(\theta)$, we can rewrite the above as

$$I = \int p(A \mid \theta)\, p(\theta)\, p(A,\theta \mid \Lambda)\, \mathrm{d}\theta\, \mathrm{d}A\,, \tag{12}$$

where we have further assumed that the population model factorizes as $p(A,\theta) = p(A \mid \theta)\, p(\theta)$.

We can now use the fact that we have N samples from the posterior θ_i to approximate the integral as

$$I \approx \int \frac{1}{N} \sum_i p(A \mid \theta_i)\, \delta(\theta - \theta_i)\, \frac{p(A, \theta \mid \Lambda)}{W(\theta)}\, \mathrm{d}\theta\, \mathrm{d}A\,, \\ = \frac{1}{N} \sum_i \int p(A \mid \theta_i)\, \frac{p(A, \theta_i \mid \Lambda)}{W(\theta_i)}\, \mathrm{d}A\,. \quad (13)$$

If we further assume that the population model factorizes as $p(A, \theta \mid \Lambda) = p(A \mid \Lambda_A)\, p(\theta \mid \Lambda_\theta)$, then

$$I \approx \frac{1}{N} \sum_i \frac{p(\theta_i \mid \Lambda_\theta)}{W(\theta_i)} \int p(A \mid \theta_i)\, p(A \mid \Lambda_A)\, \mathrm{d}A\,. \quad (14)$$

Now further assume a Gaussian population model on A with mean μ_Λ and standard deviation σ_Λ, so that $p(A \mid \Lambda_A) = \mathcal{N}(A \mid \mu_\Lambda, \sigma_\Lambda)$; then the integral over A can be performed analytically, yielding

$$I \approx \frac{1}{N} \sum_i \frac{p(\theta_i \mid \Lambda_\theta)}{W(\theta_i)} \int \mathcal{N}(A \mid \mu_i, \sigma_i)\, \mathcal{N}(A \mid \mu_\Lambda, \sigma_\Lambda)\, \mathrm{d}A\,, \\ = \frac{1}{N} \sum_i \frac{p(\theta_i \mid \Lambda_\theta)}{W(\theta_i)} \mathcal{N}(\mu_i \mid \mu_\Lambda, \sqrt{\sigma_i^2 + \sigma_\Lambda^2})\,. \quad (15)$$

This is the final expression we need to compute the hierarchical likelihood. Everything else is as in a usual populations analysis.

We will assume that there are no selection effects on A, which is a reasonable assumption (Magee et al. 2024).

Software: show your work! (Luger et al. 2021).

REFERENCES

Luger, R., Bedell, M., Foreman-Mackey, D., et al. 2021, arXiv e-prints, arXiv:2110.06271. https://arxiv.org/abs/2110.06271

Magee, R., Isi, M., Payne, E., et al. 2024, PhRvD, 109, 023014, doi: 10.1103/PhysRevD.109.023014

Payne, E., Isi, M., Chatziioannou, K., & Farr, W. M. 2023, PhRvD, 108, 124060, doi: 10.1103/PhysRevD.108.124060