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A Useful Prior for Polarization Quadratures

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ABSTRACT

We say something about marginalization of quadratures with a Gaussian likelihood and a prior that is conditionally Gaussian but marginally flat (or any other shape desired).

1. INTRODUCTION

Quadrature introduction, should probably introduce via Isi (2022).

Because it makes analytical marginalization over quadratures easy (Hogg et al. 2020), we often want to impose an isotropic Gaussian prior over polarization quadratures with some scale parameter A_0:

$$\mathbf{a} \sim N\left[0, a_0 \mathbf{I}\right]. \tag{1}$$

At fixed scale, however, this induces a prior on the amplitude $a \equiv |\mathbf{a}|$ that has a power-law behavior as $a \to 0$:

$$p\left(a \mid a_0\right) = \frac{a^{n-1}}{2^{n/2-1}\Gamma\left(\frac{n}{2}\right) a_0^n} \exp\left[-\frac{a^2}{2a_0^2}\right], \tag{2}$$

where n is the number of quadrature components (i.e. $n = 4$ for electromagnetic radiation or gravitational radiation in general relativity with a sine and cosine quadrature for each of two polarizations in each Fourier mode).

If we are trying to assess the presence of a mode this prior is inconvenient because it places zero weight on $a = 0$ (i.e. the absence of the mode). This is purely a "volume" effect, arising from the fact that the multivariate Gaussian prior in Eq. (1) places finite weight at the origin.

If we promote a_0 to a parameter, with prior $p\left(a_0\right)$ and use the n measurements—the components of $\mathbf{a}$—to constrain it, then by tuning the prior on a_0 we can control the

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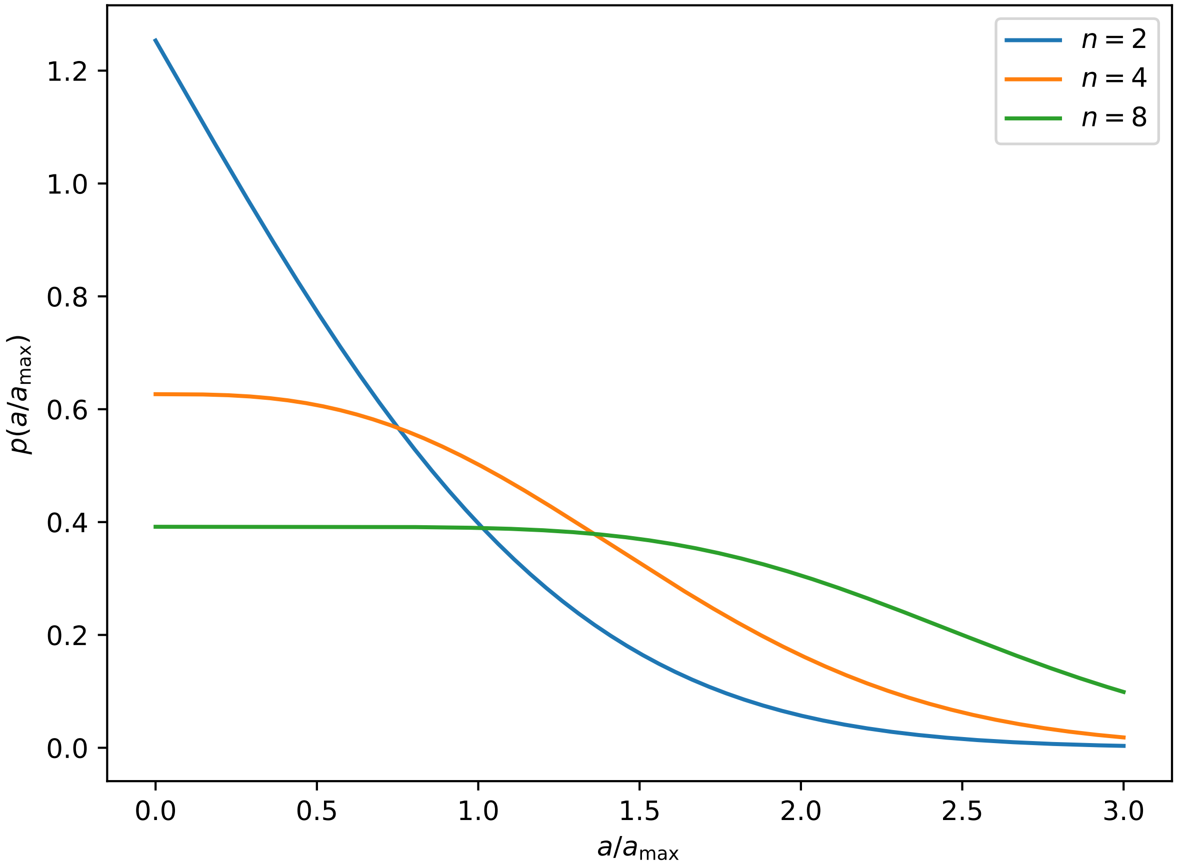

Figure 1. The prior as a function of n.

marginal prior on a. To induce, for example, a flat prior on a as $a \to 0$, let the prior on a_0 be flat up to some maximum (quadrature) scale a_{max}. Then

$$p\left(a, a_0\right) = p\left(a \mid a_0\right) p\left(a_0\right) = \frac{a^{n-1}}{2^{n/2-1}\Gamma\left(\frac{n}{2}\right) a_0^n a_{\mathrm{max}}} \exp\left[-\frac{a^2}{2a_0^2}\right] \tag{3}$$

induces a marginal prior on a that is

$$p\left(a\right) \equiv \int \mathrm{d}a_0\, p\left(a, a_0\right) = \frac{\Gamma\left(\frac{n-1}{2}, \frac{a^2}{2a_{\mathrm{max}}^2}\right)}{a_{\mathrm{max}}\sqrt{2}\Gamma\left(\frac{n}{2}\right)} = \mathrm{const} + \mathcal{O}\left(\frac{a}{a_{\mathrm{max}}}\right)^{n-1} \tag{4}$$

REFERENCES

Hogg, D. W., Price-Whelan, A. M., & Leistedt, B. 2020, arXiv e-prints, arXiv:2005.14199, doi: 10.48550/arXiv.2005.14199

Isi, M. 2022, arXiv e-prints, arXiv:2208.03372, doi: 10.48550/arXiv.2208.03372