

**Lecturer**

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**Pattern Recognition (CS254) - Sheet 1****[10 Points]**

Preliminary Discussion 27.09.2012

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This theoretical exercise does not have to be presented orally.

**Exercise 1 - Multivariate Normal Distribution****[4 Points]**

Consider a bivariate normal population with  $\mu_1 = 0, \mu_2 = -2, \sigma_{11} = 5, \sigma_{22} = 1$ , and with cross correlation coefficient,  $\rho_{12} = \frac{1}{2}$ .

- Write out the bivariate normal density.
- Write out the squared generalized distance expression  $(\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})$  as a function of  $x_1$  and  $x_2$ .
- Determine the main axes and sketch the constant-density contour of one standard deviation.

**Exercise 2 - Independence****[3 Points]**

Consider  $\mathbf{X} = [X_1, X_2, X_3]^T$  distributed according to  $\mathcal{N}(\mathbf{X} | \boldsymbol{\mu}, \boldsymbol{\Sigma})$  with

$$\boldsymbol{\mu} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

Which of the following random variables are independent? Explain.

- $X_1$  and  $X_2$
- $X_1$  and  $X_3$
- $X_2$  and  $X_3$
- $(X_1, X_3)$  and  $X_2$
- $X_1 - X_3$  and  $X_1 - 3X_2 + X_3$

**Exercise 3 - Conditional Distribution****[3 Points]**

Specify the conditional distribution of  $X_1$ , given that  $X_2 = x_2$  for the joint distribution  $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ . Compare the conditional distribution  $P(X_1 | X_2 = 1)$  to the marginal distribution  $P(X_1)$  in a plot.

$$\boldsymbol{\mu} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 3 \end{bmatrix}$$