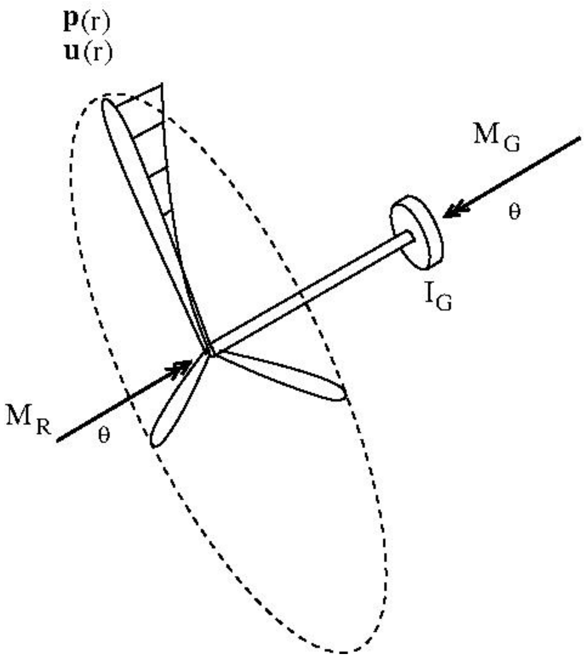
How to use R-K-N to solve the 4 DOF system from Module#10.



As long as the blades do not pitch the mass and stiffness matrix remains constant. Otherwise, you must update these before calling R-K-N algorithm

$$\mathbf{M} = \begin{bmatrix} I & \int m\cdot r\cdot u_y^{1f}(r)dr & \int m\cdot r\cdot u_y^{1e}(r)dr & m\cdot r\cdot u_y^{2f}(r)dr \\ \int m\cdot r\cdot u_y^{1f}(r)dr & GM_1 & 0 & 0 \\ \int m\cdot r\cdot u_y^{1e}(r)dr & 0 & GM_2 & 0 \\ \int m\cdot r\cdot u_y^{2f}(r)dr & 0 & 0 & GM_3 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \omega_1^2 GM_1 & 0 & 0 \\ 0 & 0 & \omega_2^2 GM_2 & 0 \\ 0 & 0 & 0 & \omega_3^2 GM_3 \end{bmatrix}$$

Runge-Kutta-Nyström integration scheme to integrate the EOMs (4th order)

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{\mathbf{g}} - \mathbf{C}\dot{\mathbf{x}} - \mathbf{K}\mathbf{x} = \mathbf{f}(\dot{\mathbf{x}}, \mathbf{x}, t)$$

$$\ddot{\mathbf{x}}^n = \mathbf{M}^{-1}\mathbf{f}(\dot{\mathbf{x}}^n, \mathbf{x}^n, t^n) = \mathbf{g}(\dot{\mathbf{x}}^n, \mathbf{x}^n, t^n)$$

g means solving for the accelerations for different positions and velocities

$$\mathbf{A} = \frac{\Delta t}{2}\ddot{\mathbf{x}}^n$$

$$\mathbf{b} = \frac{\Delta t}{2}(\dot{\mathbf{x}}^n + \tfrac{1}{2}\mathbf{A})$$

$$\mathbf{B} = \frac{\Delta t}{2}\mathbf{g}(\mathbf{x}^n + \mathbf{b}, \dot{\mathbf{x}}^n + \mathbf{A})$$

$$\mathbf{C} = \frac{\Delta t}{2}\mathbf{g}(\mathbf{x}^n + \mathbf{b}, \dot{\mathbf{x}}^n + \mathbf{B})$$

$$\mathbf{d} = \Delta t(\dot{\mathbf{x}}^n + \mathbf{C})$$

$$\mathbf{D} = \frac{\Delta t}{2}\mathbf{g}(\mathbf{x}^n + \mathbf{d}, \dot{\mathbf{x}}^n + 2\mathbf{C})$$

$$\mathbf{x}^{n+1} = \mathbf{x}^n + \Delta t(\dot{\mathbf{x}}^n + \tfrac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C}))$$

$$\dot{\mathbf{x}}^{n+1} = \dot{\mathbf{x}}^n + \tfrac{1}{3}(\mathbf{A} + 2\mathbf{B} + 2\mathbf{C} + \mathbf{D})$$

$$\ddot{\mathbf{x}}^{n+1} = \mathbf{g}(t^{n+1}, \mathbf{x}^{n+1}, \dot{\mathbf{x}}^{n+1})$$

$$t^{n+1} = t^n + \Delta t$$

E.g. at the first step we must solve for the accelerations at position $(\mathbf{x}^n + \mathbf{b}) = \begin{bmatrix} \theta^n + b_1 \\ q_1^n + b_2 \\ q_2^n + b_3 \\ q_3^n + b_4 \end{bmatrix}$ at velocity $\dot{\mathbf{x}}^n + \mathbf{A} = \begin{bmatrix} \dot{\theta}^n + A_1 \\ \dot{q}_1^n + A_2 \\ \dot{q}_2^n + A_3 \\ \dot{q}_3^n + A_4 \end{bmatrix}$

Note: $\mathbf{x}^n, \dot{\mathbf{x}}^n, \mathbf{b}, \mathbf{A}$ are all known at this stage

$$\mathbf{B} = \frac{\Delta t}{2}\mathbf{g}(\mathbf{x}^n + \mathbf{b}, \dot{\mathbf{x}}^n + \mathbf{A})$$

Equations of motion: $\mathbf{g}(\mathbf{x}^n + \mathbf{b}, \dot{\mathbf{x}}^n + \mathbf{A}) = \mathbf{M}^{-1}(-\mathbf{K}\cdot(\mathbf{x}^n + \mathbf{b}) + \mathbf{GF}(\dot{\mathbf{x}}^n + \mathbf{A}))$

To solve this, we need to update the generalized force vector **GF** for a velocity $\dot{\mathbf{x}}^n + \mathbf{A}$

The aerodynamic loads p_y and p_z are first found from the velocity triangle and not updating the induced wind

$$\begin{pmatrix} V_{rel,y} \\ V_{rel,z} \end{pmatrix} = \begin{pmatrix} V_{o,y} \\ V_{o,z} \end{pmatrix} + \begin{pmatrix} W_y \\ W_z \end{pmatrix} + \begin{pmatrix} -(\dot{\theta}^n + A_1)\cdot x \\ 0 \end{pmatrix} - (\dot{q}_1^n + A_2)\begin{pmatrix} u_y^{1f} \\ u_z^{1f} \end{pmatrix} + (\dot{q}_2^n + A_3)\begin{pmatrix} u_y^{1e} \\ u_z^{1e} \end{pmatrix} + (\dot{q}_3^n + A_4)\begin{pmatrix} u_y^{2f} \\ u_z^{2f} \end{pmatrix}$$

That is, you must call the routine that calculates the aerodynamic loads with this updated relative velocity

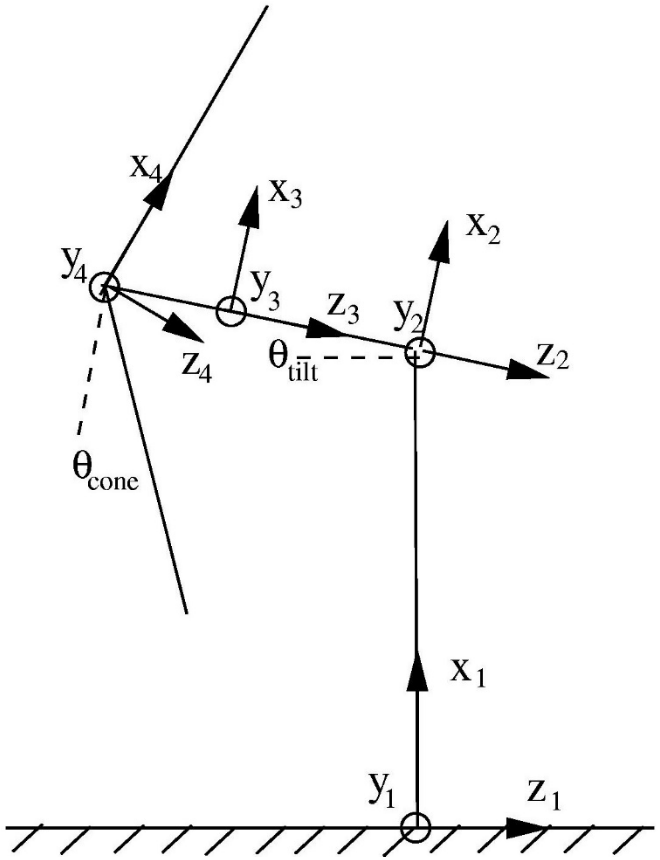
Now you can update the generalized force vector and solve for the accelerations to find $\mathbf{B}$

$$\mathbf{GF} = \begin{bmatrix} \sum_{n=1}^{NB} r p_y(r) dr - M_G(\dot{\theta}^n + A_1) \\ \int p_y u_y^{1f} dr + \int p_z u_z^{1f} dr \\ \int p_y u_y^{1e} dr + \int p_z u_z^{1e} dr \\ \int p_y u_y^{2f} dr + \int p_z u_z^{2f} dr \end{bmatrix}$$

$$\mathbf{B} = \frac{\Delta t}{2} \mathbf{g}(\mathbf{x}^n + \mathbf{b}, \dot{\mathbf{x}}^n + \mathbf{A})$$

$$\mathbf{g}(\mathbf{x}^n + \mathbf{b}, \dot{\mathbf{x}}^n + \mathbf{A}) = \mathbf{M}^{-1}(-\mathbf{K} \cdot (\mathbf{x}^n + \mathbf{b}) + \mathbf{GF}(\dot{\mathbf{x}}^n + \mathbf{A}))$$

You have to do this 3 times more to find **C, D** and $\ddot{\mathbf{x}}^{n+1}$
You may keep the induced wind constant during the entire R-K-N call



To add the gravitational load on the blade

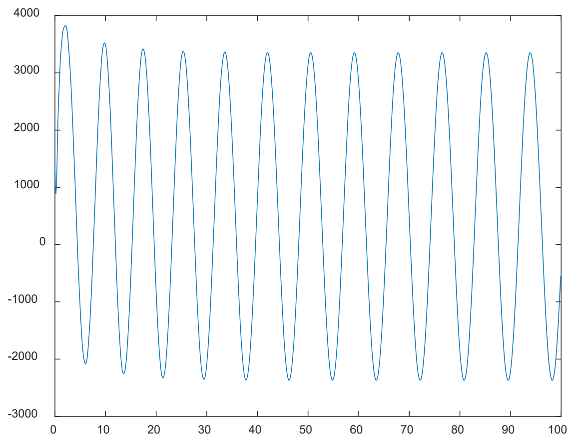
$$\mathbf{p}_{\mathbf{grav}}(r) = \begin{bmatrix} p_{grav,x} \\ p_{grav,y} \\ p_{grav,z} \end{bmatrix} = \mathbf{a}_{14} \cdot \begin{bmatrix} -m(r)g \\ 0 \\ 0 \end{bmatrix}$$

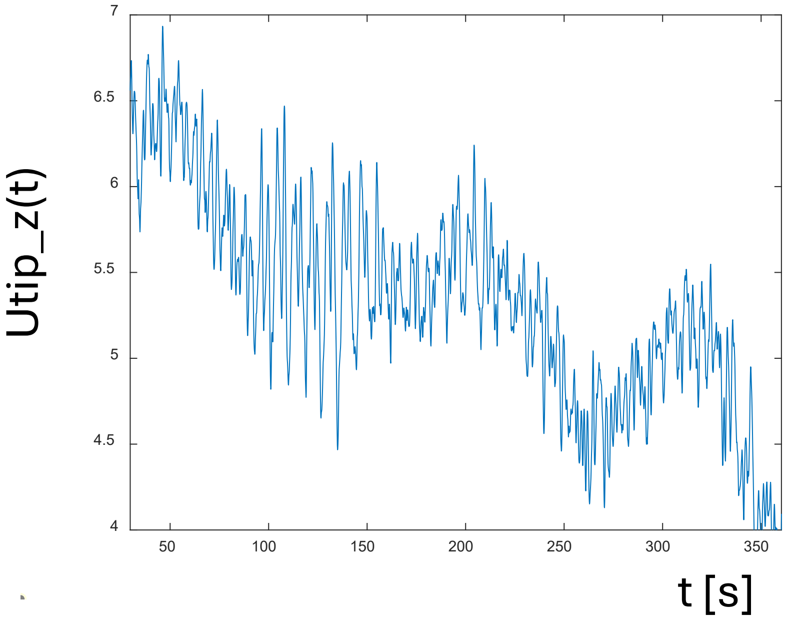
These are then added to the aerodynamic loads

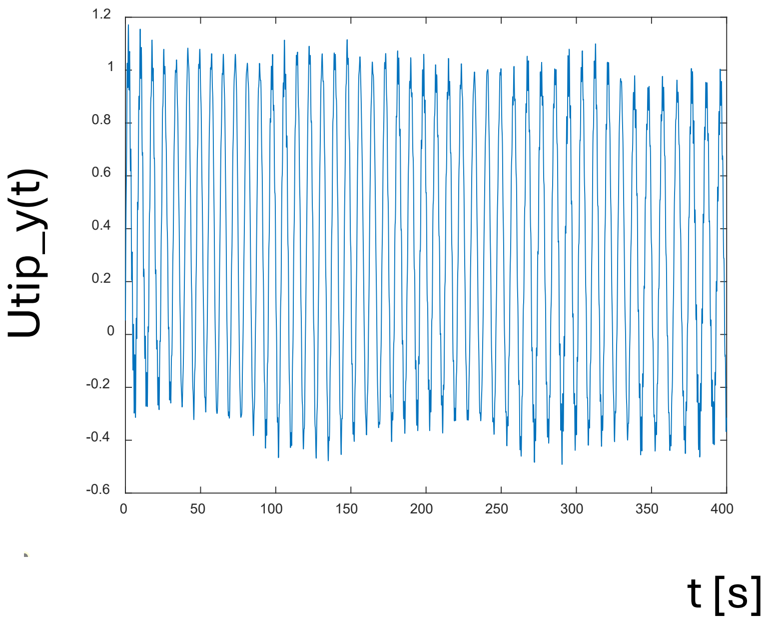
$$p_y(r) = p_{y,aero}(r) + p_{grav,y}$$

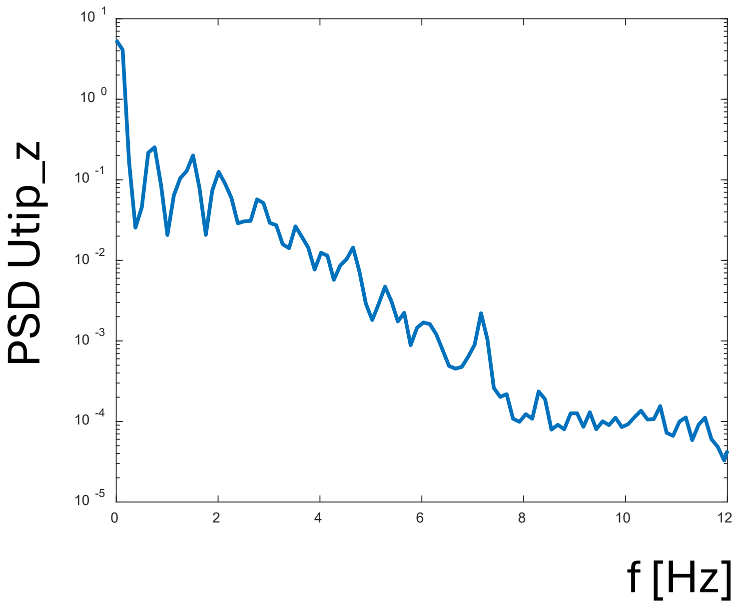
$$p_z(r) = p_{z,aero}(r) + p_{grav,z}$$

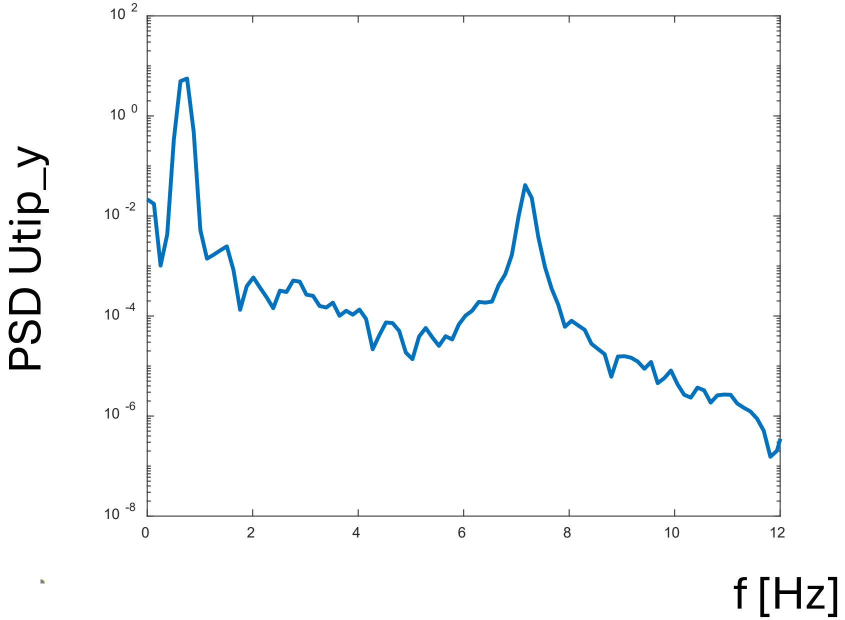
Tangential load as function of time at r= 58.5 m for V=8m/s constant




Utip_z(t)
t [s]
7
6.5
6
5.5
5
4.5
4
50
100
150
200
250
300
350


Utip_y(t)
t [s]
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
50
100
150
200
250
300
350
400


PSD Utip_z
f [Hz]
10 1
10 0
10 -1
10 -2
10 -3
10 -4
10 -5
0
2
4
6
8
10
12


PSD Utip_y
f [Hz]
10 2
10 0
10 -2
10 -4
10 -6
10 -8
0
2
4
6
8
10
12