Unsteady Blade Element Momentum Theory:
Dynamic inflow and dynamic stall model

# Module 3

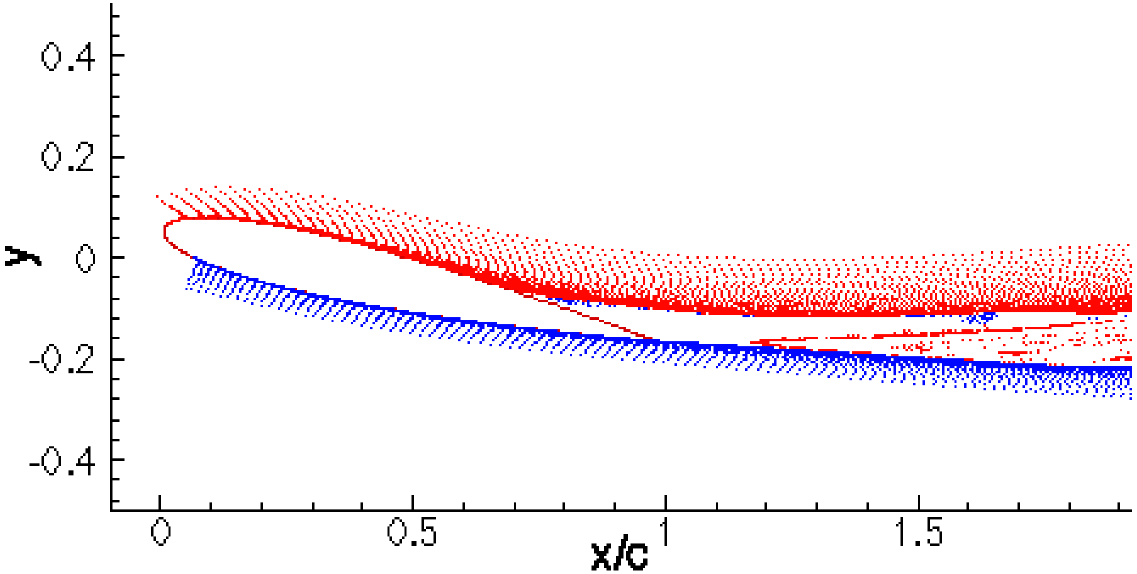
# Agenda

Morning:

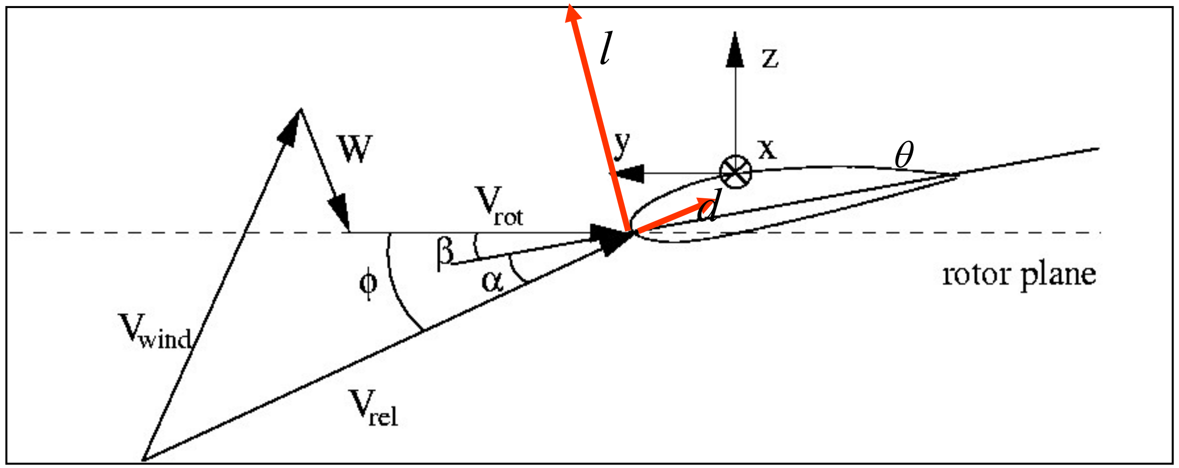
- Recap of induction
- Lecture on dynamic wake and stall
- Hands-on work

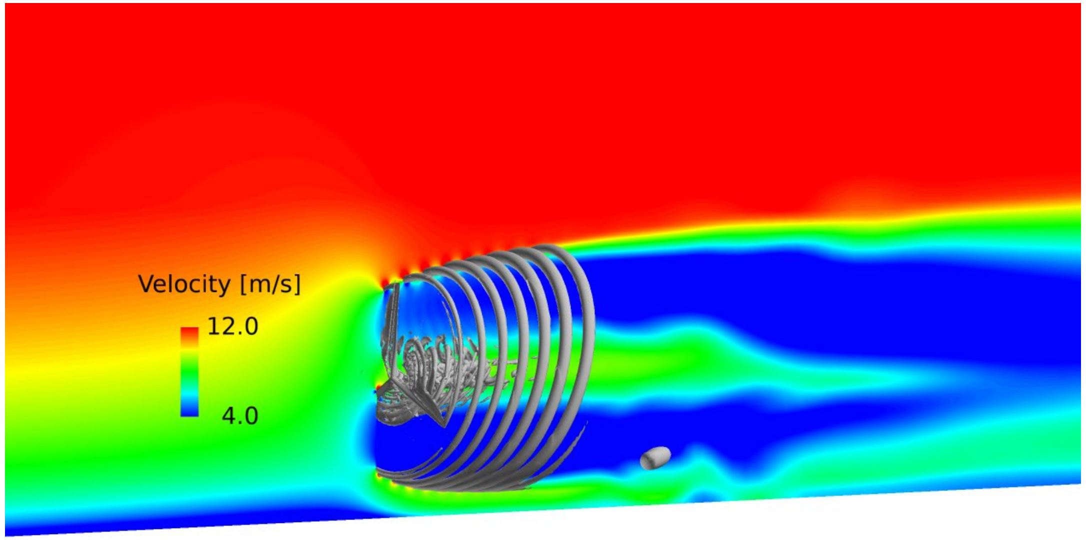
Afternoon:

- Turbo-intro to ASHES
- Hands-on work



# Recap – Quasi steady induced wind speed




We need the induced wind speed **W** to compute the aerodynamic loads

$$\begin{pmatrix} V_{rel,y} \\ V_{rel,z} \end{pmatrix} = \begin{pmatrix} V_{o,y} \\ V_{0,z} \end{pmatrix} + \begin{pmatrix} -\omega \cdot x \\ 0 \end{pmatrix} + \begin{pmatrix} W_y \\ W_z \end{pmatrix}$$

$$l = \tfrac{1}{2}\rho \left|V_{rel}\right|^2 cC_l(\alpha)$$

$$d = \tfrac{1}{2}\rho \left|V_{rel}\right|^2 cC_d(\alpha)$$

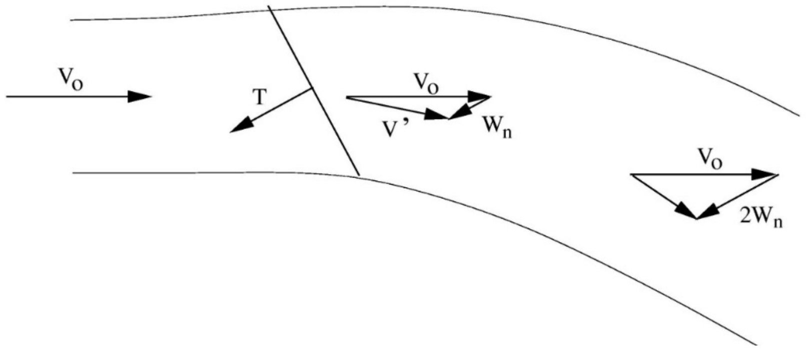
# Quasi steady induced velocities calculated as:

Quasi steady means when the induced wind is in equilibrium with the aerodynamic loads.

$$W_z^{qs} = \frac{-Bl\cos\phi}{4\pi\rho rF\left|\mathbf{V_o} + f_g\mathbf{W}_n\right|}$$

$$W_y^{qs} = \frac{-Bl\sin\phi}{4\pi\rho rF\left|\mathbf{V_o} + f_g\mathbf{W}_n\right|}$$

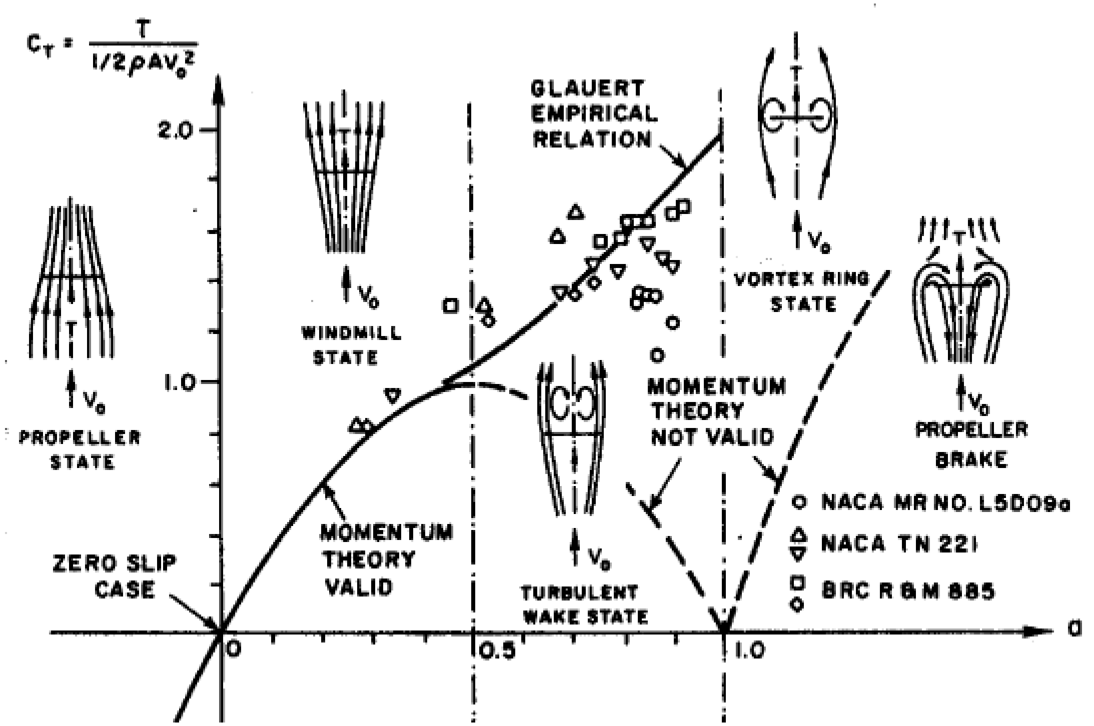
- $f_g$ is the Glauert correction and $F$ is the Prandtl's tip loss correction, see later.
- $\mathbf{W}_n$ is the induced wind normal to the rotor plane as a vector
- $r$ is the radial position along the blade and B the number of blades



# Glauert correction $f_g$

$$a = \frac{-W_z}{V_o}$$

$$f_g = \begin{cases} 1 & for\ a \le \frac{1}{3} \\ \frac{1}{4}(5-3a) & for\ a > \frac{1}{3} \end{cases}$$




# Prandtl's tip loss correction

$$F = \frac{2}{\pi}\cos^{-1}\left(\exp\left(-\frac{B}{2}\frac{R-r}{r\sin|\phi|}\right)\right)$$

## For every element on every blade:

$$V_{rel,y}^n = V_{o,y}^n + W_y^{n-1} - \omega x \cos\theta_{cone}$$

$$V_{rel,z}^n = V_{o.z}^n + W_z^{n-1}$$

$$\left|\mathbf{V_{rel}}\right| = \sqrt{(V_{rel,y}^n)^2 + V_{rel,z}^n)^2}$$

$$\tan\phi^n = \frac{V_{rel,z}^n}{-V_{rel,y}^n}, \quad \alpha^n = \phi^n - (\beta(r) + \theta_p^n)$$

$$C_l(\alpha^n), C_d(\alpha^n)$$

$$l^n = \tfrac{1}{2}\rho\left|\mathbf{V_{rel}}\right|^2 c C_l(\alpha^n)$$

$$d^n = \tfrac{1}{2}\rho\left|\mathbf{V_{rel}}\right|^2 c C_d(\alpha^n)$$

$$p_z^n = l^n\cos\phi^n + d^n\sin\phi^n$$

$$p_y^n = l^n\sin\phi^n - d^n\cos\phi^n$$

estimate the global axial induction $a \approx \dfrac{-\overline{W}_z^{n-1}(r \approx 0.7R)}{V_o(\text{hub})}$ ($\overline{W}$ means the mean from all blades)

Or just use: $a = \dfrac{-W_z^{n-1}(i,b)}{V_o\,(\text{hub})}$

calculate $f_g$ and $F$

$$\left|\mathbf{V}_o + f_g\mathbf{W}_n\right| = \sqrt{(V_{o,y}^n)^2 + (V_{o,z}^n + f_g W_z^{n-1})^2}$$

$$W_z^{qs} = \frac{-Bl^n\cos\phi^n}{4\pi\rho r F\left|\mathbf{V}_o + f_g\mathbf{W}_n\right|}$$

$$W_y^{qs} = \frac{-Bl^n\sin\phi^n}{4\pi\rho r F\left|\mathbf{V}_o + f_g\mathbf{W}_n\right|}$$

Apply dynamic wake filter to go from $(W_y^{qs}, W_z^{qs})$ to $(W_y^n, W_z^n)$

## After blade loop, integrate to find power and thrust

# Please find here some pitfalls to avoid when programming your unsteady BEM

1) Remember to store the induced wind for each element on each blade either in time or as old/new variables, so that you can access the previous value at the specific location.

2) Remember that the induction factor should be positive (0-1), so put a minus sign in front of the axial induced velocity which is negative in it's definition.

$$a = \frac{-W_n}{V_0}$$

3) Also use the induced wind from the previous time step when estimating the term in the denominator when computing the new quasi steady values for the induced wind. This is because at this step the "new" value is not known

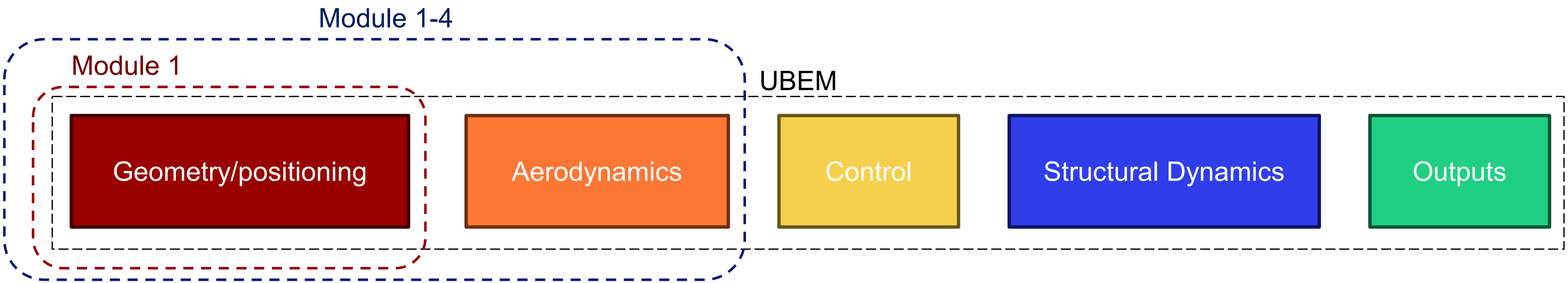
$$\left|\mathbf{V}_o + f_g \mathbf{W}_n\right| = \sqrt{(V_{o,y}^n)^2 + (V_{o,z}^n + f_g W_z^{n-1})^2}$$

4) No need to calculate the last element of the blade at the tip, since Prandtl's tip loss correction might become unstable. Just force the lift and drag to 0 at the tip position.

# Status on our codes – what is it we're doing?

- First modules meant to get a basis code working from input settings to loads and power/thrust
  - After module 2, you can get same results as your steady BEM code from 46300
  - You should also be able to do Q1 and Q2 (ish) in the assignment
  - After today you can do Q3
  - After next time you can do Q4

- Assignment is due on March 10th, but please try to have as much ready as possible for module 5 (7th) where we'll do presentations and peer review of reports



# Learning objectives

**Course LO's**

- Write a computer code to determine the unsteady aerodynamic loads
- Implement dynamic wake and dynamic stall models
- Use a software to generate atmospheric turbulence
- Implement a pitch controller
- Establish the equations of motion for a wind turbines
- Integrate the equations of motion including the coupling with the aerodynamic loads

- **Understand** concept of dynamic inflow, and why it is needed
- **Implement** a dynamic inflow/wake model
- **Analyse** the effect of having the model in the UBEM code

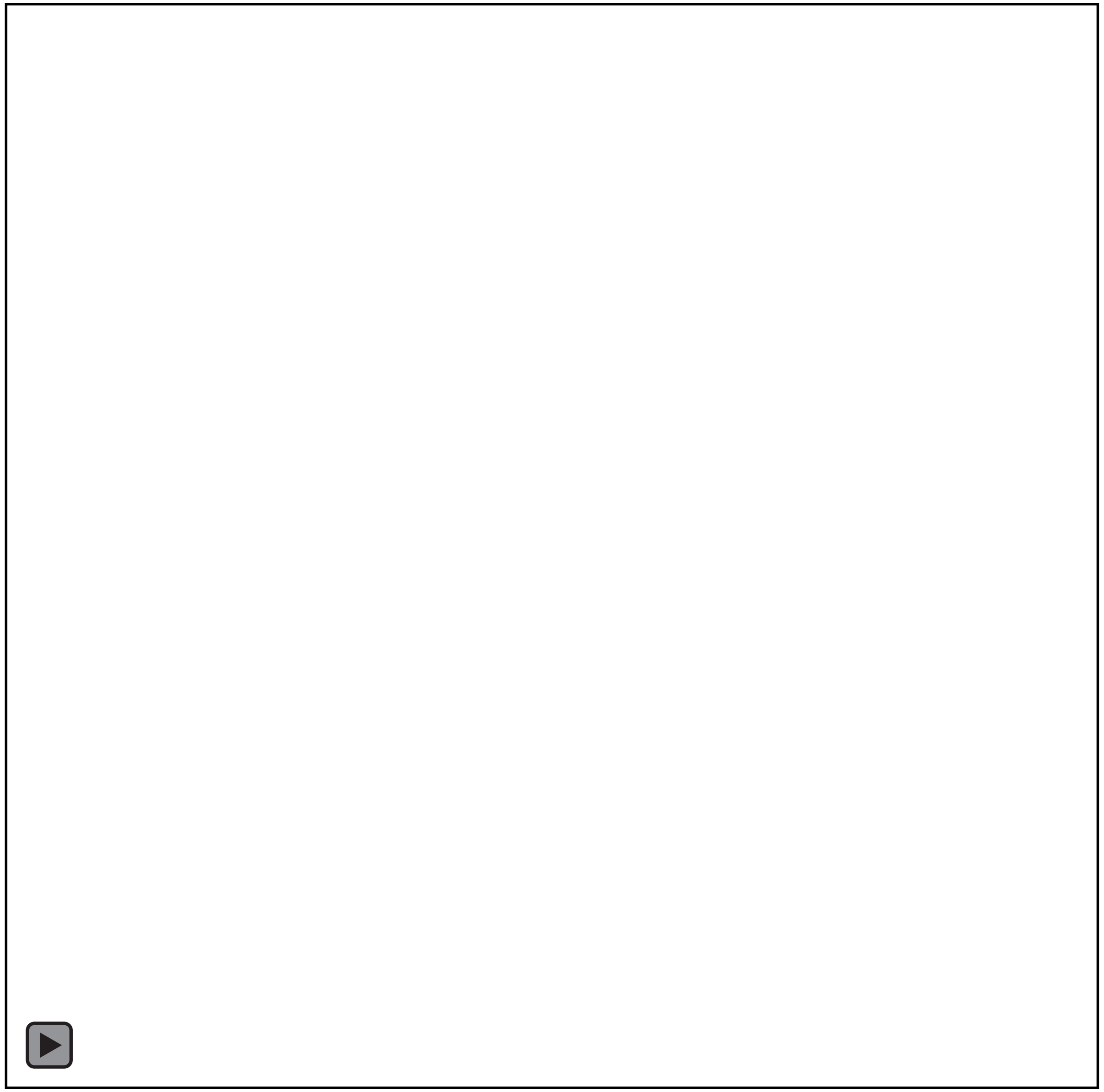
- **Understand** concept of dynamic stall, and why it is needed
- **Implement** a dynamic stall model
- **Analyse** the effect of having the model in the UBEM code

# Are aerodynamics changing instantaneously?

https://www.youtube.com/watch?v=RgUtFm93Jfo

# Are aerodynamics changing instantaneously?

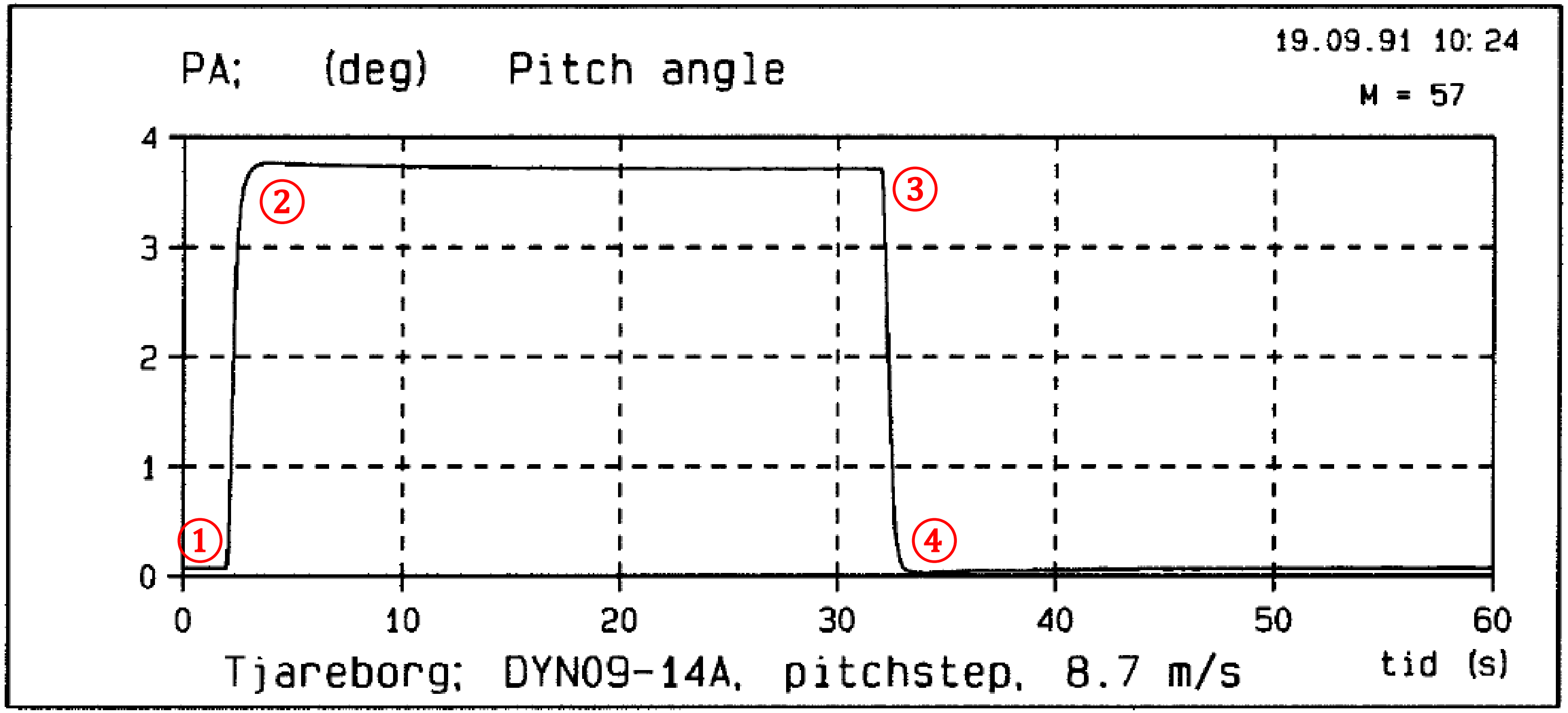
# Dynamic wake model



# Dynamic wake model

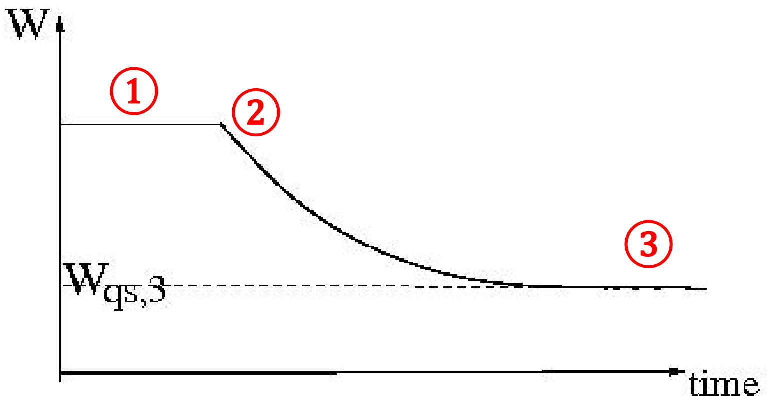
Let's consider the case where the step change of the pitch angle shown below is considered.

**What do you think happens to the power?**

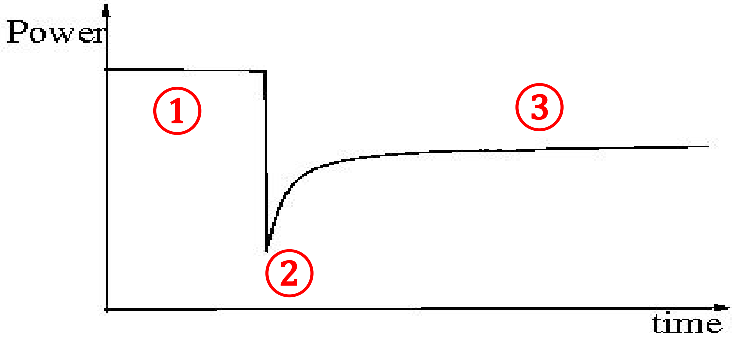


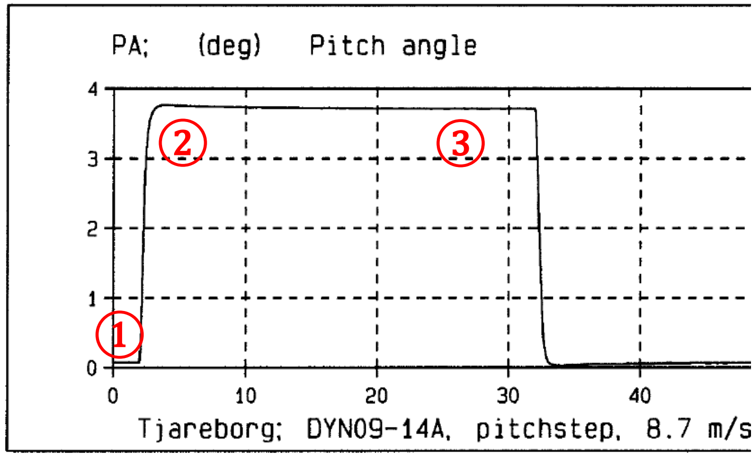
# Dynamic wake model

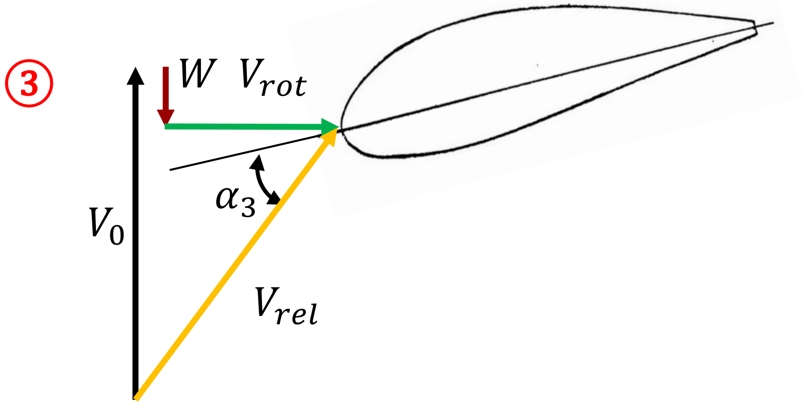
- It takes time for the wake to settle to a new equilibrium and thus the induced velocities to adapt to a thrust value.

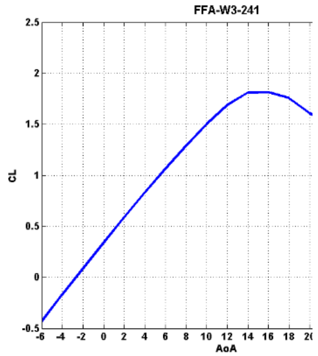



$$\alpha_1 > \alpha_3 > \alpha_2$$

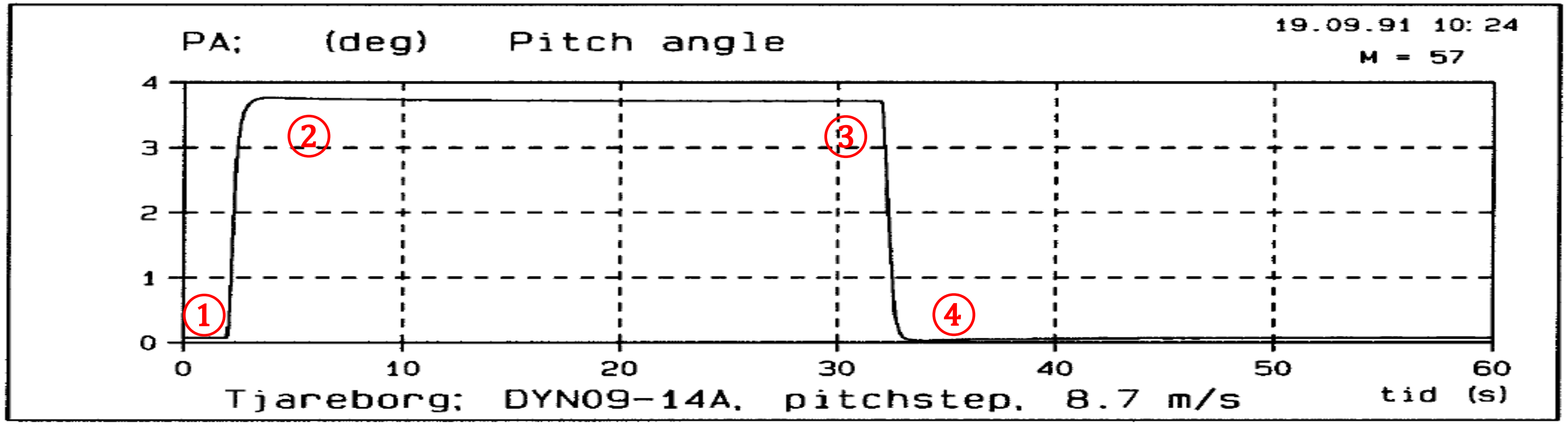


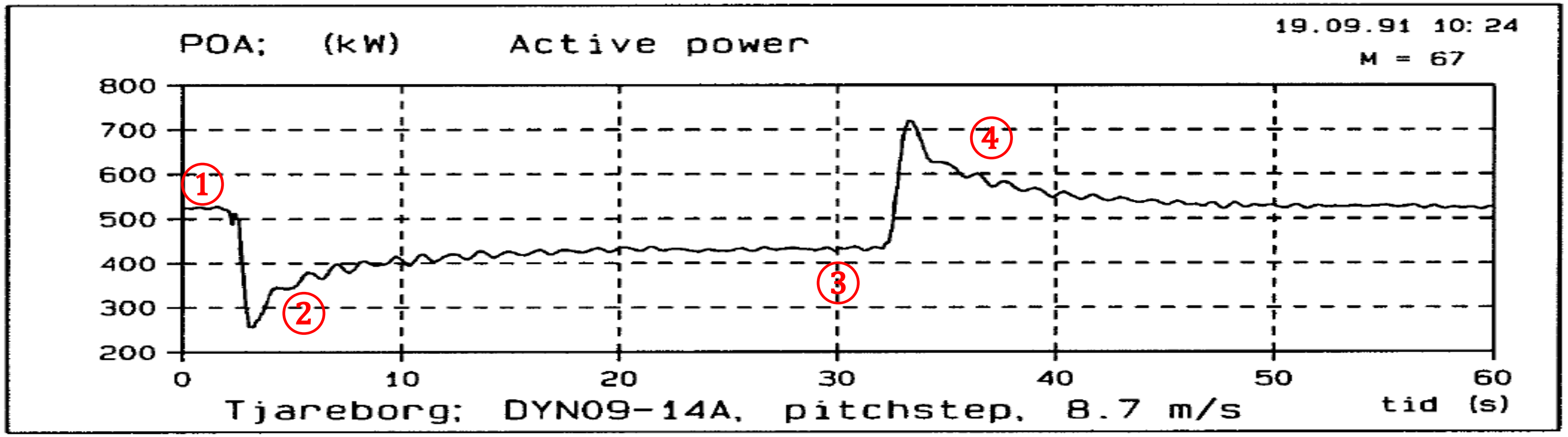



# Dynamic wake model



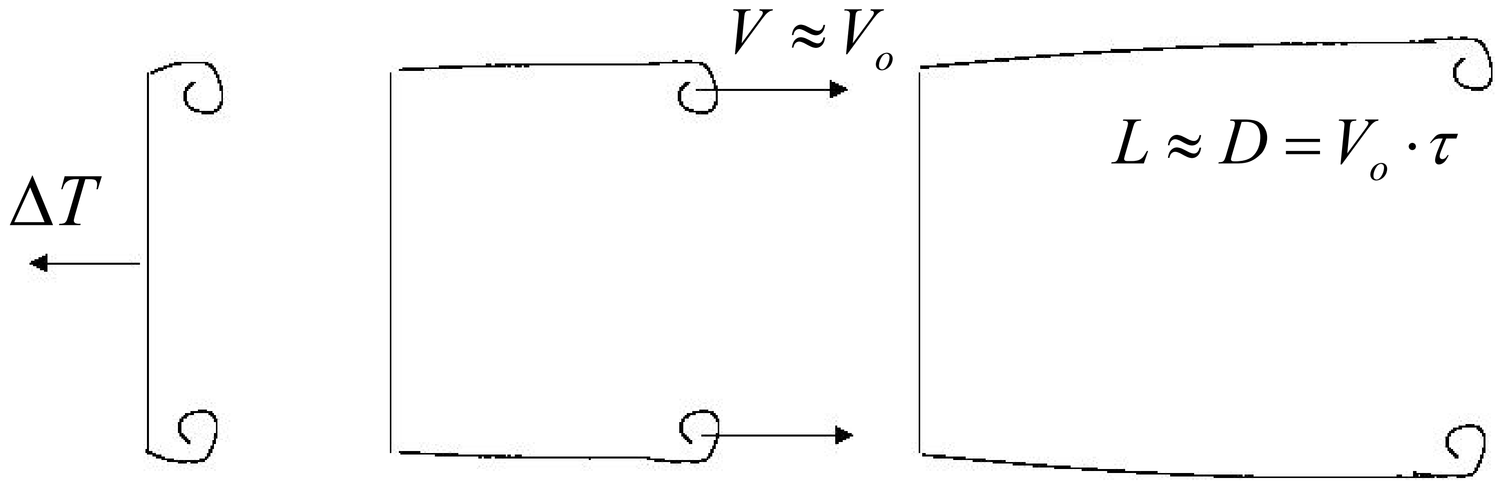



# Dynamic wake model

- When operating conditions of the wind turbine are changed such as sudden change in pitch angle change the flow field cannot adjust instantaneously to the new flow state because of the inertia of the air mass.

- This results in a time delay in the changes of the induced velocity.

- The time scale for this event is the approximately $D/V_0$ which is in the order of 1 to 10 seconds, approximately 100 times longer than the time scale for the dynamic stall.



# Dynamic wake model

$$W_n = \frac{-BL\cos\emptyset}{4\rho\pi rF\left|V_0 + f_g W_n\right|} \quad [1] \qquad W_t = \frac{-BL\sin\emptyset}{4\rho\pi rF\left|V_0 + f_g W_n\right|} \quad [2]$$

- The Eqs. (1) and (2) are only valid when the flow/wake is in equilibrium with the thrust.
- It takes times for the flow to adapt to changing loads.
- Following model has been proposed by S. Øye if the right side is assumed known, it is two first order differential equation with known analytical solution.

$$W_{\text{int}} + \tau_1 \frac{dW_{\text{int}}}{dt} = W_{qs} + k \cdot \tau_1 \frac{dW_{qs}}{dt} \quad [3]$$

$$W + \tau_2 \frac{dW}{dt} = W_{\text{int}} \quad [4]$$

where $k$=0.6 and the time constant $\boldsymbol{\tau_1}$ depends on the axial induction factor *a*. $a = \frac{-W_n}{V_0}$

***Note***: *u*sing Eq. (6), *a* is not allowed to exceed 0.5 (min(a,0.5)).

$$\tau_1 = \frac{1.1}{(1-1.3a)} \cdot \frac{R}{V_o} \quad [5] \qquad \tau_2 = (0.39 - 0.26\left(\frac{r}{R}\right)^2) \cdot \tau_1. \quad [6]$$

# Dynamic wake model: Apply it into the computer code

- Step1:calculate ${W_{qs}}^{i}$ using Eqs. (1) and (2)

- Step2: Estimate right hand side of Eq. (3) using backward difference,

$$H = {W_{qs}}^{i} + k\boldsymbol{\tau}_1 \frac{{W_{qs}}^{i} - {W_{qs}}^{i-1}}{\Delta t}$$

$$W_{\mathrm{int}} + \tau_1 \frac{dW_{\mathrm{int}}}{dt} = W_{qs} + k \cdot \tau_1 \frac{dW_{qs}}{dt} \quad [3]$$

- Step3: Solve Eq. (3) analytically,

$${W_{int}}^{i} = H + \left({W_{int}}^{i-1} - H\right)\exp\left(\frac{-\Delta t}{\boldsymbol{\tau}_1}\right)$$

- Step4: Solve Eq. (4) analytically,

$$W + \tau_2 \frac{dW}{dt} = W_{\mathrm{int}} \quad [4]$$

$$W^{i} = {W_{int}}^{i} + \left(W^{i-1} - {W_{int}}^{i}\right)\exp\left(\frac{-\Delta t}{\boldsymbol{\tau}_2}\right)$$

- In each time step, some values must be stored in each element

${W_{qs}}^{i-1}$, ${W_{int}}^{i-1}$, and $W^{i-1}$

# Dynamic wake model:
# Apply it into the computer code

$$V^n_{rel,y} = V^n_y + W^{n-1}_y - \omega x \cos\theta_{cone}$$

$$V^n_{rel,z} = V^n_z + W^{n-1}_z$$

$$\tan\phi^n = \frac{V^n_{rel,z}}{-V^n_{rel,y}}, \quad \alpha^n = \phi^n - (\beta + \theta^n_p)$$

$$C_l(\alpha^n), C_d(\alpha^n)$$

$$\left|\mathbf{V_{rel}}\right| = \sqrt{(V^n_{rel,y})^2 +'(V^n_{rel,y})^2}$$

$$L^n = \tfrac{1}{2}\rho\left|\mathbf{V_{rel}}\right|^2 c C_l(\alpha^n)$$

$$D^n = \tfrac{1}{2}\rho\left|\mathbf{V_{rel}}\right|^2 c C_d(\alpha^n)$$

$$p^n_z = L^n\cos\phi^n + D^n\sin\phi^n$$

$$p^n_y = L^n\sin\phi^n - D^n\cos\phi^n$$

$$\left|\mathbf{V}_o + f_g\mathbf{W_n}\right| = \sqrt{V^2_{o,y} + (V_{o,z} + f_g W^{n-1}_z)^2}$$

$$\mathbf{W_n} = \begin{pmatrix} 0 \\ W_z \end{pmatrix}, \quad \mathbf{V_o} = \begin{pmatrix} V_{0,y} \\ V_{0,z} \end{pmatrix}$$

$$W^{qs}_z = \frac{-BL^n\cos\phi^n}{4\pi\rho r F\left|\mathbf{V}_o + f_g\mathbf{W_n}\right|}$$

$$W^{qs}_y = \frac{-BL^n\sin\phi^n}{4\pi\rho r F\left|\mathbf{V}_o + f_g\mathbf{W_n}\right|}$$

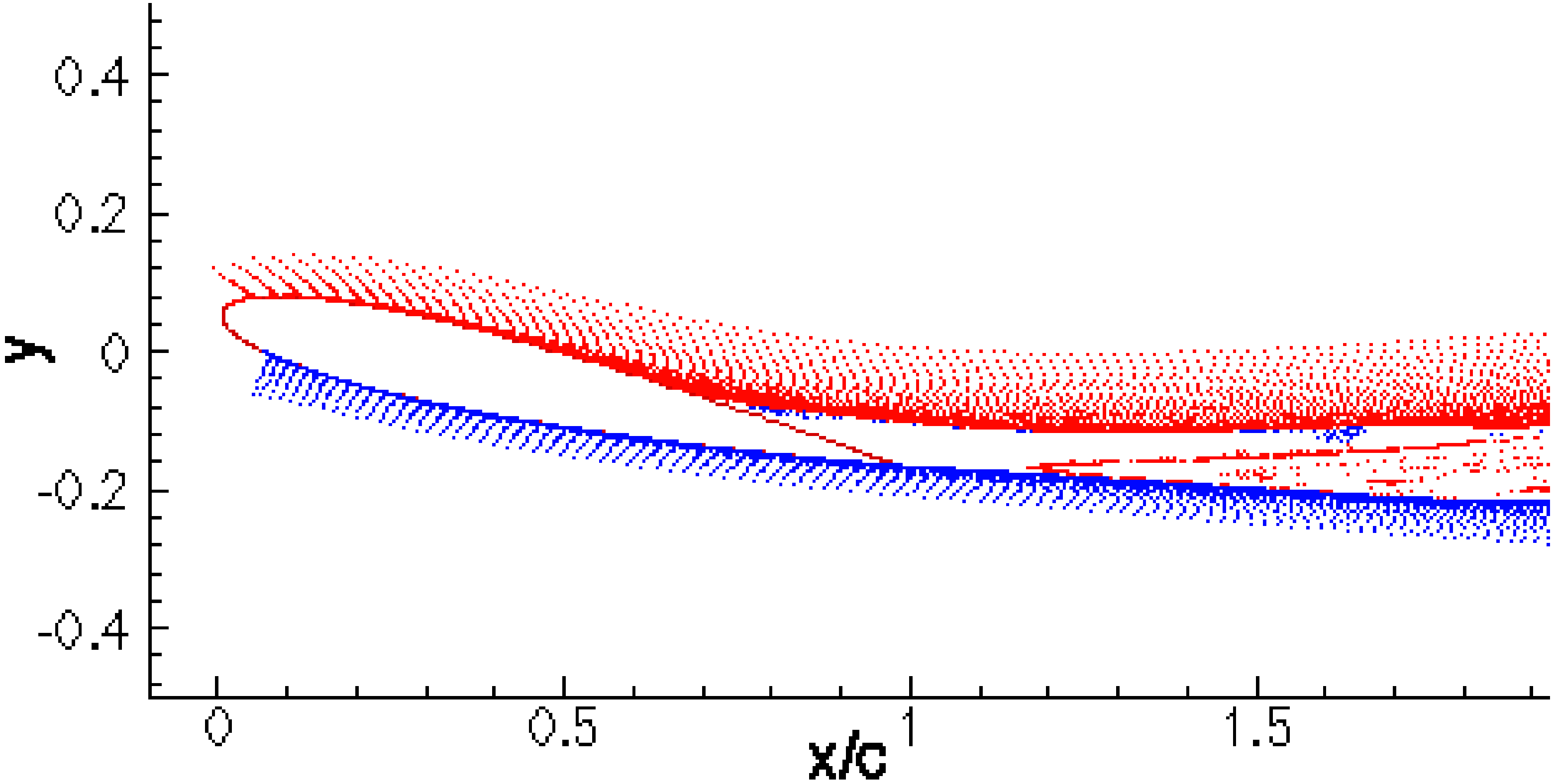
Dynamic wake filter

$W^n_z$ and $W^n_y$ before yaw/tilt

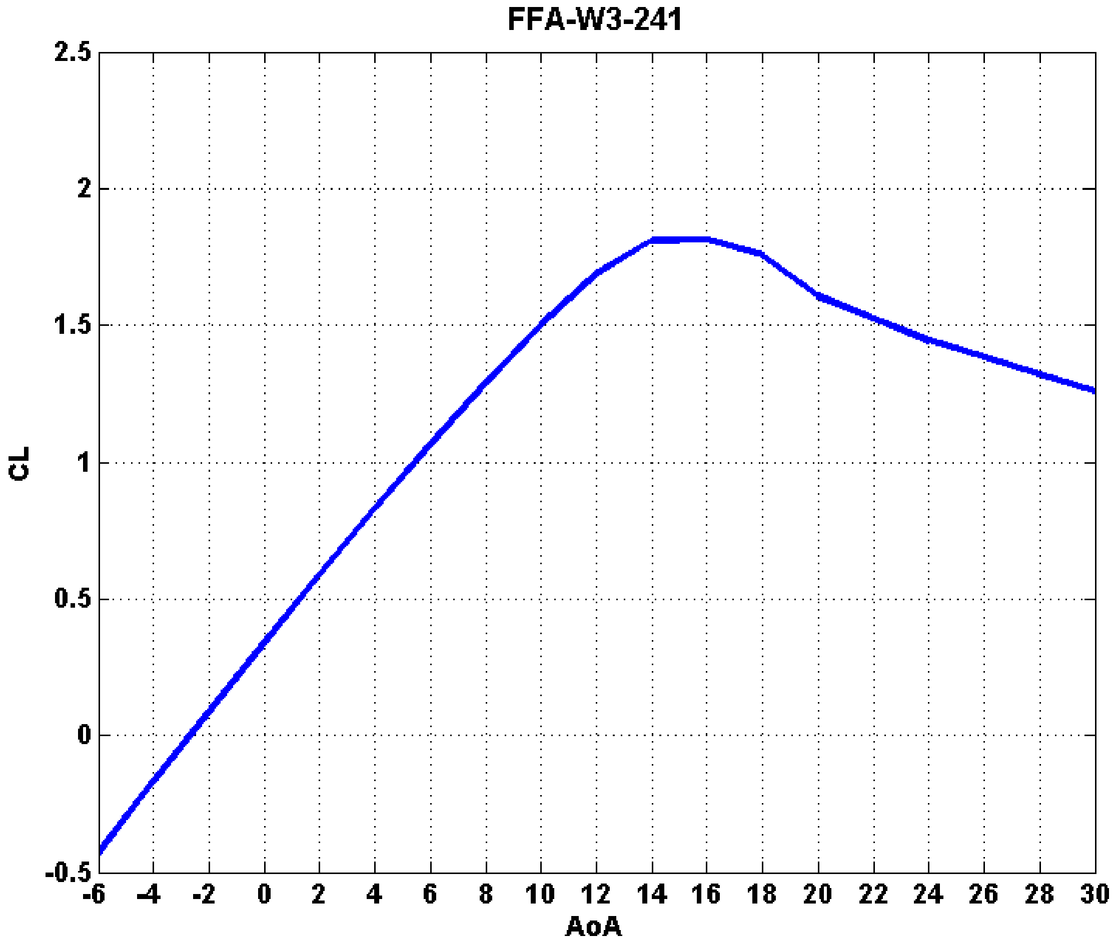
Remember to update the used inductions to actual induction not quasi-steady induction.

# Dynamic stall

# What happens when you change the angle of attack?



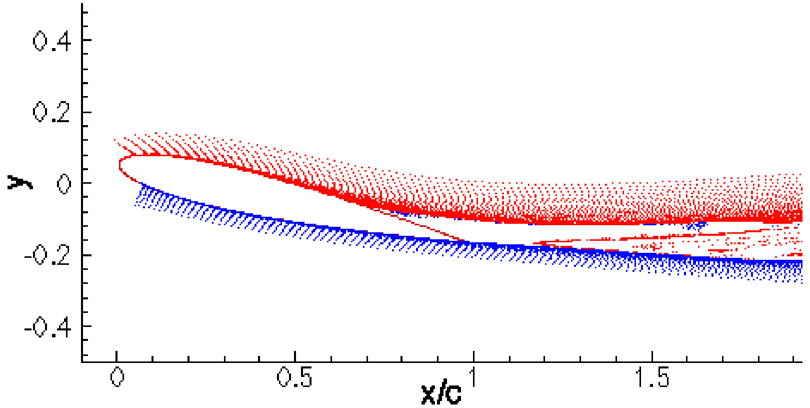
# Lift curve

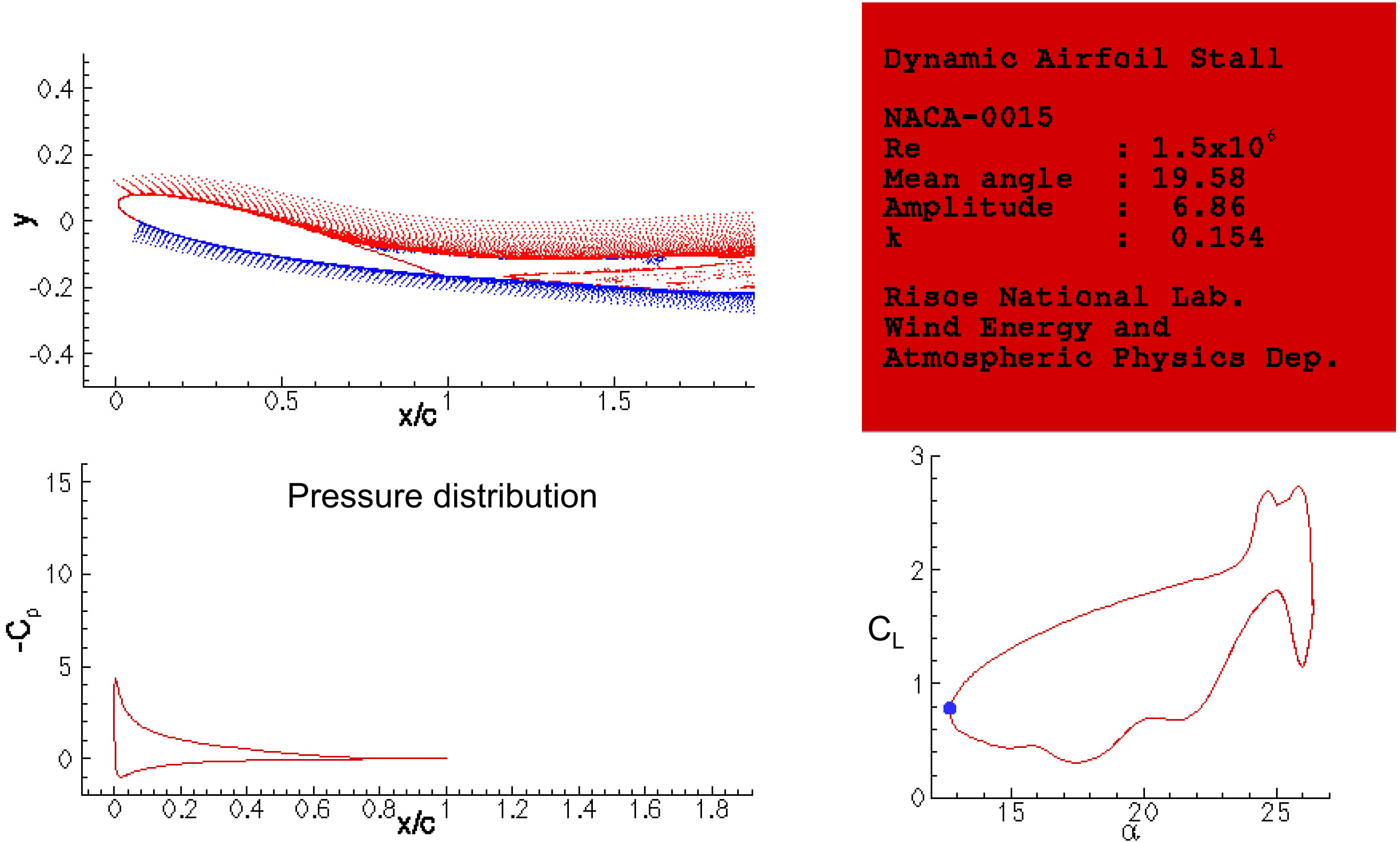



What happens when you have:

1) AoA variation between 0 to 6 deg

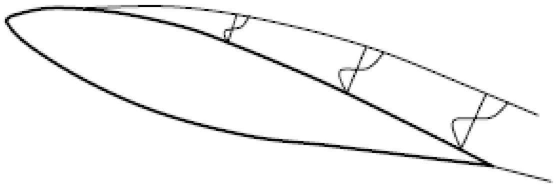
2) AoA variation between 13 to 20 deg



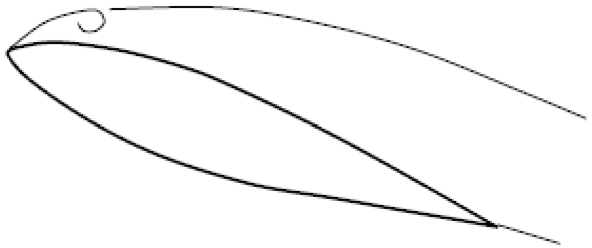


Dynamic Airfoil Stall
NACA-0015
Re : 1.5x10^6
Mean angle : 19.58
Amplitude : 6.86
k : 0.154
Risoe National Lab.
Wind Energy and
Atmospheric Physics Dep.
Pressure distribution
y
x/c
-C_p
C_L
α

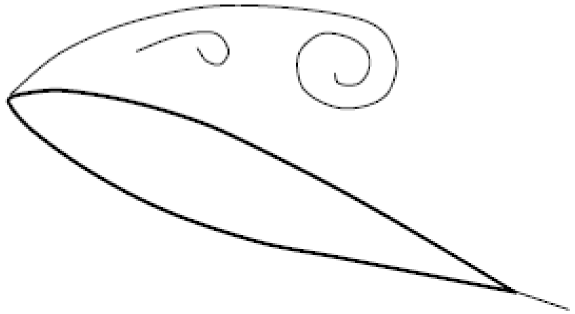
Stage 1: Aerofoil exceeds $\alpha_s$ and flow reversal occurs in upper boundary layer

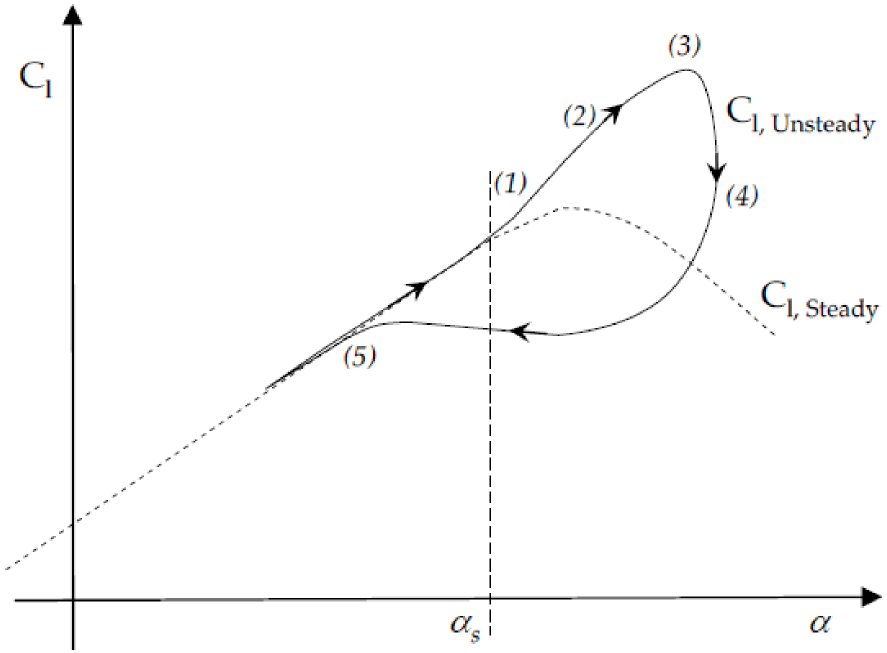


Stage 2: Formation of vortex at leading edge

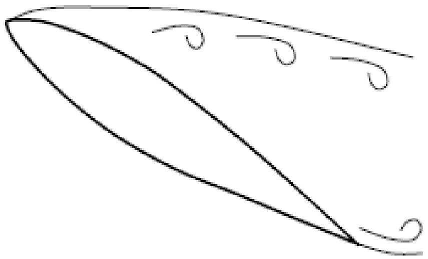


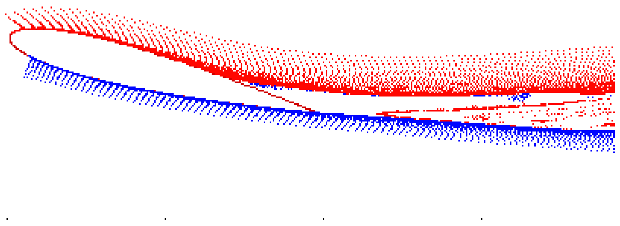
Stage 2 - 3: Vortex convects towards trailing edge, causing extra lift



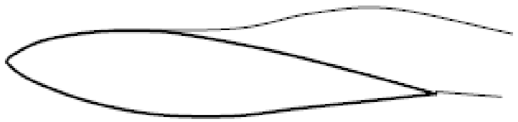



Stage 3-4: Lift stall occurs. After vortex reaches trailing edge, full separation occurs.





Stage 5: When angle of attack decreases again, boundary layer reattaches front to rear



When α(t) (e.g. $\alpha(t)=\alpha_o+A\sin(\omega t)$)
a so called stall loop develops depending on angle of attack, amplitude and frequency

Ref: Tonio Sant, "Improving BEM-based Aerodynamic Models in Wind Turbine Design Codes," PhD Thesis, TUDelft

# Dynamic stall

- Dynamic stall models
  - Stig Øye model: trailing edge stall, very simple and working fine for wind turbine (original paper uploaded to LEARN)

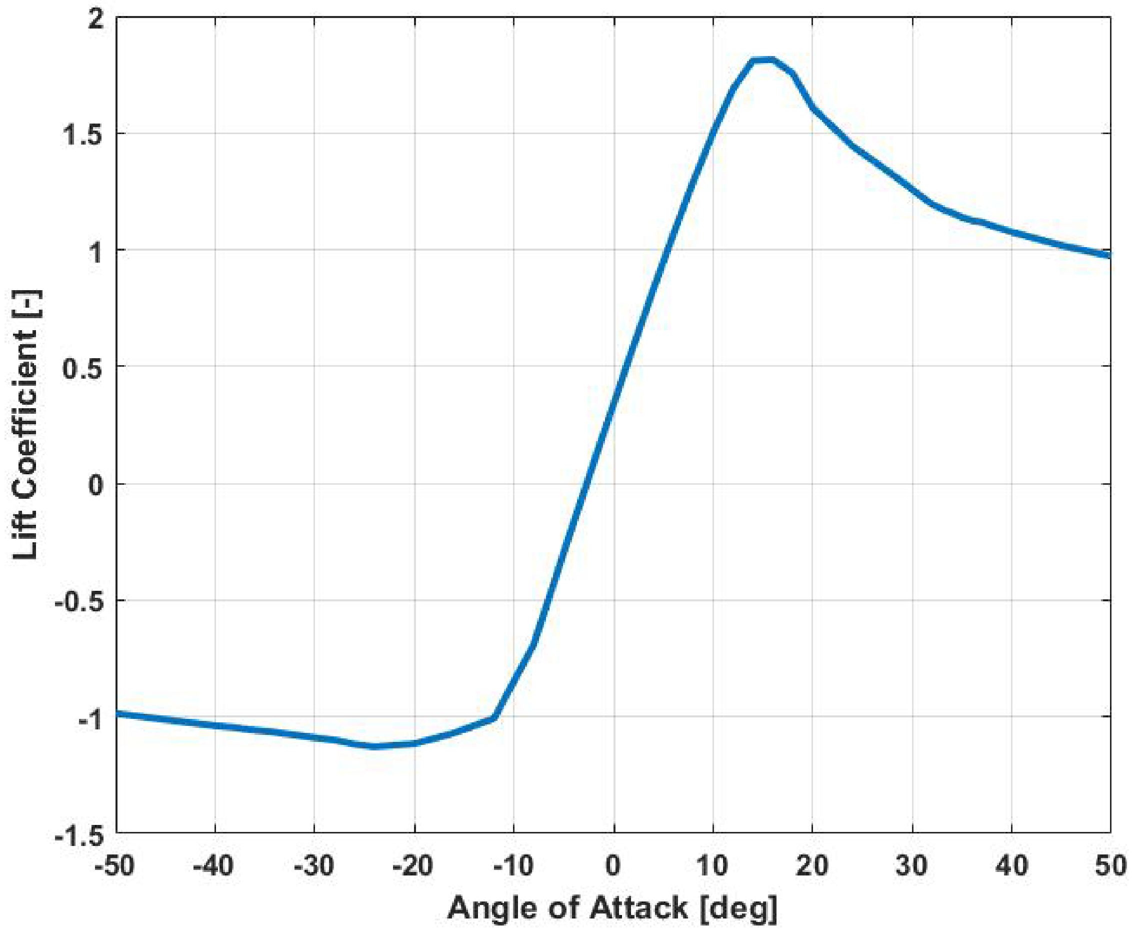
  - Beddoes-Leishman model: leading edge separation, attached flow, compressibility, considering Cd and Cm, but complex
    - BL-paper
  - IAG model
    - IAG-paper

- S. Øye model
  - For the trailing edge stall, the degree of stall is described via $f_s$ which is separation function.
  - The beauty of this model is that the dynamic stall model is represented with **only one function, $f_s$**.

$$C_l(t) = f_s(t) C_{l,inv}(\alpha) + (1 - f_s(t)) C_{l,fs}(\alpha), \qquad [7]$$

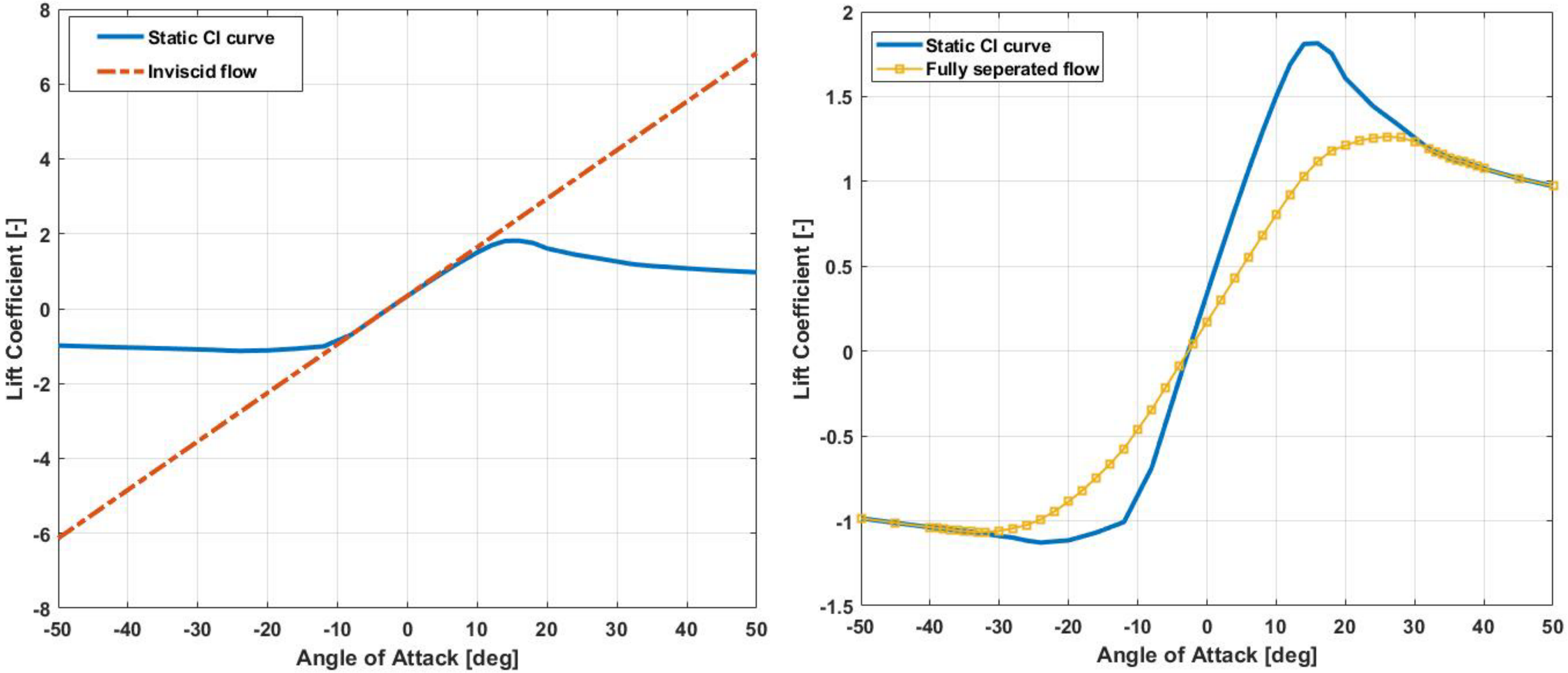
# S. Øye dynamic stall model

- Consider a static Cl curve (FFA-W3-241)



# S. Øye dynamic stall model

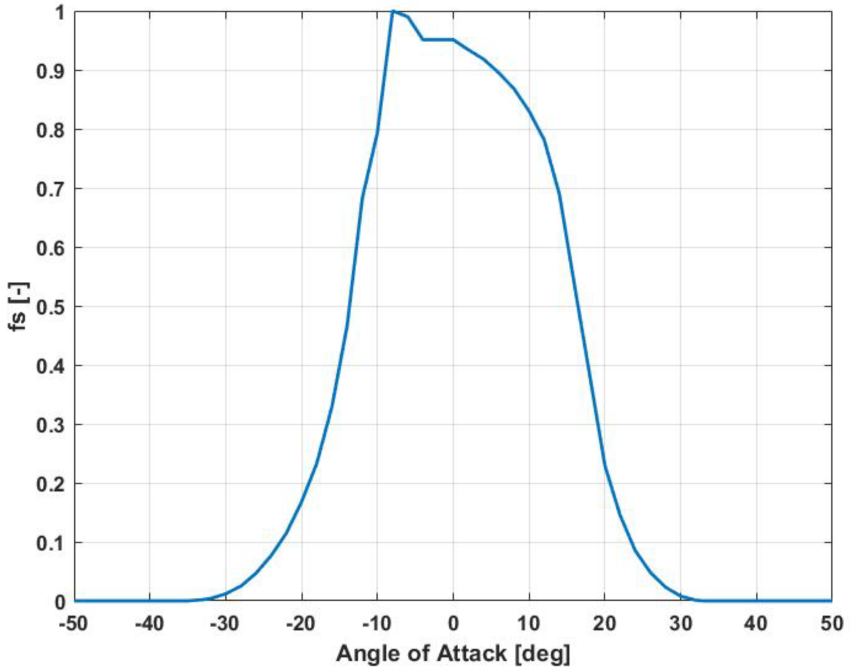
- Consider a static Cl curve (FFA-W3-241)
- Inviscid flow region, $C_{l,inv}$
- Fully separated flow region, $C_{l,fs}$

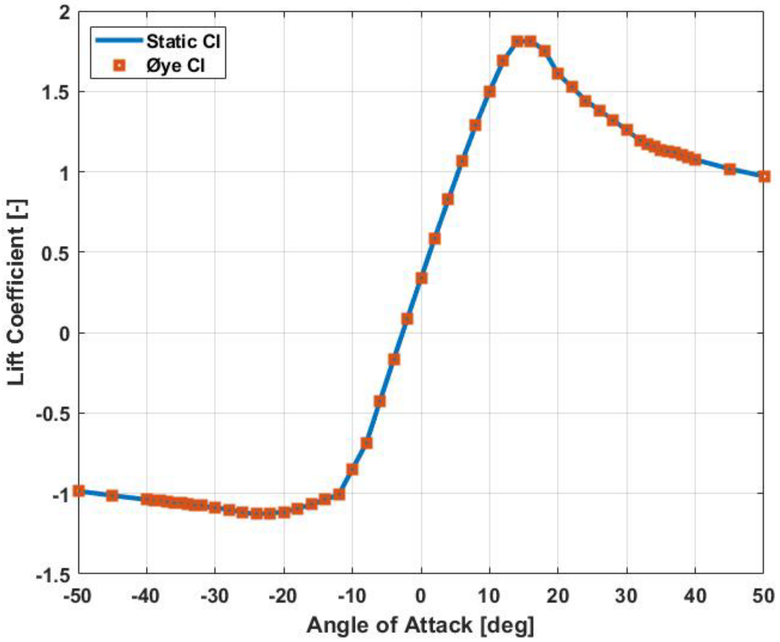


# S. Øye dynamic stall model

- Consider a static Cl curve (FFA-W3-241)
- Inviscid flow region, $C_{l,inv}$
- Fully separated flow region, $C_{l,fs}$
- Separation function, $f_s^{st}$
- $f_s^{st}$ is the value of $f_s$ that yields the static lift.

$$C_l(t) = f_s(t)C_{l,inv}(\alpha) + (1 - f_s(t))C_{l,fs}(\alpha)$$



# S. Øye dynamic stall model

- Assumption
  - $f_s$ is always try to go back to the static value. It means that if there is an angle of attack which is an unbalance, $f_s$ will go toward $f_s^{st}$.

$$\frac{df_s}{dt} = \frac{f_s^{st} - f_s}{\tau} \qquad [8]$$

where $\tau = Ac/V_{rel}$, typically $A$ = *4.*

  - Eq. (8) can be analytically integrated as follows:

$$\int_{f_s(t)}^{f_s(t+\Delta t)} \frac{1}{f_s^{st} - f_s} df_s = \int_t^{t+\Delta t} \frac{1}{\tau} dt \qquad [9]$$

$$\Rightarrow \quad f_s(t + \Delta t) = f_s^{st} + \left(f_s(t) - f_s^{st}\right)\exp(-\frac{\Delta t}{\tau}) \qquad [10]$$

$$C_l(t) = f_s(t)C_{l,inv}(\alpha) + (1 - f_s(t))C_{l,fs}(\alpha)$$

# Apply it into the computer code

- Airfoil data needed as input for the BEM code
  - $C_{l,inv}, C_{l,fs}$, and $f_s{}^{st}$
- Make it simple: relevant airfoil data is given.
  - In the “Assignment#1”, **<u>xxxx._ds.txt</u>** file, is given.

$$\alpha, C_{l,stat}, C_{d,stat}, C_{m,stat}, f_{stat}, C_{l,inv}, C_{l,fs}$$

```
for i = time
    for j = Nblade
        for k = Nsec

            Calculating AoA

            find C_l,inv, f_s, C_l,fs by using linear interpolation with given data

            tau = 4*c/Vrel

            updating fs(i) = fs_,stat + (fs(i-1)-fs_,stat)*exp(-deltaT/tau)     (eq.10)

            calculating Cl(i) = fs(i)* C_l,inv + (1-fs(i))*C_l,fs               (eq. 7)

        end
    end
end
```

# S. Øye dynamic stall model

- Example: $\alpha = 15 + 5\sin(25t)$ with $\tau = 0.24s$ and initial condition $f_s(0) = f_s{}^{st}$.