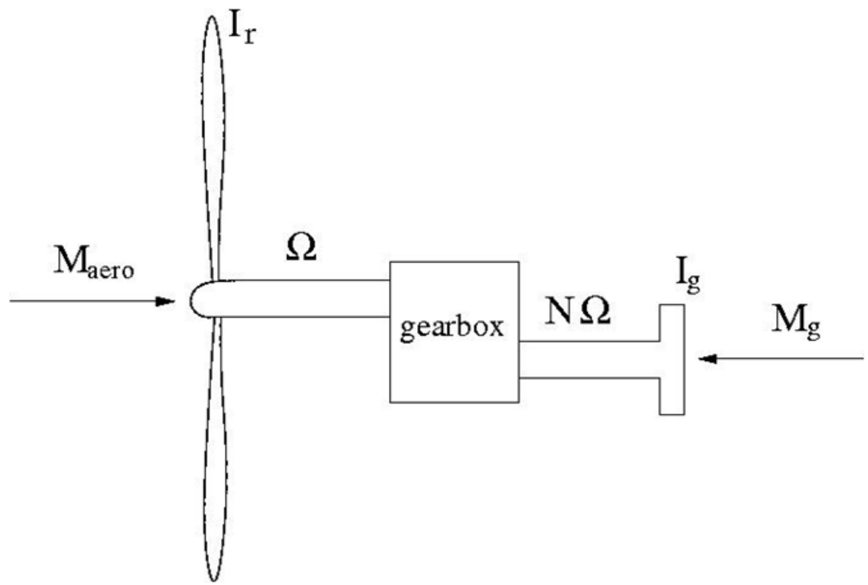
Basic collective pitch control

And how to simulate it using your uBEM code

This is the equation governing the rotational speed of a wind turbine

$$I\frac{d\omega}{dt} = M_{aero}(\omega, \theta_p) - M_G(\omega)$$



We can today control the generator torque (electrically)
We can control the aerodynamic torque by changing the pitch

In **Region 1 MPPT** we track maximum power $C_p = C_{p,max}$
Meaning the tip speed ratio is $\lambda = \lambda_{\mathrm{opt}}$ and the pitch is $\theta_p = \theta_{p,opt}$
This then gives that the generator torque must be controlled as $M_G(\omega) = \mathrm{const} \cdot \omega^2$

$$P = M\omega = \tfrac{1}{2}\rho V_o^3 A C_p(\lambda, \theta_p)$$

$$V_o = \frac{\omega R}{\lambda}$$

$$M = \tfrac{1}{2}\rho \frac{R^3}{\lambda^3} A C_p(\lambda, \theta_p)\omega^2$$ if $\lambda = \lambda_{opt}$ and $\theta_{\mathrm{p}} = \theta_{p,opt}$ then $C_p = C_{p,opt}$ and

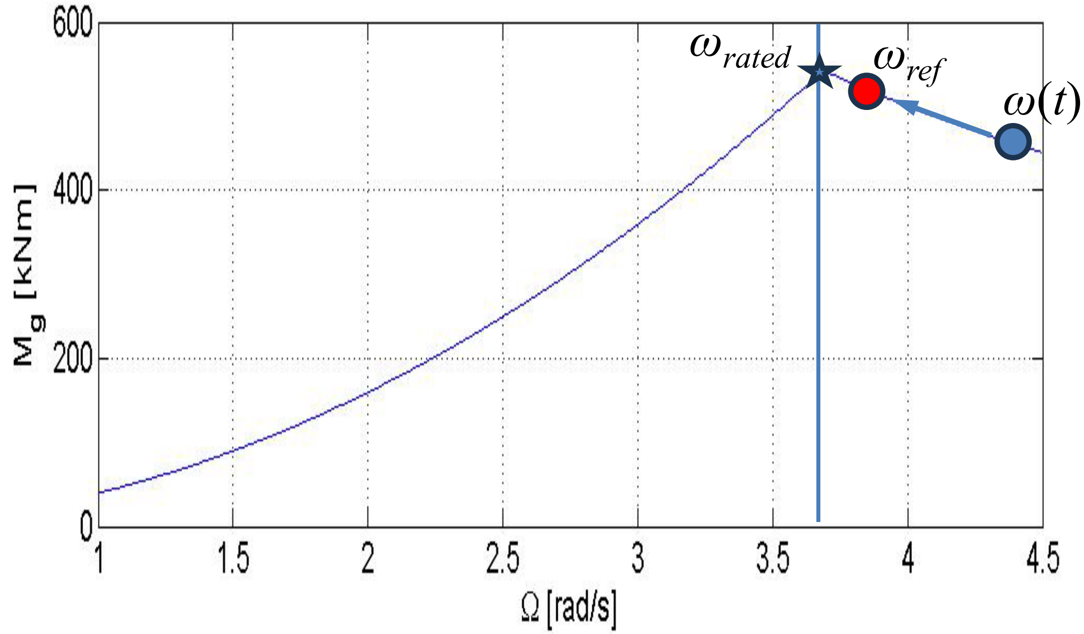
$$M_{opt} = const \cdot \omega^2$$

$$const = \tfrac{1}{2}\rho \frac{R^3}{\lambda_{opt}^3} A C_{p,opt}(\lambda_{opt}, \theta_{p,opt})$$ and can be found using BEM

Region 3 after rated wind speed

The power should be constant $P=P_{rated}$ and $\omega=const=\omega_{ref}$

This can be achieved by varying the pitch angle and controlled by rotational speed $\theta_p(\omega)$



$$M_G(\omega) = \begin{cases} const \cdot \omega^2 & \omega < \omega_{rated} \\ \dfrac{P_{rated}}{\omega} & \omega > \omega_{rated} \end{cases}$$

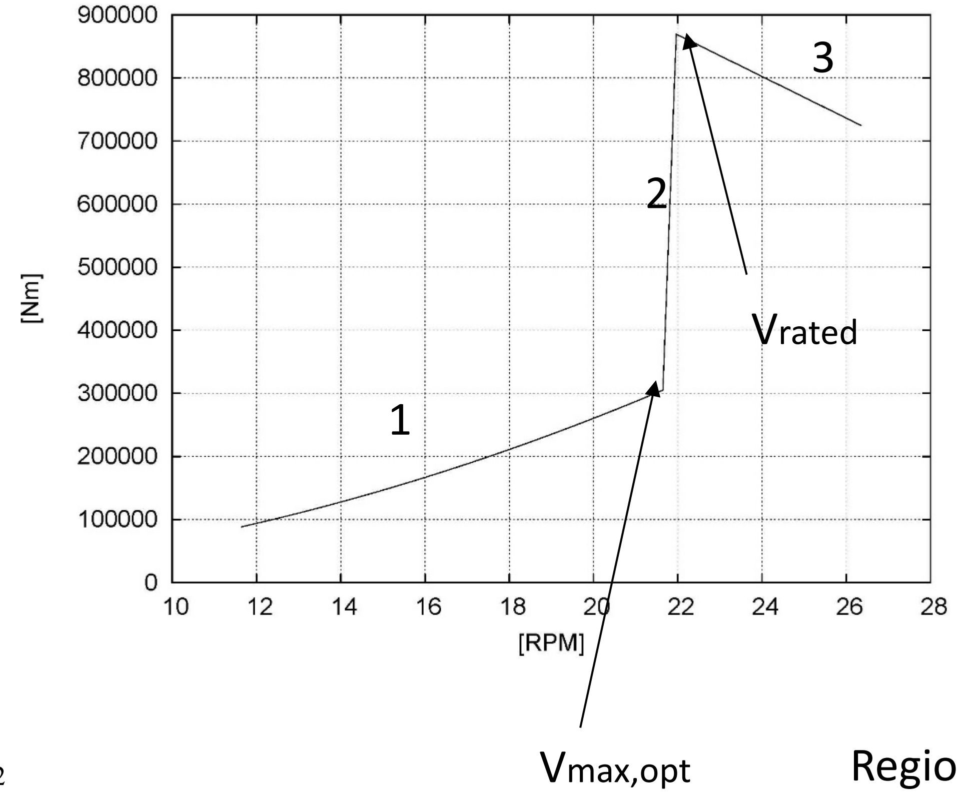
Generator characteristic $M_G(\omega)$ constant power after rated

The wind turbine is constantly measuring the rotational speed, and if it is too high the pitched is changed to decrease the aerodynamic torque to reduce $\omega(t)$ to ω_{ref}

$$I\frac{d\omega}{dt} = M_{aero}(\omega,\theta_p) - M_G(\omega)$$

There may be a **region 2** depending on how high omega you allow

$M_G(\omega)$



Region 1

$$M_{opt} = const \cdot \omega^2$$

Region 3

Constant power

In this course we are not simulating the generator, but assume that the torque can be made to always fit the characteristic $M_G(\omega)$

How does the wind turbine control the aerodynamic torque to obtain almost constant rotational speed and rated power at high wind speeds **(Region 3)**

$$I\frac{d\omega}{dt} = M_{aero}(\omega, \theta_p) - M_G(\omega)$$

The rotational speed is constantly measured and transferred to a controller calculating the necessary pitch and transferring this signal to the pitch actuator

PI controller, see next slide

PI controller (this is what goes on in the control computer)

$$\theta_p^P(t) = GK \cdot K_p \cdot (\omega - \omega_{ref})$$

$$\frac{d\theta_p^I}{dt} = GK \cdot K_I \cdot (\omega - \omega_{ref}) \Rightarrow \theta_p^I(t) = \theta_p^I(t - \Delta t) + GK \cdot K_I(\omega - \omega_{ref})\Delta t$$

$$GK(\theta_p) = \frac{1}{1 + \theta_p / KK}$$

$$\theta_p^{setpoint}(t) = \theta_p^P(t) + \theta_p^I(t)$$

The gains the for DTU 10MW RWT are estimated to **Kp=1.5rad/(rad/s), KI=0.64rad/rad)**
and the gain reduction to **KK=14 deg.**
Set ω_{ref} sligthly higher than ω_{rated}

Both the integral and the setpoint(total) term must be limited between the physical stops.

$$\theta_p^I = \max(\theta_p^I, \theta_{p,\min})$$

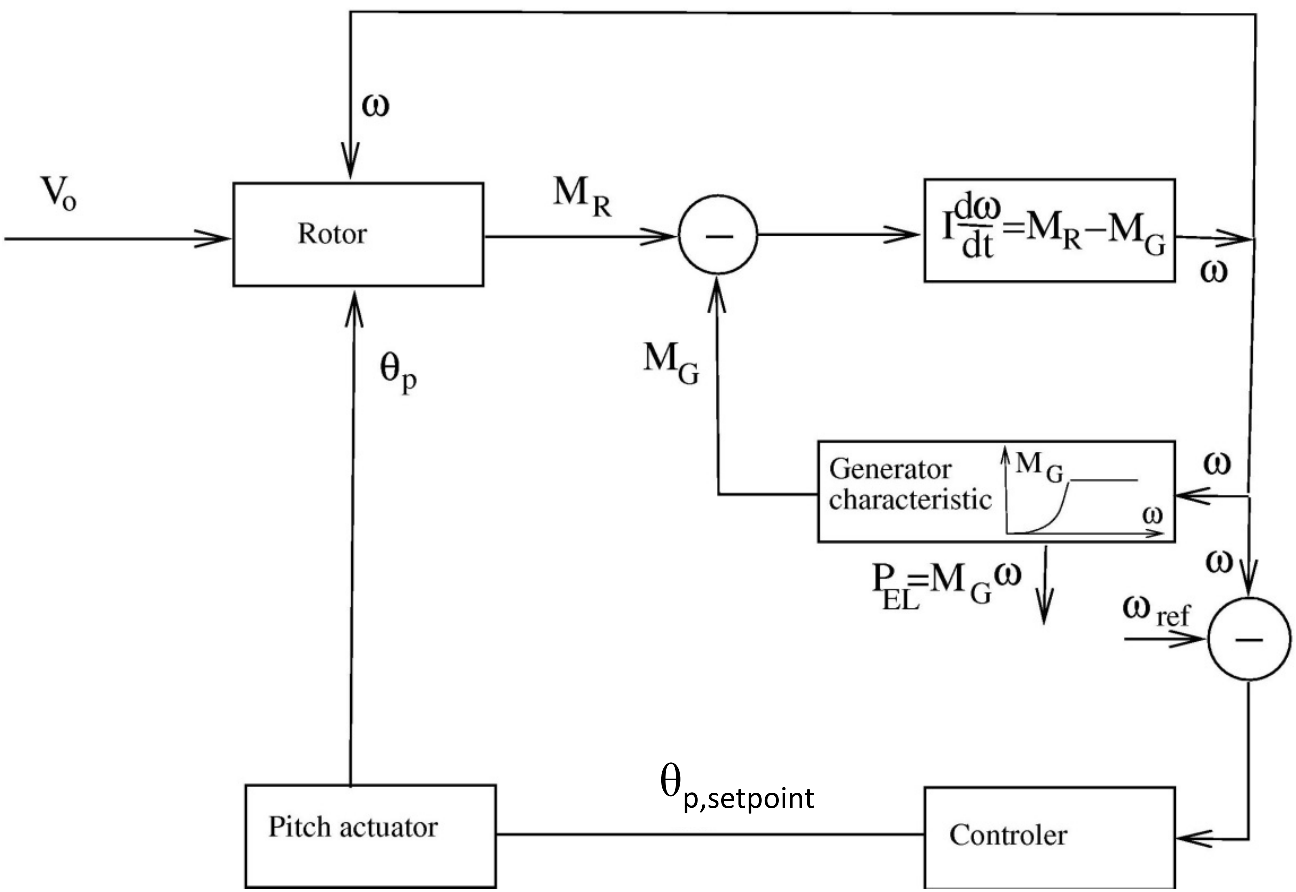
$$\theta_p^I = \min(\theta_p^I, \theta_{p,\max})$$

and

$$\theta_p^{setpoint} = \max(\theta_p^{setpoint}, \theta_{p,\min})$$

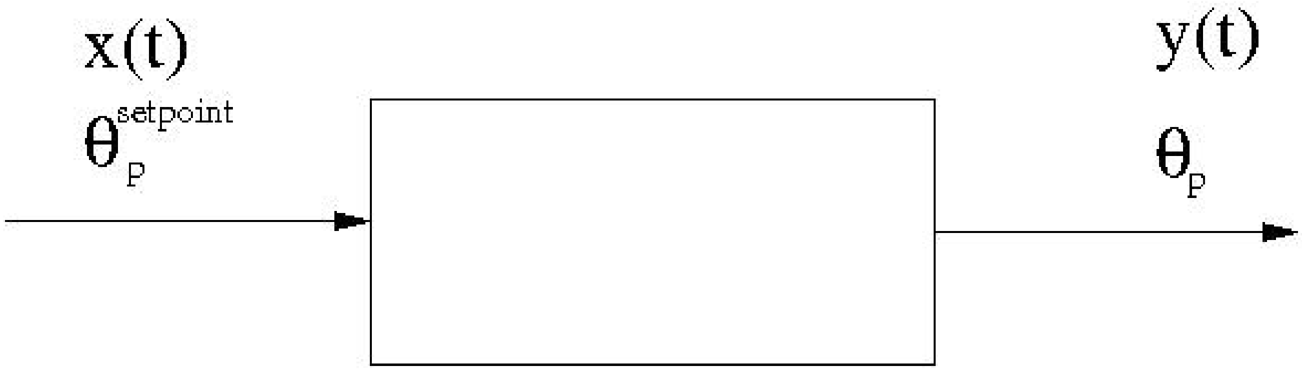
$$\theta_p^{setpoint} = \min(\theta_p^{setpoint}, \theta_{p,\max})$$

Sketch of WT control



To simulate more realistically one should also take inertia of the pitch system into account when estimating numerically the final pitch angle from the set point angle. This can be done by putting the set point angle, X(t), through a second order filter to give the more realistic output Y(t)

Not mandatory



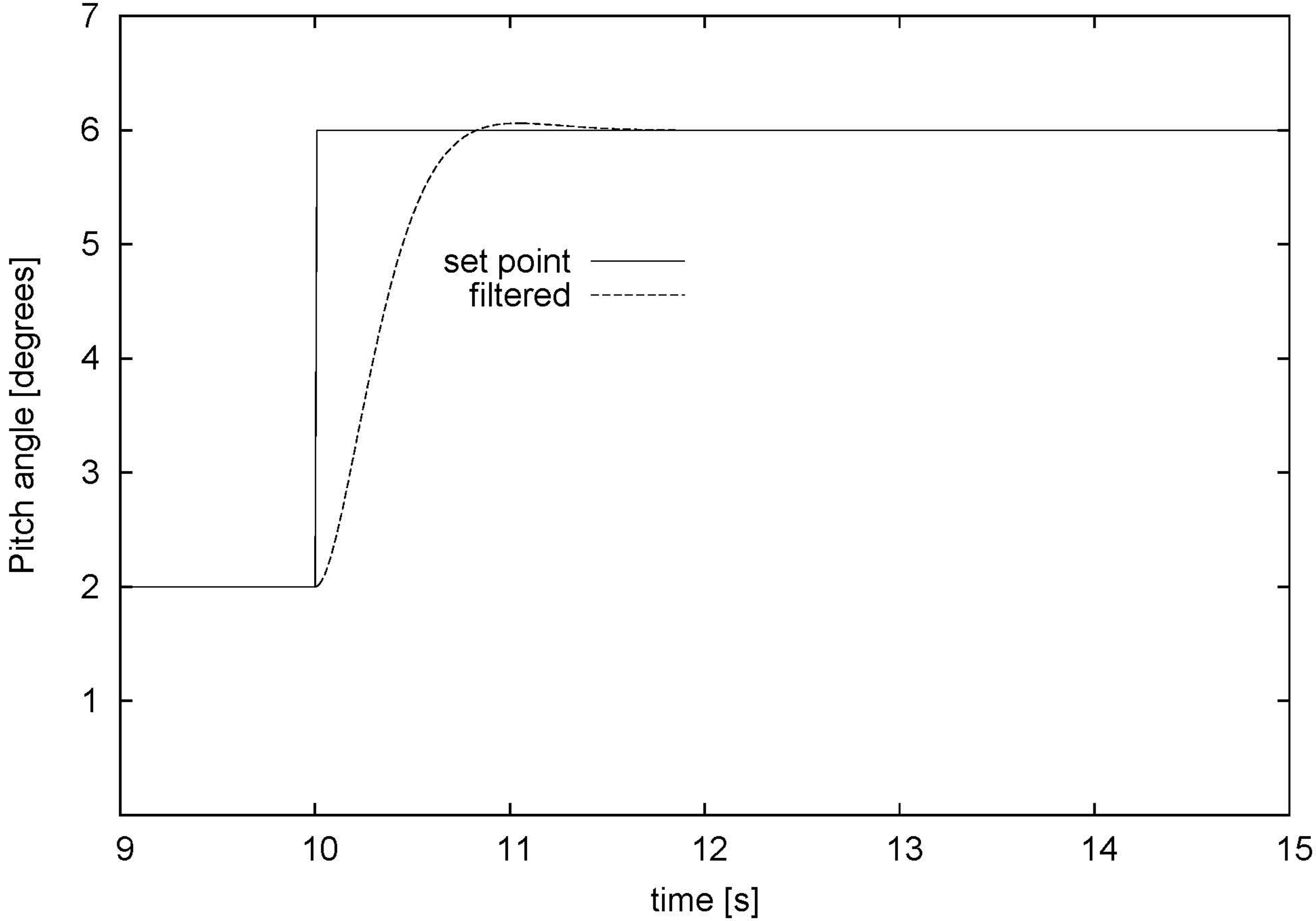
Can be modelled as a second order filter (has been tuned to real Tjaereborg rotor)

ω_o=8 and $\xi=0.7$

$$\frac{1}{\omega_o^2}\ddot{y}+\frac{2\xi}{\omega_o}\dot{y}+y=x$$

$$y^{n+1}=\frac{\omega_o^2\Delta t^2 x^n+(2-\omega_o^2\Delta t^2)y^n+(\xi\omega_o\Delta t-1)y^{n-1}}{1+\xi\omega_o\Delta t}$$

Due to inertia, one cannot immediately change the pitch to a new value



Apply it to the computer code

Initialize (blade positions, wind speed, pitch angle, angular velocity, induced wind speed, etc.)
For n=1,N (time)

For each blade (blade)

For each element/section (section)

$$V_{rel,y}^{n} = V_{y}^{n} + W_{y}^{n-1} - \omega x \cos\theta_{cone}$$

$$V_{rel,z}^{n} = V_{z}^{n} + W_{z}^{n-1}$$

$$\tan\varphi^{n} = \frac{V_{rel,z}^{n}}{-V_{rel,y}^{n}}, \quad \alpha^{n} = \varphi^{n} - (\beta + \theta_{p}^{n})$$

$$\vdots$$

end (section)

end (blade)

$$\theta_{p}^{P}(n+1) = GK \cdot K_{p} \cdot (\omega(n) - \omega_{ref})$$

$$\theta_{p}^{I}(n+1) = \theta_{p}^{I}(n) + GK \cdot K_{I}(\omega(n) - \omega_{ref})\Delta t$$

$$\theta_{p}^{setpoint} = \theta_{p}^{P}(n+1) + \theta_{p}^{I}(n+1)$$

$$\text{If } (\theta_{p}^{n+1} > \theta_{p}^{n} + \dot{\theta}_{\max}\Delta t) \text{ then } \theta_{p}^{n+1} = \theta_{p}^{n} + \dot{\theta}_{\max}\Delta t$$

$$\text{If } (\theta_{p}^{n+1} < \theta_{p}^{n} - \dot{\theta}_{\max}\Delta t) \text{ then } \theta_{p}^{n+1} = \theta_{p}^{n} - \dot{\theta}_{\max}\Delta t$$

$$\text{If } (\theta_{p}^{n+1} \geq \theta_{p,\max}) \text{ then } \theta_{p}^{n+1} = \theta_{p,\max}$$

$$\text{If } (\theta_{p}^{n+1} \leq \theta_{p,\min}) \text{ then } \theta_{p}^{n+1} = \theta_{p,\min}$$

$$\omega(n+1) = \omega(n) + \frac{(M_{aero}(n) - M_{G}(n))}{I_{rotor}}\Delta t$$

Note that the electrical power is

$$P_{EL}(t) = M_G(t) \cdot \omega(t)$$

And is different from the mechanical power

$$P_{mech}(t) = M_{aero}(t) \cdot \omega(t)$$

Examples

- Uniform wind condition: no shear, no tower shadow, no tilt, no cone

$$\omega_{ref} = 1.01 \text{ rad/s}$$

$$I = 1.6 \cdot 10^8 \text{ kgm}^2$$

$$K_p = 1.50 \text{ s}$$

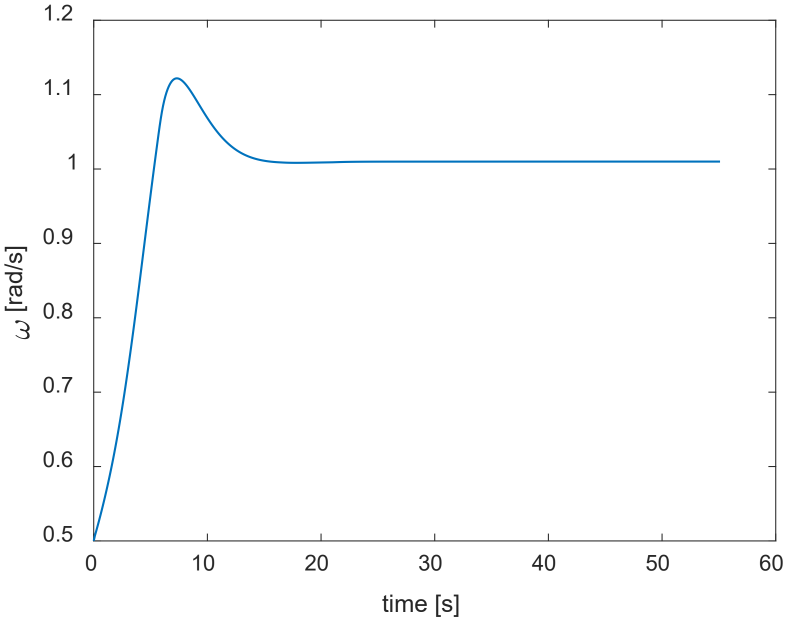
$$K_I = 0.64$$

$$M_G = \begin{cases} const \cdot \omega^2 & \text{for } \omega < \omega_{rated} \\ P_{rated} / \omega & \text{for } \omega > \omega_{rated} \end{cases}$$

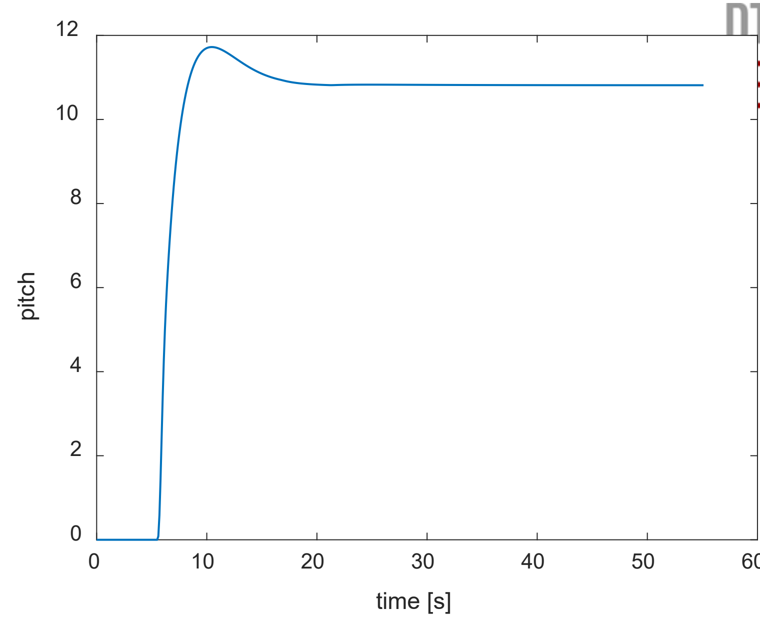
$$V_o = 15 \text{ m/s}$$

$$\omega(t = 0 \text{ s}) = 0.5 \text{ rad/s}$$

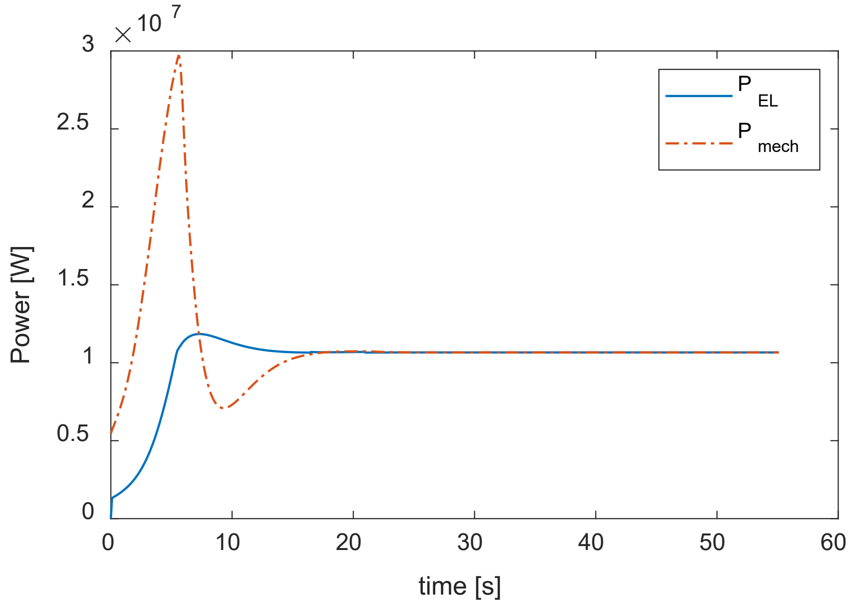
$$\theta_p(t = 0 \text{ s}) = 0 \text{ rad}$$



ω [rad/s]
time [s]

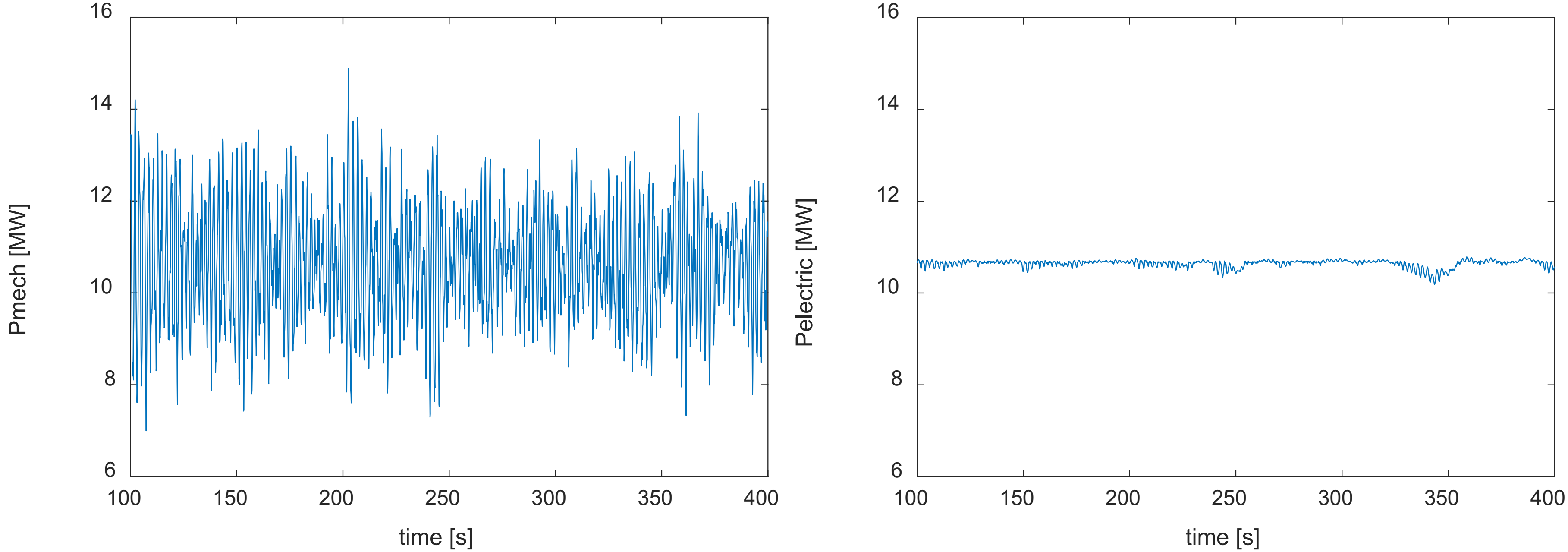


pitch
time [s]



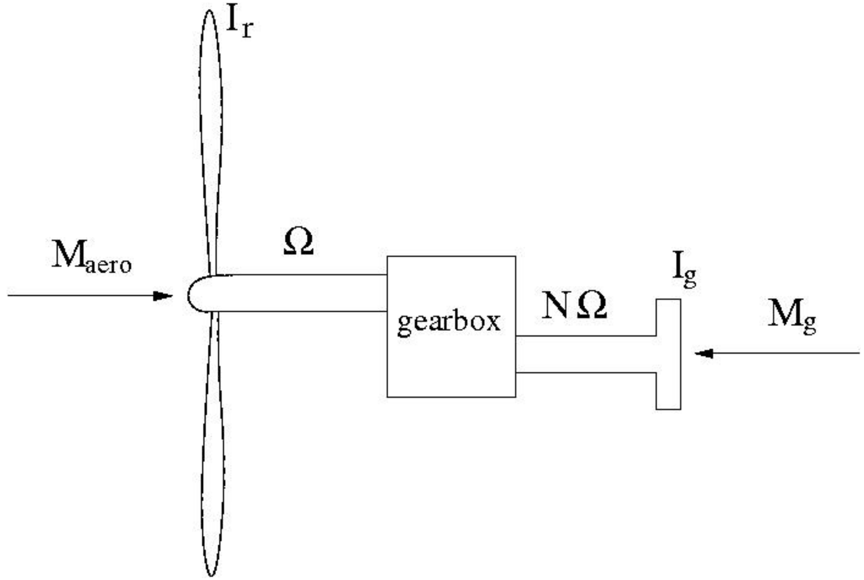
×10 7
Power [W]
time [s]
P EL
P mech

Same input, but including atmospheric turbulence



Theory behind the equations for PI pitch controller

In the following you may read yourself how the gains can be estimated and is not mandatory or part of this course and taught much more in the LAC course 46320 that some of you may also take



$$(I_r + N^2 I_g)\frac{d}{dt}(\Omega_o + \Delta\Omega) = I\Delta\dot{\Omega} = M_{aero}(\theta_o + \Delta\theta, \Omega_o + \Delta\Omega) - N \cdot M_g(\Omega_o + \Delta\Omega)$$

$$M_g = \frac{P_o}{N\Omega_o} + \frac{1}{N}\frac{d}{d\Omega}\left(\frac{P_o}{\Omega}\right) \cdot \Delta\Omega = \frac{P_o}{N\Omega_o} - \frac{P_o}{N\Omega_o^2}\Delta\Omega$$

$$M_{aero} \approx \frac{P_o + \left.\frac{dP}{d\theta}\right|_o \Delta\theta}{\Omega_o} = \frac{P_o}{\Omega_o} + \frac{1}{\Omega_o}\left.\frac{dP}{d\theta}\right|_o \Delta\theta$$

$$I\Delta\dot{\Omega} = \frac{1}{\Omega_o}\left.\frac{dP}{d\theta}\right|_o \Delta\theta + \frac{P_o}{\Omega_o^2}\Delta\Omega$$

PI controller

$$\Delta\theta = \theta - \theta_o = K_p \cdot N \cdot \Delta\Omega + K_I \cdot N \cdot \Delta\Omega\Delta t$$

Introducing new variable $\dot{\varphi} = \Delta\Omega = (\Omega - \Omega_o)$

$\ddot{\varphi} = \Delta\dot{\Omega}$ and $\Delta\varphi = \varphi - \varphi_o = \varphi = \Delta\Omega\Delta t$

Now inserting

$$\Delta\theta = K_p \cdot N \cdot \Delta\Omega + K_I \cdot N \cdot \Delta\Omega\Delta t$$

$$\varphi = \Delta\Omega\Delta t$$

$$\dot{\varphi} = \Delta\Omega$$

$$\ddot{\varphi} = \Delta\dot{\Omega}$$

into

$$I\Delta\dot{\Omega} = \frac{1}{\Omega_o}\left.\frac{dP}{d\theta}\right|_o \Delta\theta + \frac{P_o}{\Omega_o^2}\Delta\Omega$$

$$I\ddot{\varphi} - \frac{1}{\Omega_o}\left.\frac{dP}{d\theta}\right|_o (K_p \cdot N \cdot \dot{\varphi} + K_I \cdot N \cdot \varphi) - \frac{P_o}{\Omega_o^2}\dot{\varphi} = 0$$

$$I\ddot{\varphi} + (\frac{1}{\Omega_o}\left(-\left.\frac{dP}{d\theta}\right|_o\right)(K_p \cdot N - \frac{P_o}{\Omega_o^2})\dot{\varphi} + \frac{1}{\Omega_o}\left(-\left.\frac{dP}{d\theta}\right|_o\right)K_I \cdot N \cdot \varphi = 0$$

$$I\ddot{\varphi}+D\dot{\varphi}+K\varphi=0$$

$$I\ddot{\varphi}+\left(\frac{1}{\Omega_o}\left(-\left.\frac{dP}{d\theta}\right|_o\right)\left(K_p\cdot N-\frac{P_o}{\Omega_o^2}\right)\dot{\varphi}+\frac{1}{\Omega_o}\left(-\left.\frac{dP}{d\theta}\right|_o\right)K_I\cdot N\cdot\varphi=0\right.$$

$$I\ddot{\varphi}+D\dot{\varphi}+K\varphi=0$$

$$D=\frac{1}{\Omega_o}\left(-\left.\frac{dP}{d\theta}\right|_o\right)\left(K_p\cdot N-\frac{P_o}{\Omega_o^2}\right)$$

$$K=\frac{1}{\Omega_o}\left(-\left.\frac{dP}{d\theta}\right|_o\right)K_I\cdot N$$

A second order response with a resonans frequency and damping ratio

$$\omega_o=\sqrt{\frac{K}{I}}\quad\text{and}\quad\xi=\frac{D}{2I\omega_o}$$

Experience show that the system behaves good if $\omega_o=0.6$ and $\xi=0.6\text{ - }0.7$

And if further the negative damping from the generator is ignored

$$K_I = \frac{\Omega_o \omega_o^2 I}{\left(-\left.\frac{dP}{d\theta}\right|_o\right) N}$$

$$K_p = \frac{2\xi I \omega_o \Omega_o}{\left(-\left.\frac{dP}{d\theta}\right|_o\right) N}$$

KK is the angle where $-\frac{dP}{d\theta}$ has doubled from the value at rated

The slope $dP/d\theta$ can be computed using a steady BEM code but keeping the induced velocity constant when changing the pitch $W(\theta + \Delta\theta, V_o) = W(\theta - \Delta\theta, V_o) = W(\theta, V_o)$

$$\frac{dP}{d\theta}(V_o) \approx \frac{P(\theta + \Delta\theta, V_o) - P(\theta - \Delta\theta, V_o)}{2\Delta\theta}, \qquad \left.\frac{dP}{d\theta}\right|_o = \frac{dP}{d\theta}(V_{rated})$$