```
Looking in indexes: <a href="https://pypi.org/simple">https://us-python.pkg.dev/colab-wheels/public/simple/</a>
 Collecting symforce
   Downloading symforce-0.7.0-cp38-cp38-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (4.4 MB)
                                        1 4.4 MB 5.4 MB/s
 Requirement already satisfied: numpy in /usr/local/lib/python3.8/dist-packages (from symforce) (1.21.6)
 Collecting skymarshal==0.7.0
   Downloading skymarshal-0.7.0-py3-none-any.whl (82 kB)
                                      82 kB 312 kB/s
 Collecting sympy~=1.11.1
   Downloading sympy-1.11.1-py3-none-any.whl (6.5 MB)
                                     6.5 MB 42.5 MB/s
 Requirement already satisfied: scipy in /usr/local/lib/python3.8/dist-packages (from symforce) (1.7.3)
 Collecting clang-format
   Downloading clang_format-15.0.4-py2.py3-none-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (1.5 MB)
                                        1.5 MB 23.6 MB/s
 Collecting black
   Downloading black-22.10.0-cp38-cp38-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (1.5 MB)
 Requirement already satisfied: jinja2 in /usr/local/lib/python3.8/dist-packages (from symforce) (2.11.3)
 Collecting symforce-sym==0.7.0
   Downloading symforce_sym-0.7.0-py3-none-any.wh1 (70 kB)
 | 70 kB 4.6 MB/s
Requirement already satisfied: graphviz in /usr/local/lib/python3.8/dist-packages (from symforce) (0.10.1)
 Requirement already satisfied: six in /usr/local/lib/python3.8/dist-packages (from skymarshal==0.7.0->symforce) (1.15.0)
   Downloading ply-3.11-py2.py3-none-any.whl (49 kB)
 Collecting argh
Downloading argh-0.26.2-py2.py3-none-any.whl (30 kB)
 Requirement already satisfied: mpmath>=0.19 in /usr/local/lib/python3.8/dist-packages (from sympy~=1.11.1->symforce) (1.2.1)
Requirement already satisfied: typing-extensions>=3.10.0.0 in /usr/local/lib/python3.8/dist-packages (from black->symforce) (4.1.1)
 Requirement already satisfied: tomli>=1.1.0 in /usr/local/lib/python3.8/dist-packages (from black->symforce) (2.0.1) Collecting platformdirs>=2
   Downloading platformdirs-2.5.4-py3-none-any.whl (14 kB)
 Collecting pathspec>=0.9.0
   Downloading pathspec-0.10.2-py3-none-any.whl (28 kB)
 Collecting click>=8.0.0
   Downloading click-8.1.3-py3-none-any.whl (96 kB)
                                     96 kB 3.9 MB/s
 Collecting mypy-extensions>=0.4.3
 Downloading mypy_extensions-0.4.3-py2.py3-none-any.whl (4.5 kB)

Requirement already satisfied: MarkupSafe>=0.23 in /usr/local/lib/python3.8/dist-packages (from jinja2->symforce) (2.0.1)
 Installing collected packages: ply, platformdirs, pathspec, mypy-extensions, click, argh, sympy, symforce-sym, skymarshal, clang-format Attempting uninstall: click
     Found existing installation: click 7.1.2
     Uninstalling click-7.1.2:
       Successfully uninstalled click-7.1.2
   Attempting uninstall: sympy
     Found existing installation: sympy 1.7.1
     Uninstalling sympy-1.7.1:
Successfully uninstalled sympy-1.7.1
 ERROR: pip's dependency resolver does not currently take into account all the packages that are installed. This behaviour is the source flask 1.1.4 requires click<8.0,>=5.1, but you have click 8.1.3 which is incompatible.
 Successfully installed argh-0.26.2 black-22.10.0 clang-format-15.0.4 click-8.1.3 mypy-extensions-0.4.3 pathspec-0.10.2 platformdirs-2.5
import numpy as np
import os
import symforce
symforce.set_symbolic_api("symengine")
symforce.set log level("warning")
# https://symforce.org/tutorials/epsilon tutorial.html
symforce.set epsilon to symbol()
from symforce import codegen
from symforce.codegen import codegen util
from symforce import ops
import symforce.symbolic as sf
```

```
inputs.add(sf.Symbol("foo"))
display(inputs)
     Values(
        x: x,
        y: <Rot2 <C real=c_re, imag=c_im>>,
        foo: foo,
x, y = sf.symbols("x y")
expr = x ** 2 + sf.sin(y) / x ** 2
inputs["states"] = Values(p=expr)
display(inputs)
     Values(
        x: x,
        y: <Rot2 <C real=c_re, imag=c_im>>,
       foo: foo,
states: Values(
         p: x^{**2} + \sin(y)/x^{**2},
        ),
      )
display(inputs.to_storage())
      \left[x,\ c_{re},\ c_{im},\ foo,\ x^2+rac{\sin\left(y
ight)}{x^2}
ight]
display(inputs.items_recursive())
     [('x', x),
  ('y', <Rot2 <C real=c_re, imag=c_im>>),
       ('foo', foo),
       ('states.p', x^{**2} + \sin(y)/x^{**2})]
display(inputs.keys_recursive())
display(inputs.values_recursive())
      ['x', 'y', 'foo', 'states.p']
      [x, \langle Rot2 \langle C real=c_re, imag=c_im \rangle \rangle, foo, x^{**2} + \sin(y)/x^{**2}]
```

```
item_index = inputs.index()["states"].item_index
assert item_index == inputs["states"].index()
inputs["states.blah"] = 3
display(inputs)
     Values(
       x: x,
       y: <Rot2 <C real=c_re, imag=c_im>>,
       foo: foo,
states: Values(
         p: x^{**2} + \sin(y)/x^{**2},
         blah: 3,
       ),
assert inputs["states.p"] is inputs["states"]["p"] is inputs.attr.states.p
display(inputs.attr.states.p)
     x^2+\frac{\sin{(y)}}{x^2}
with sf.scope("params"):
    s = sf.Symbol("cost")
display(s)
     params.cost
v = Values()
v.add(sf.Symbol("x"))
with sf.scope("foo"):
    v.add(sf.Symbol("x"))
    with sf.scope("bar"):
        v.add(sf.Symbol("x"))
display(v)
display(v.attr.foo.bar.x)
     Values(
       x: x,
              Values(
       foo:
        oo:
x: foo.x,
Values(
           x: foo.bar.x,
         ),
       ),
     foo.bar.x
```

```
v = Values()
    with v.scope("hello"):
          v["y"] = x ** 2
          v["z"] = sf.Symbol("z")
           Values(
              hello: Values(
                 y: x**2,
                 z: hello.z,
              ),
           )
    lie_vals = Values()
    lie_vals["scalar"] = sf.Symbol("x")
    lie_vals["rot3"] = sf.Rot3.symbolic("rot")
    sub_lie_vals = Values()
    sub_lie_vals["pose3"] = sf.Pose3.symbolic("pose")
    sub_lie_vals["vec"] = sf.V3.symbolic("vec")
    lie_vals["sub_vals"] = sub_lie_vals
    display(lie_vals)
    Values(
      rot3: <Rot3 <Q xyzw=[rot_x, rot_y, rot_z, rot_w]>>,
       pose3: <Pose3 R=<Rot3 <Q xyzw=[pose.R_x, pose.R_y, pose.R_z, pose.R_w]>>, t=(pose.t0, pose.t1, pose.t2)>,
    [vec0],
    [vec1],
    [vec2]]),
display(lie_vals.tangent_dim())
display(len(lie_vals.to_tangent()))
    13
    13
display(lie_vals.storage_D_tangent())
                                         0
                                                  0
                                                        0 0 0 0 0 0
         \begin{array}{c} \frac{rot_w}{2} \\ \frac{rot_z}{2} \\ -\frac{rot_y}{2} \\ -\frac{rot_x}{2} \end{array}
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                                0
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      0
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                                                        0 0 0
                                                                 0
                                                                    0 0
                                                \frac{pose.R_y}{2}
                              pose.R_w
                                      -\frac{pose.R_z}{}
      0
                  0
                                                        0 0 0
                                                                 0
                                                                    0 0
                                       \frac{\overline{\frac{2}{2}}}{2}
                              pose.R_z
                                               -\frac{pose.R_x}{2} 0 0 0 0 0 0
      0
          0
                  0
                        0
                                      -\frac{\frac{2}{pose.R_x}}{\frac{2}{pose.R_y}}
                             -\frac{pose.R_y}{}
                                                pose.R_w
      0
          0
                  0
                        0
                                                        0 0 0 0 0 0
                             \frac{1}{2} \frac{2}{pose.R_x}
                                               -\frac{pose.R_z}{2}
      0
          0
                                                        0 0 0 0 0 0
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                        0
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                                                 0 0 0 0 0 1 0
     0
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                                0
                                                        0 0 0 0 0 1
```

```
residual = sf.Matrix(6, 1)
residual[0:3, 0] = lie_vals["rot3"] * lie_vals["sub_vals.vec"]
residual[3:6, 0] = lie_vals["sub_vals.pose3"] * lie_vals["sub_vals.vec"]
display(residual)
                                                                                                                                                                                                                                                vec_{0}\left(-2rot_{y}^{2}-2rot_{z}^{2}+1\right)+vec_{1}\left(-2rot_{w}rot_{z}+2rot_{x}rot_{y}\right)+vec_{2}\cdot\left(2rot_{w}rot_{y}+2rot_{x}rot_{z}\right)\\vec_{0}\cdot\left(2rot_{w}rot_{z}+2rot_{x}rot_{y}\right)+vec_{1}\left(-2rot_{x}^{2}-2rot_{z}^{2}+1\right)+vec_{2}\left(-2rot_{w}rot_{x}+2rot_{y}rot_{z}\right)\\vec_{0}\left(-2rot_{w}rot_{y}+2rot_{x}rot_{z}\right)+vec_{1}\cdot\left(2rot_{w}rot_{x}+2rot_{y}rot_{z}\right)+vec_{2}\left(-2rot_{x}^{2}-2rot_{y}^{2}+1\right)
                                              \begin{array}{l} pose.t0 + vec_0\left(-2pose.R_y^2 - 2pose.R_z^2 + 1\right) + vec_1\left(-2pose.R_wpose.R_z + 2pose.R_xpose.R_y\right) + vec_2\cdot\left(2pose.R_wpose.R_y + 2pose.R_ypose.R_z\right) \\ pose.t1 + vec_0\cdot\left(2pose.R_wpose.R_z + 2pose.R_xpose.R_y\right) + vec_1\left(-2pose.R_z^2 - 2pose.R_z^2 + 1\right) + vec_2\left(-2pose.R_wpose.R_x + 2pose.R_z\right) \\ pose.t2 + vec_0\left(-2pose.R_wpose.R_y + 2pose.R_xpose.R_z\right) + vec_1\cdot\left(2pose.R_wpose.R_x + 2pose.R_ypose.R_z\right) + vec_2\left(-2pose.R_z^2 - 2pose.R_z\right) \\ pose.t2 + vec_0\left(-2pose.R_wpose.R_z\right) + vec_1\cdot\left(2pose.R_wpose.R_x + 2pose.R_ypose.R_z\right) + vec_2\cdot\left(-2pose.R_z^2 - 2pose.R_z\right) \\ pose.t3 + vec_0\left(-2pose.R_wpose.R_z\right) + vec_1\cdot\left(2pose.R_wpose.R_z\right) + vec_2\cdot\left(-2pose.R_z\right) \\ pose.t4 + vec_0\left(-2pose.R_wpose.R_z\right) + vec_1\cdot\left(2pose.R_wpose.R_z\right) + vec_2\cdot\left(-2pose.R_z\right) \\ pose.t5 + vec_0\left(-2pose.R_wpose.R_z\right) + vec_1\cdot\left(2pose.R_wpose.R_z\right) \\ pose.t6 + vec_0\left(-2pose.R_wpose.R_z\right) + vec_1\cdot\left(2pose.R_wpose.R_z\right) \\ pose.t7 + vec_0\left(-2pose.R_wpose.R_z\right) + vec_1\cdot\left(2pose.R_wpose.R_z\right) \\ pose.t8 + vec_0\left(-2pose.R_wpose.R_z\right) + vec_1\cdot\left(2pose.R_wpose.R_z\right) \\ pose.t9 + vec_0\left(-2pose.R_wpose.R_z\right) + vec_0\left(-2pose.R_wpose.R_z\right) \\ pose.t9 + vec_0\left(-2pose.R_wpose.R_z\right) \\ pose
residual_D_tangent = residual.jacobian(lie_vals)
display(residual_D_tangent.shape)
display(residual_D_tangent)
                                          (6, 13)
                                                         vec_0\left(-2rot_wrot_y-2rot_xrot_z\right)+vec_2\left(rot_w^2+rot_x^2-rot_z^2\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 vec_0 \cdot (2rot_w rot_x - 2rot_y rot_z) + vec_2 \cdot (2rot_w rot_z + 2rot_y rot_z) + vec_0 \cdot (-rot_w^2 + rot_x^2 + rot_y^2 - rot_z^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_z^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_z^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + rot_y^2 - rot_y^2) + vec_2 \cdot (-2rot_w rot_y + ro
                                                                                                                                                                                                                                                                                                                                                       0
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```