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1 Problem 1

Consider the two-dimensional plane with the usual coordinate axes x and y

Solution

For solutions for Problem 1 (a),(b), and (d) see Figure 1

(c)

$$\vec{P} = 2\vec{i} + 3\vec{j} \tag{1}$$
$$\vec{Q} = 5\vec{i} + 1\vec{j} \tag{2}$$

(e)

$$\begin{aligned}
\left|\left|\vec{Q} - \vec{P}\right|\right| &= \left|\left|(5\vec{i} + 1\vec{j}) - (2\vec{i} + 3\vec{j})\right|\right| \\
&= \left|\left|(5-2)\vec{i} + (1-3)\vec{j}\right|\right| \\
&= \left|\left|3\vec{i} - 2\vec{j}\right|\right| \\
&= \sqrt{(3)^2 + (-2)^2} \\
&= \sqrt{13}
\end{aligned}$$
$$\implies \left|\left|\vec{Q} - \vec{P}\right|\right| = \sqrt{13} \tag{3}$$

(f)

$$|PQ| = \sqrt{(5-2)^2 + (1-3)^2}$$
$$|PQ| = \sqrt{13} \tag{4}$$

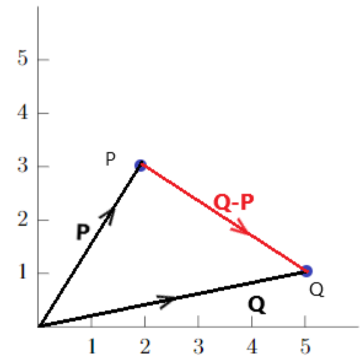

Figure 1: Vectors $\vec{P}$ and $\vec{Q}$

2 Problem 2

For $v = -2i + 6j$ and $w = 10j - 3i$,
(a) Calculate $||v + w||$
(b) What are the coordinates of the points in the plan with position vectors v and w?
(c) Calculate $v \cdot w$
(d) Show that $||v + w|| \leq ||v|| + ||w||$
(e) Calculate $\cos\theta$ where θ is the angle between v and w

Solution

(a)

$$\begin{aligned}
||v+w|| &= \left|\left|(-2\vec{i} + 6\vec{j}) + (-3\vec{i} + 10\vec{j})\right|\right| \\
&= \left|\left|(-2-3)\vec{i} + (6+10)\vec{j}\right|\right| \\
&= \left|\left|-5\vec{i} + 16\vec{j}\right|\right| \\
&= \sqrt{(-5)^2 + (16)^2} \\
&= \sqrt{281}
\end{aligned}$$

$$||v+w|| = \sqrt{281} \tag{5}$$

(b) $v = (-2, 6) w = (-3, 10)$ (c)

$$\vec{v} = \begin{pmatrix} -2 & 6 & 0 \end{pmatrix} \tag{6}$$

$$\vec{w} = \begin{pmatrix} -3 & 10 & 0 \end{pmatrix} \tag{7}$$

$$v \cdot w = \begin{pmatrix} -2 & 6 & 0 \end{pmatrix} \cdot \begin{pmatrix} -3 & 10 & 0 \end{pmatrix}$$

$$v \cdot w = (-2)(-3) + (6)(10)$$

$$v \cdot w = 66 \tag{8}$$

(d)

$$||v+w|| \leq ||v|| + ||w||$$

$$\begin{aligned}
(5)...\ \sqrt{281} &\leq ||v|| + ||w|| \\
&\leq \left|\left|-2\vec{i} + 6\vec{j}\right|\right| + \left|\left|-3\vec{i} + 10\vec{j}\right|\right| \\
&\leq \sqrt{(-2)^2 + (6)^2} + \sqrt{(-3)^2 + (10)^2} \\
&\leq \sqrt{40} + \sqrt{109}
\end{aligned}$$

$$\sqrt{281} \approx 16.763 \leq 2\sqrt{10} + \sqrt{109} \approx 16.765 \blacksquare \tag{9}$$

(e)

$$\theta = \arctan\left(\frac{-3\vec{i}}{10\vec{j}}\right) - \arctan\left(\frac{-2\vec{i}}{6\vec{j}}\right)$$

$$\theta \approx 1.736$$

$$\implies \cos\theta = \cos(1.736) \approx 1 \tag{10}$$