PYU11P10 - Physics of Motion: Weekly Problems 1

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1 Problem 1

A particle travels along the line $y = 2x + 1$ with uniform speed $\vec{\mathbf{V}}$. Find the components of its velocity parallel to the axes of x and y.

Solution

The x and y components of a vector $\vec{\mathbf{V}}$ are denoted as $\vec{\mathbf{V_x}}$ and $\vec{\mathbf{V_y}}$ respectively.
It follows from Pythagoras' Theorem that the x and y components of the vector $\vec{\mathbf{V}}$ could be written in the forms

$$\vec{\mathbf{V_x}} = \vec{\mathbf{V}} \cos\theta \tag{1}$$

$$\vec{\mathbf{V_y}} = \vec{\mathbf{V}} \sin\theta \tag{2}$$

Let $\theta =$ The angle between the line y and the x axis.
Using $tan\theta = m$ where m = the slope of the line $y = 2x + 1$ in the form $y = mx + c$. We find that:

$$\begin{aligned} \tan\theta &= 2 \\ \arctan(2) &= \theta \\ \theta &\approx\ 63.435^\circ \end{aligned} \tag{3}$$

From inserting the value of θ from Eq.3 into the values for $\vec{\mathbf{V_x}}$ and $\vec{\mathbf{V_y}}$ from Eq.1 and Eq.2 respectively:

$$\vec{\mathbf{V_x}} = \vec{\mathbf{V}} \cos 63.435^\circ, \vec{\mathbf{V_y}} = \vec{\mathbf{V}} \sin 63.435^\circ$$

2 Problem 2

The velocities of the two particles at time t are $2t\vec{\mathbf{i}} + 12\vec{\mathbf{j}}$ and $4\vec{\mathbf{i}} + (3 - 2t)\vec{\mathbf{j}}$, respectively. Find the instant when the particles are moving in perpendicular directions.

Solution

Let $\vec{\mathbf{A}} = 2t\vec{\mathbf{i}} + 12\vec{\mathbf{j}}$ and $\vec{\mathbf{B}} = 4\vec{\mathbf{i}} + (3 - 2t)\vec{\mathbf{j}}$. We can rewrite these vectors in the form:

$$\vec{\mathbf{A}} = \begin{pmatrix} 2t & 12 & 0 \end{pmatrix} \tag{4}$$

$$\vec{\mathbf{B}} = \begin{pmatrix} 4 & 3 - 2t & 0 \end{pmatrix} \tag{5}$$

To find the time in which the vectors are moving in perpendicular directions, we must find the time t such that the dot product

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0 \tag{6}$$

$$\begin{aligned} \begin{pmatrix} 2t & 12 & 0 \end{pmatrix} \cdot \begin{pmatrix} 4 & 3 - 2t & 0 \end{pmatrix} &= 2t \cdot 4 + 12 \cdot (3 - 2t) \\ &= 8t + 36 - 24t \\ &= 36 - 16t \end{aligned}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 36 - 16t \tag{7}$$

$$Eq.6 = Eq.7$$

$$0 = 36 - 16t$$

$$16t = 36$$

$$t = \frac{36}{12}$$

$$t = \frac{9}{4} \tag{8}$$

From Equation 8 we can see that the two particles are moving in perpendicular directions when $t = \frac{9}{4}$units of time.