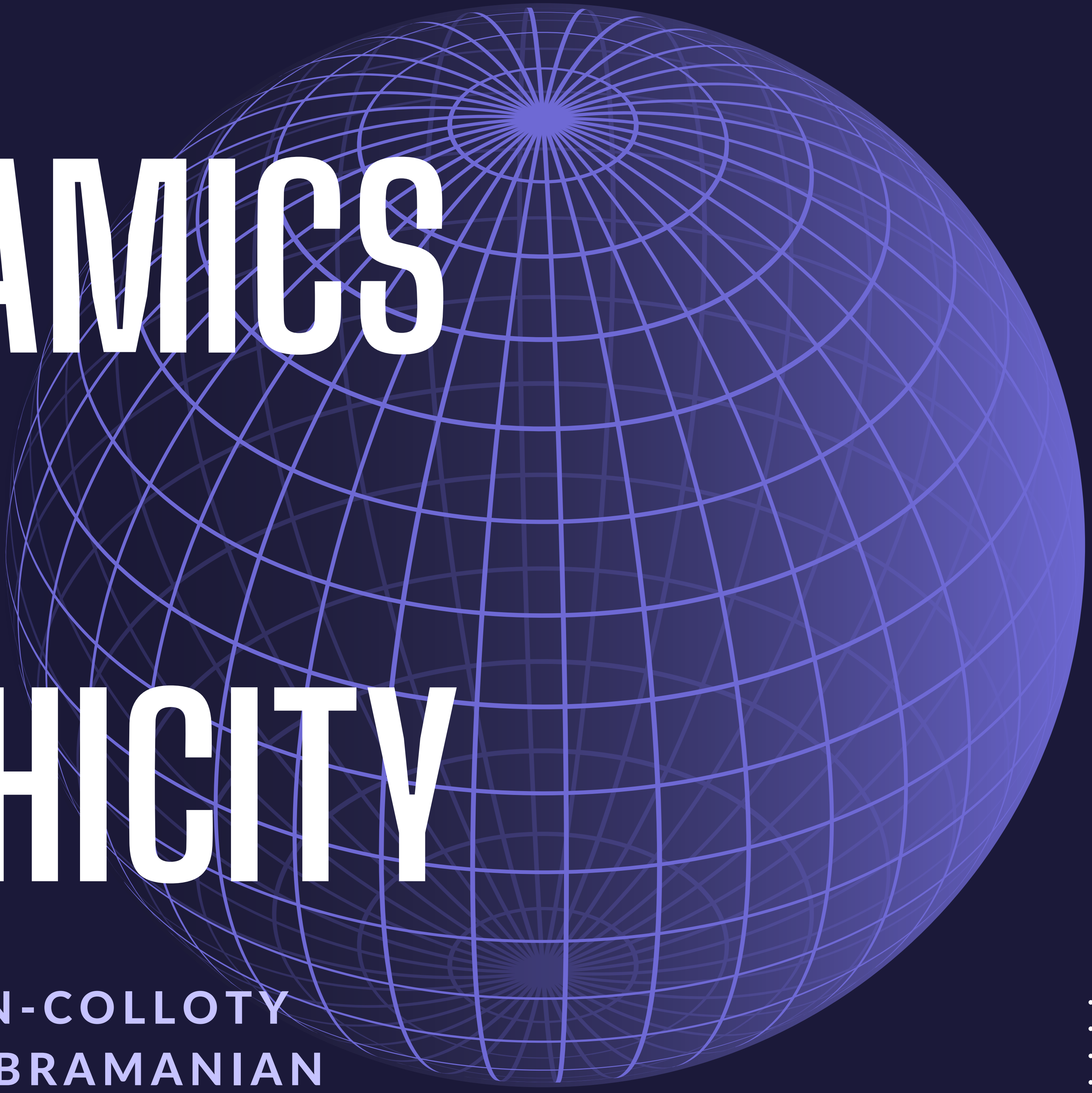
HYDRODYNAMICS AND HOLOMORPHICITY



DANIEL BARRON - CASEY FARREN-COLLOTY
EDWARD HEENEY - VEDANTH SUBRAMANIAN

PYU33TP10 - COMMUNICATIONS

INTRODUCTION

BREAKDOWN

VEDANTH (RELEVANCE)

DANIEL (SPECIAL REL)

ED (MATHS)

CAS (SIMULATIONS)

WHAT IS HYDRODYNAMICS?

Hydrodynamics is the study of the motion of liquids

OR IS IT???

RELEVANCE OF HYDRODYNAMICS

FLUID FLOW IN PIPES

Z. INSTRUMENT, "KEY CONSIDERATIONS IN PIPELINE DESIGN," ZERO INSTRUMENT. HTTPS://ZEROINSTRUMENT.COM/KEY-CONSIDERATIONS-IN-PIPELINE-DESIGN/ (ACCESSED JAN. 27, 2025)

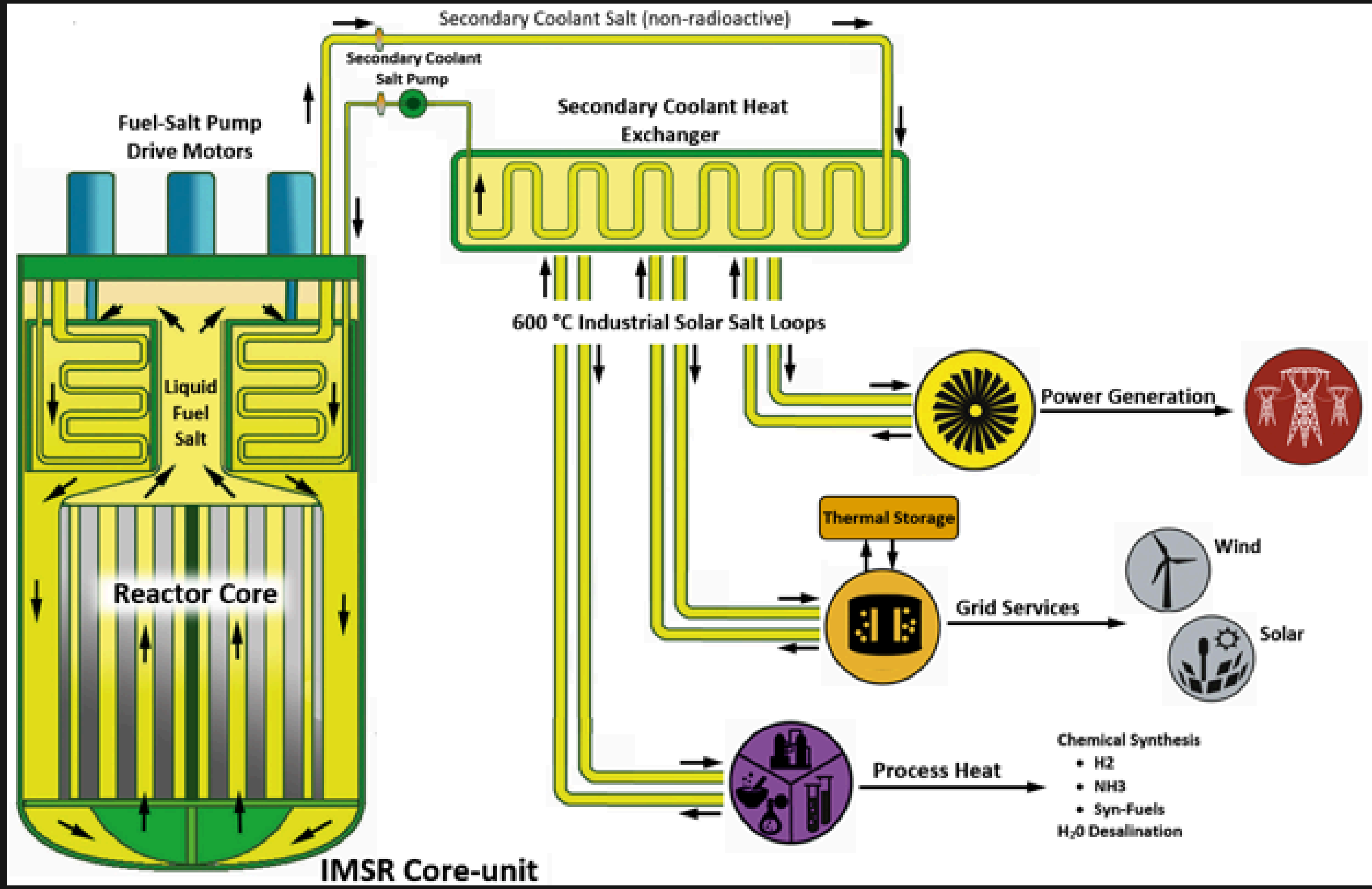
METEOROLOGY/ATMOSPHERIC PHYSICS

RELEVANCE OF HYDRODYNAMICS

NASA, "ASTRONAUT PHOTOGRAPH OF EARTH," FLICKR. Z. INSTRUMENT, "KEY CONSIDERATIONS IN PIPELINE DESIGN," ZERO INSTRUMENT. HTTPS://ZEROINSTRUMENT.COM/KEY-CONSIDERATIONS-IN-PIPELINE-DESIGN/ (ACCESSED JAN. 27, 2025) (ACCESSED JAN. 27, 2025).

HYDRODYNAMICS AND HOLOMORPHICITY

MOLTEN SALT REACTORS



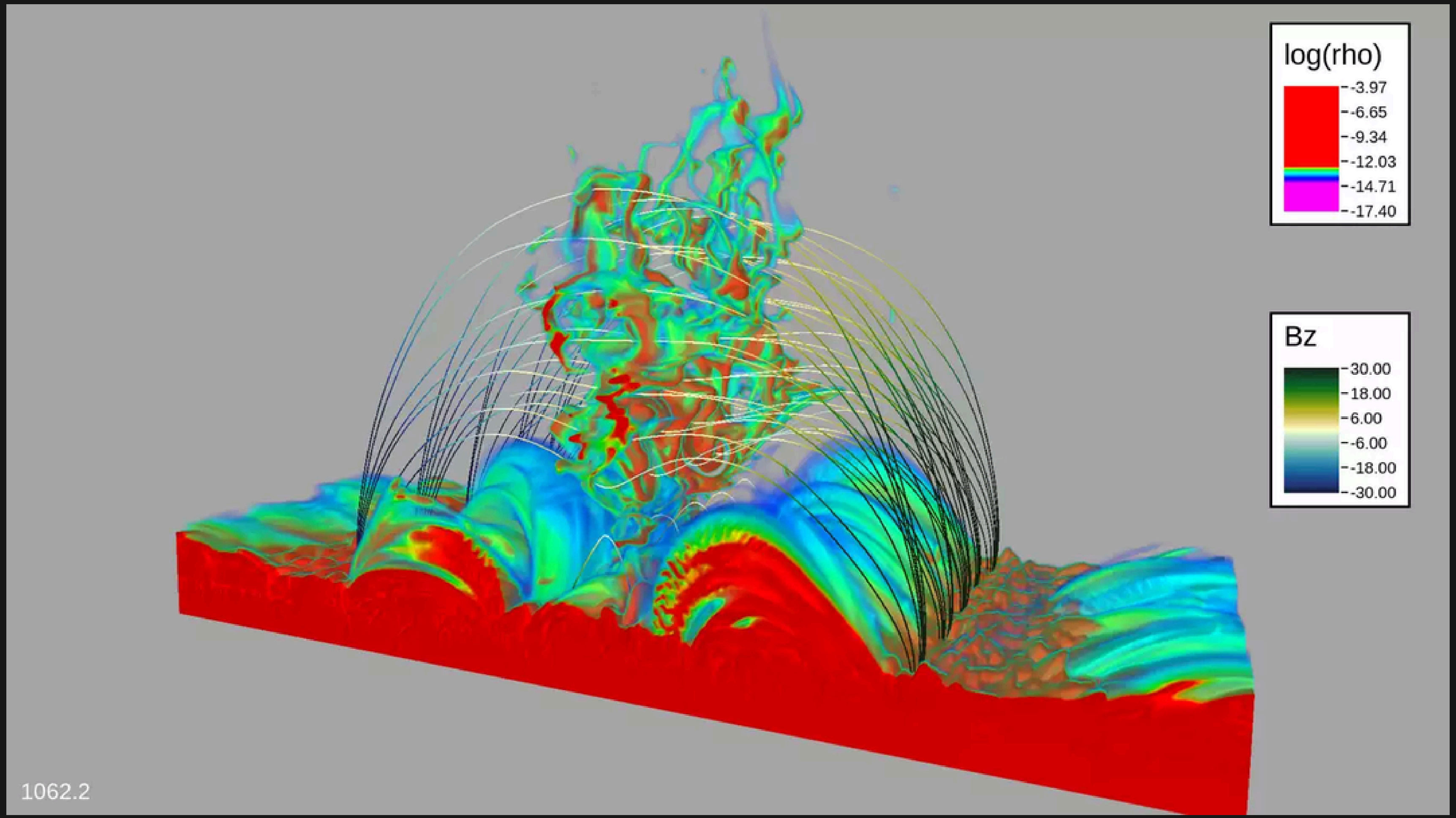
RELEVANCE OF HYDRODYNAMICS

"INTEGRAL MOLTEN SALT REACTOR," WIKIPEDIA. NASA, "ASTRONAUT PHOTOGRAPH OF EARTH," FLICKR. Z. INSTRUMENT, "KEY CONSIDERATIONS IN PIPELINE DESIGN," ZERO INSTRUMENT. HTTPS://ZEROINSTRUMENT.COM/KEY-CONSIDERATIONS-IN-PIPELINE-DESIGN/ (ACCESSED JAN. 27, 2025) (ACCESSED JAN. 27, 2025). (ACCESSED JAN. 29, 2025).

MAGNETOHYDRODYNAMICS (MHD)

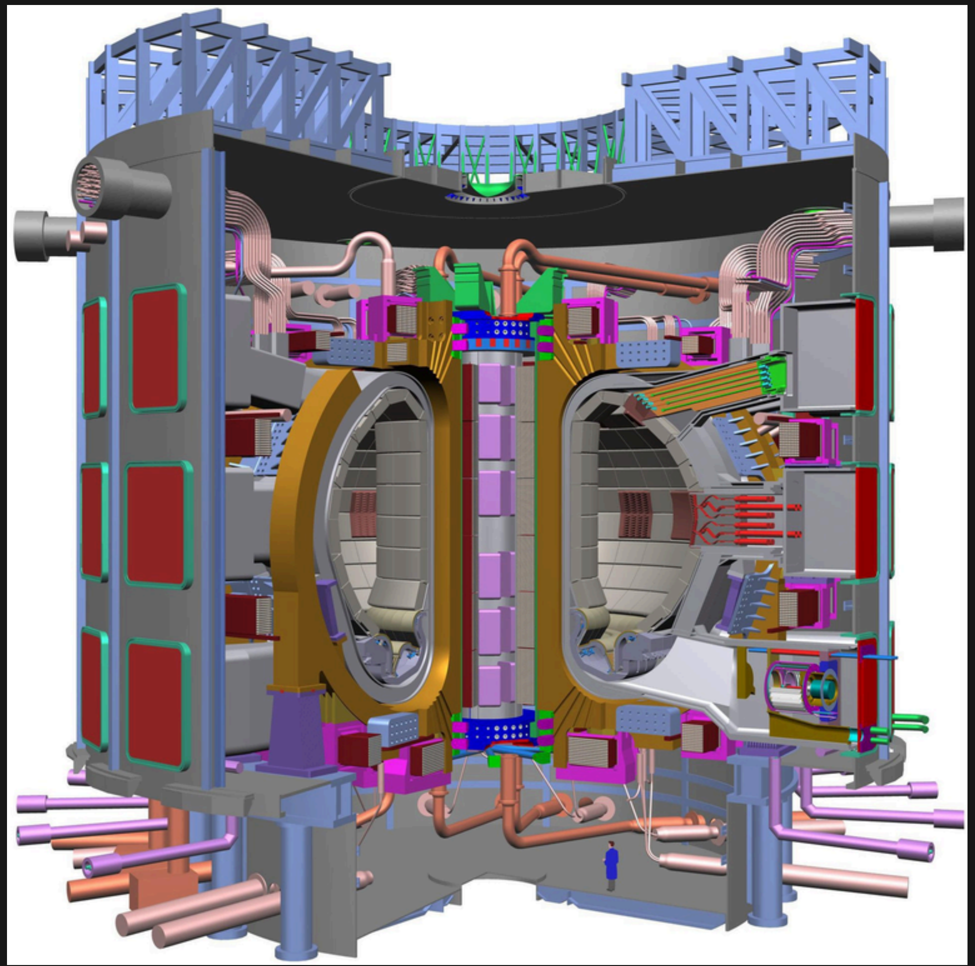
Study of charged conductive fluids

SOLAR PHYSICS



MAX PLANCK INSTITUTE FOR SOLAR SYSTEM RESEARCH, "SOLAR AND STELLAR MAGNETOHYDRODYNAMICS," MPS. YOUR PARAGRAPH TEXT (ACCESSED JAN. 27, 2025).

MAGNETIC CONFINEMENT REACTORS



U.S. DEPARTMENT OF ENERGY, "NUCLEAR ENERGY IMAGE," FLICKR. HTTPS://WWW.FLICKR.COM/PHOTOS/DEPARTMENTOFENERGY/9786811206/ (ACCESSED JAN. 27, 2025).

WHAT IS HYDRODYNAMICS, REALLY?

Hydrodynamics is a low-energy effective description of many-body dynamical systems.

A RELATIVISTIC APPROACH

- We can use the following definitions from relativity to build up the picture:

$$u^\mu = \gamma(c, \vec{v}), \quad \partial_\mu = (\frac{1}{c}\partial_t, \vec{\nabla})$$

- The EoM's are given by the following:

$$\partial_t \rho + \vec{\nabla} \cdot \vec{J} = 0, \quad \partial_\mu T^{\mu\nu} = 0$$

USING SYMMETRIES

- Using the symmetries of relativity, we can say the following:

$$J^\mu \sim u^\mu, \quad T^{\mu\nu} \sim u^\mu u^\nu, \eta^{\mu\nu}$$

- The zeroth order expansion of the current density and stress energy tensor are given by:

$$J^\mu_{(0)} = \rho u^\mu, \quad T^{\mu\nu}_{(0)} = \frac{\epsilon + P}{c^2} u^\mu u^\nu + P\eta^{\mu\nu}$$

THE MATHS : COMPLEX ANALYSIS

Holomorphic Functions

Holomorphic functions : complex differentiable at every point in domain

Satisfy Cauchy-Reimann equations

$$f = u + iv$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

LAPLACE EQUATION

If the Cauchy-Reimann equations are satisfied then both the real and imaginary part satisfied the Laplace equation

$$\nabla^2 \psi = 0$$

PARTICLE PLANAR FLOW POTENTIAL

COMPLEX POTENTIAL

$$f = \phi + i\psi$$

PARTICLE PLANAR FLOW POTENTIAL

COMPLEX POTENTIAL

$$f = \phi + i\psi \quad \&$$

PARTICLE PLANAR FLOW POTENTIAL

COMPLEX POTENTIAL

$$f = \phi + i\psi$$

&

POSITION VECTOR

$$z = re^{i\theta}$$

PARTICLE PLANAR FLOW POTENTIAL

COMPLEX POTENTIAL

$$f = \phi + i\psi$$

&

POSITION VECTOR

$$z = re^{i\theta}$$

=

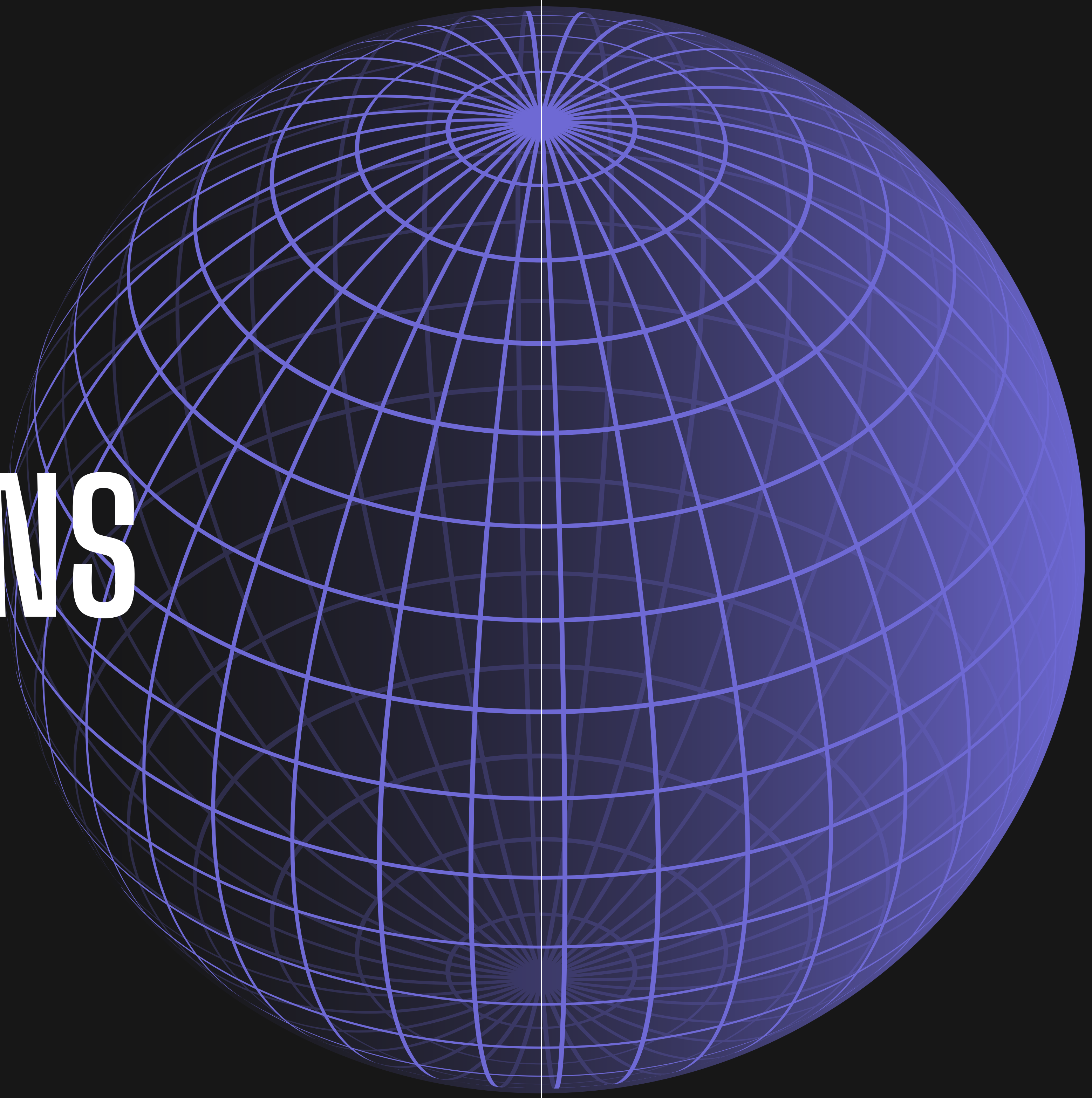
PARTICLE PLANAR FLOW POTENTIAL

FLUID FLOW AROUND CYLINDER

$$f(z) = U\left\{z + \frac{R^2}{z}\right\}$$

SIMULATIONS

SOME 2D EXAMPLES

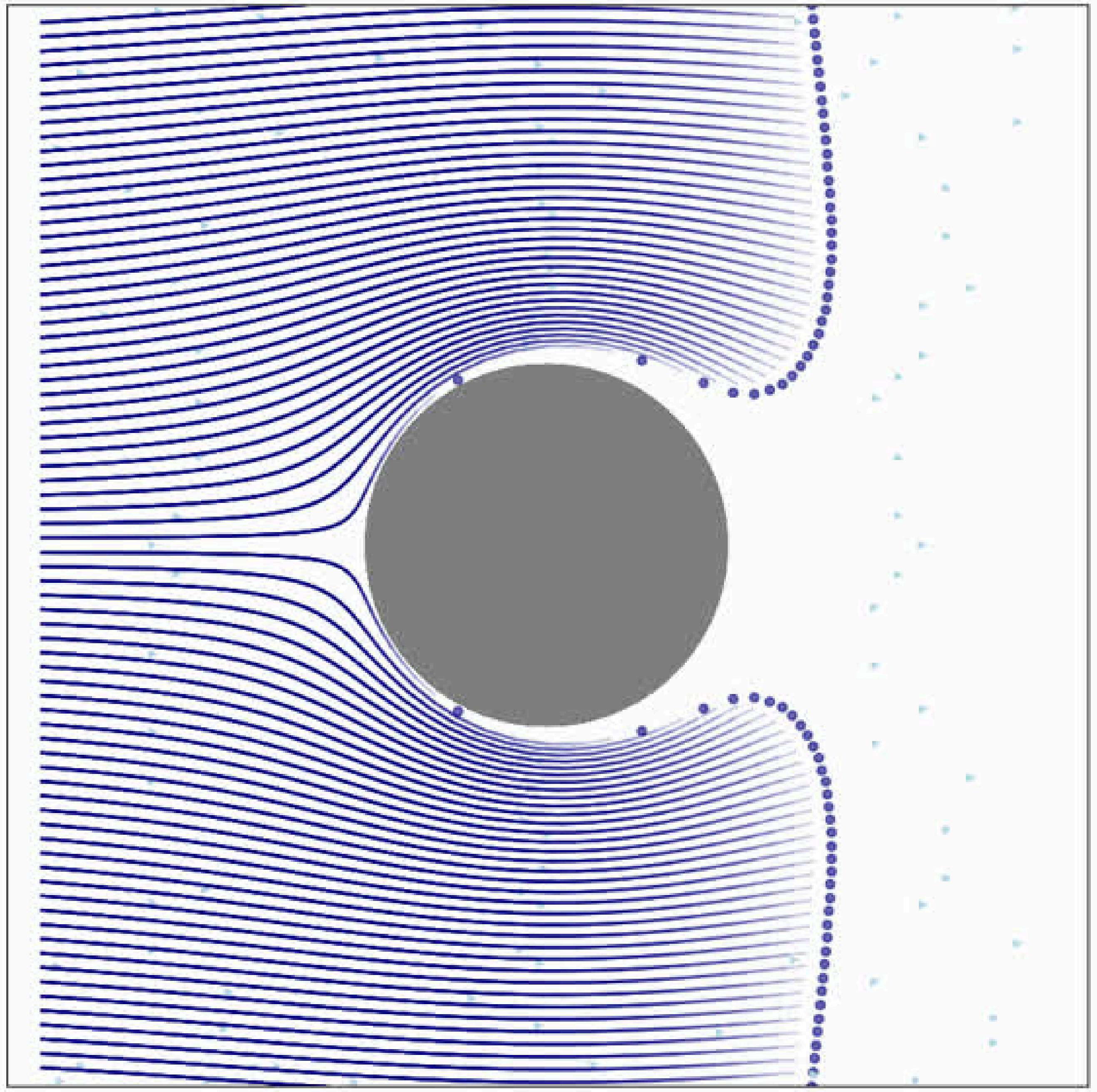


HYDRODYNAMICS AND HOLOMORPHICITY

Water Flow in 2D

$$W(z) = U\left(z + \frac{R^2}{z}\right)$$

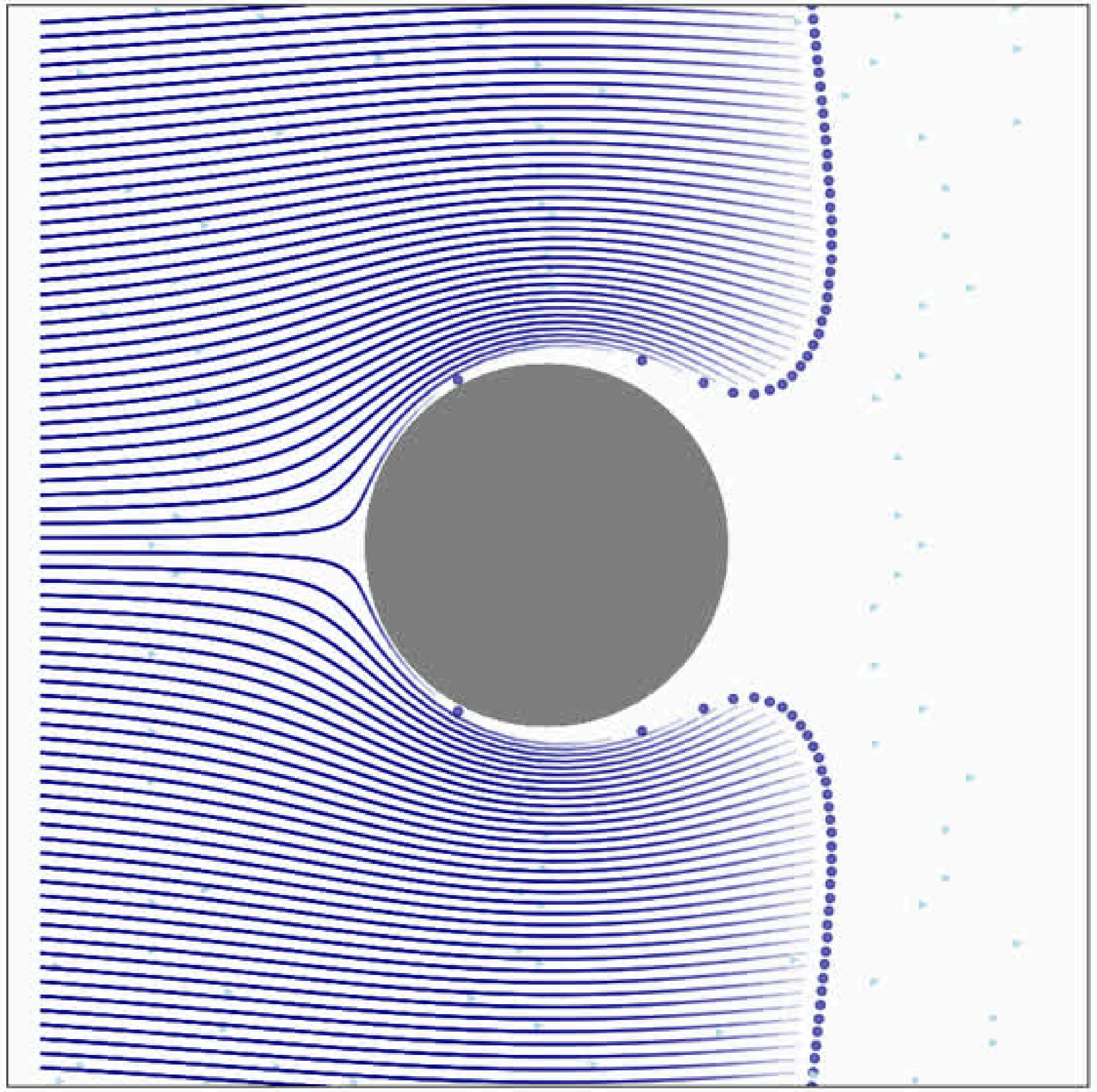
Water Flow in 2D



$$W(z) = U\left(z + \frac{R^2}{z}\right)$$

Copilot was used in some capacity

Water Flow in 2D



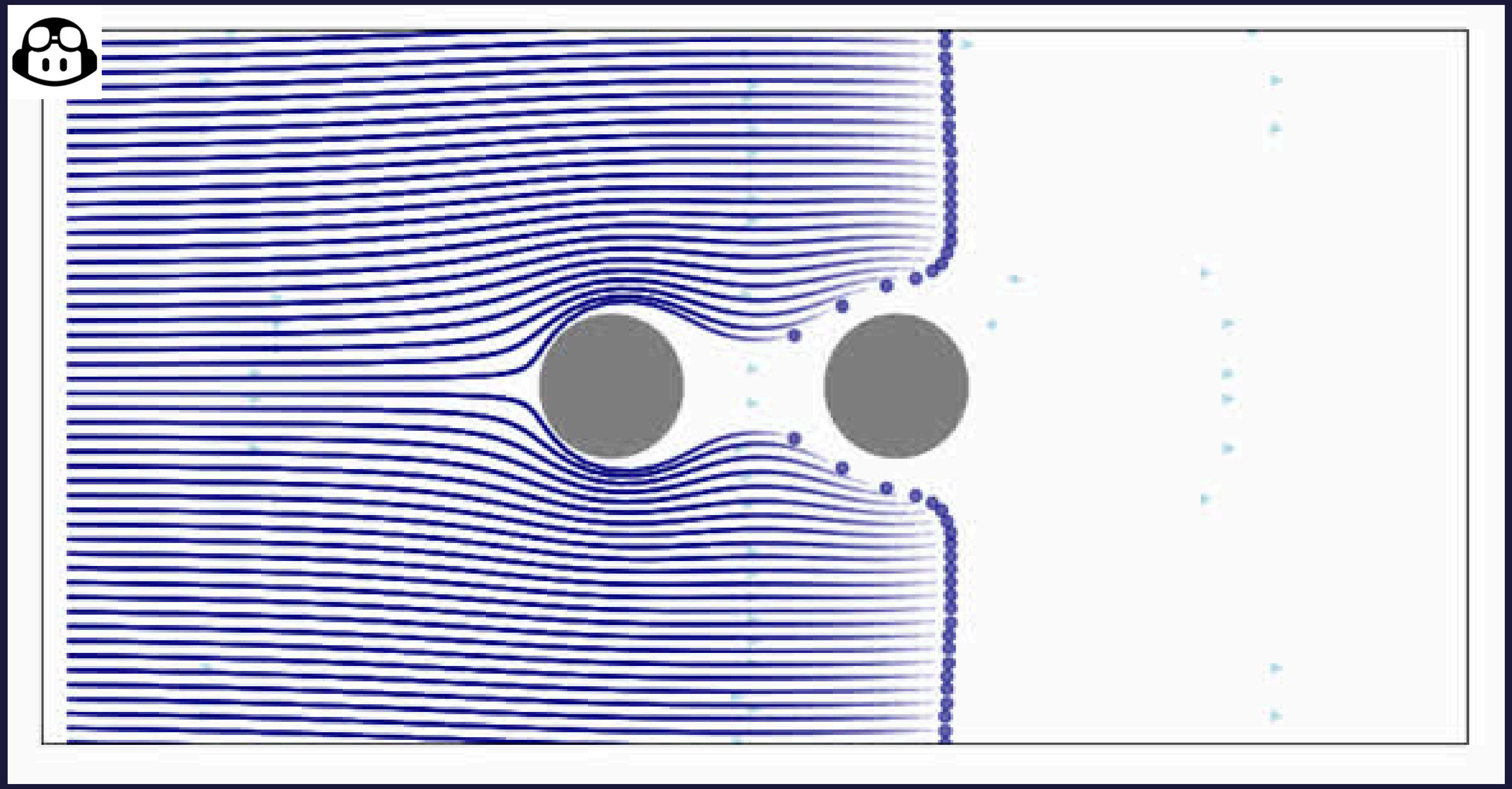
$$W(z) = U\left(z + \frac{R^2}{z}\right)$$

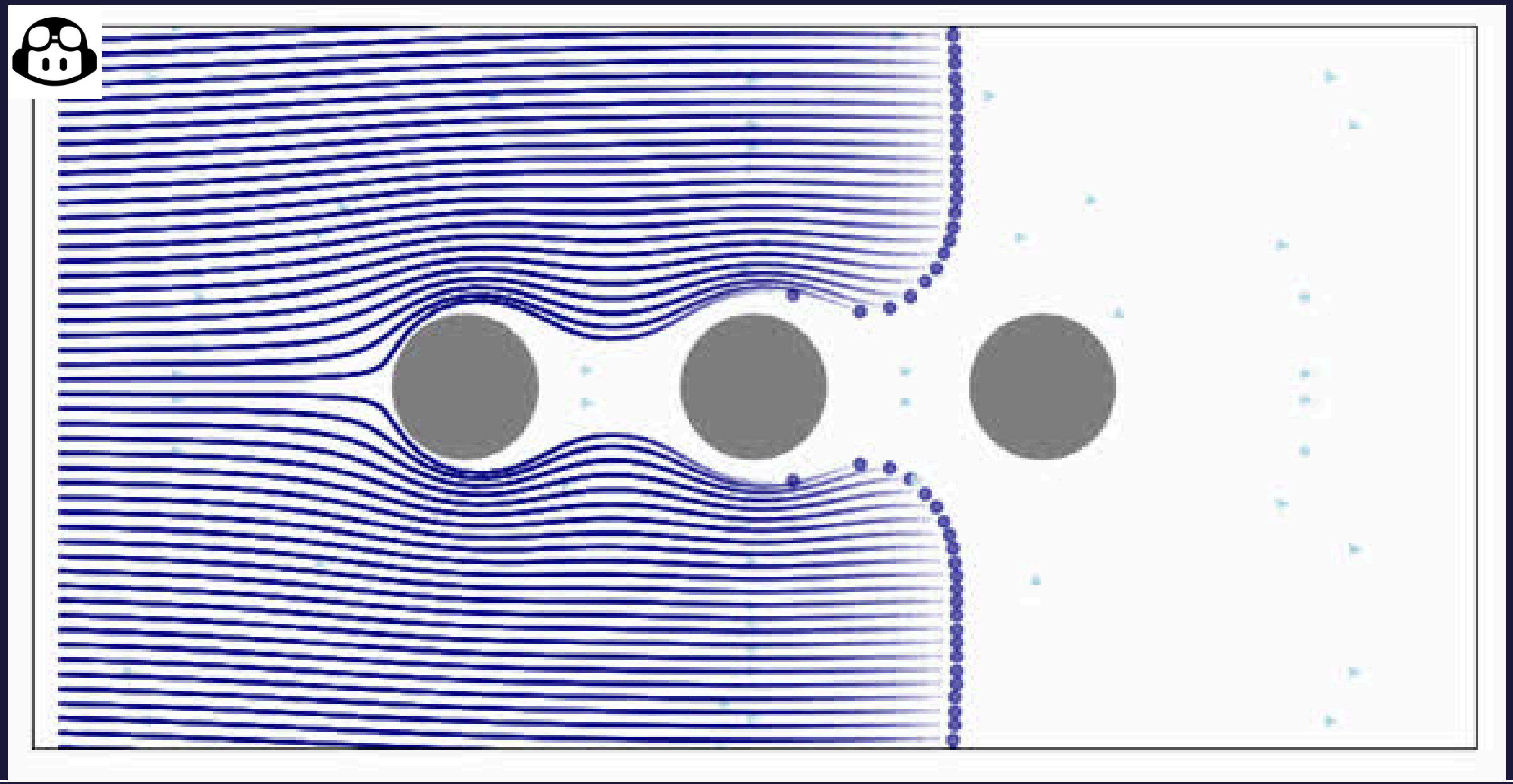
WHAT ABOUT MORE **COMPLEX** EXAMPLES?

THE FULL SOLUTION FOR TWO CYLINDERS

Crowdy, D. G. (2006). "Analytical solutions for uniform potential flow past multiple cylinders."
European Journal of Mechanics B/Fluids, 25(4), 459-470.

$$W = \frac{Ua}{\beta}\zeta\beta^{-1} \cdot \partial_{\zeta\beta^{-1}} \log\left((1-\zeta\beta^{-1}) \prod_{k=1}^{\infty} (1-q^{2k}\zeta\beta^{-1})(1-q^{2k}\zeta^{-1}\beta) \right)$$
$$-\frac{Ua}{\beta}\zeta\beta^{-1} \cdot \zeta\beta\partial_{\zeta\beta} \log\left((1-\zeta\beta) \prod_{k=1}^{\infty} (1-q^{2k}\zeta\beta^{-1})(1-q^{2k}\zeta^{-1}\beta) \right)$$

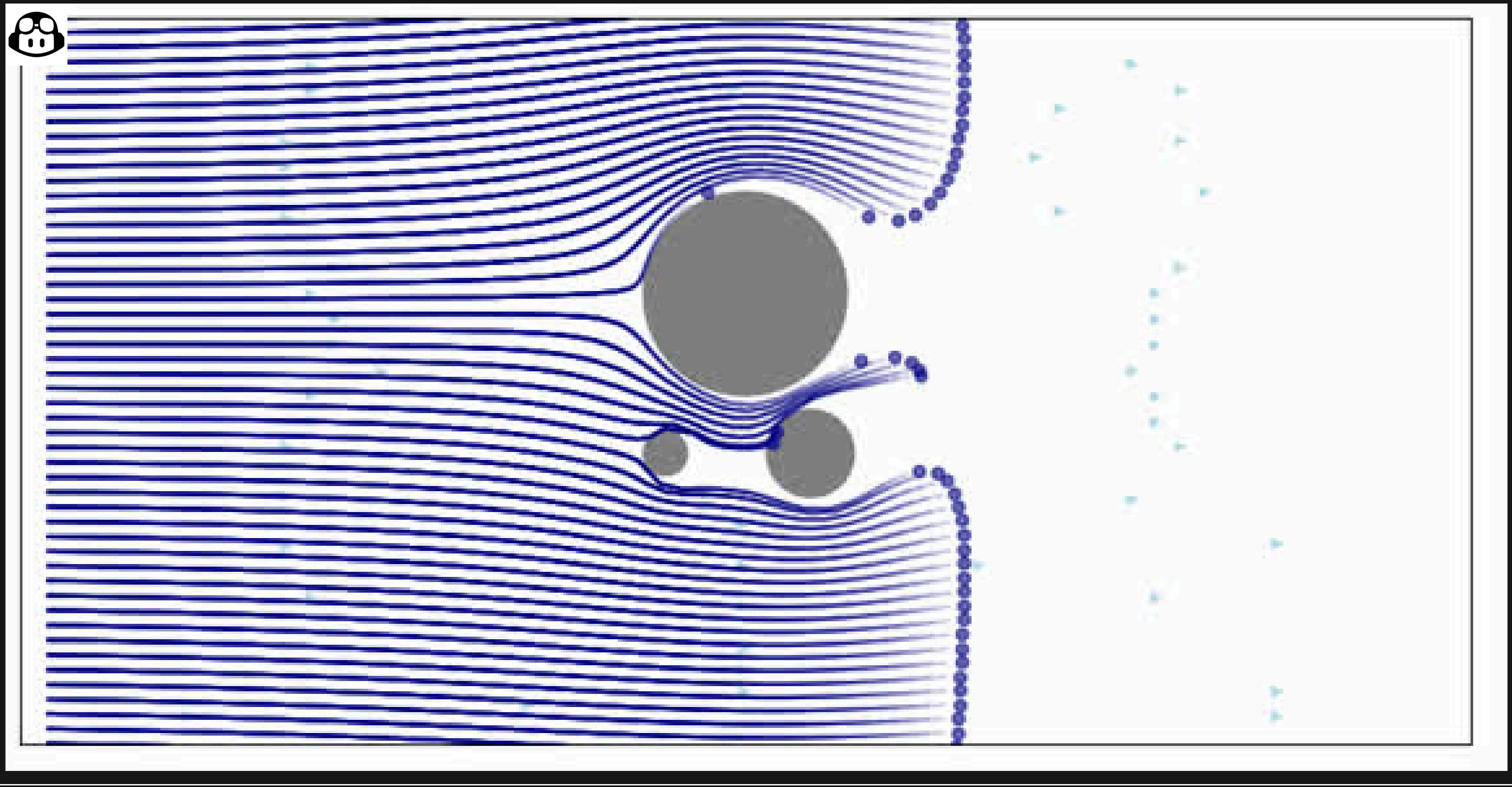




MORE CYLINDERS

Laplace is Linear

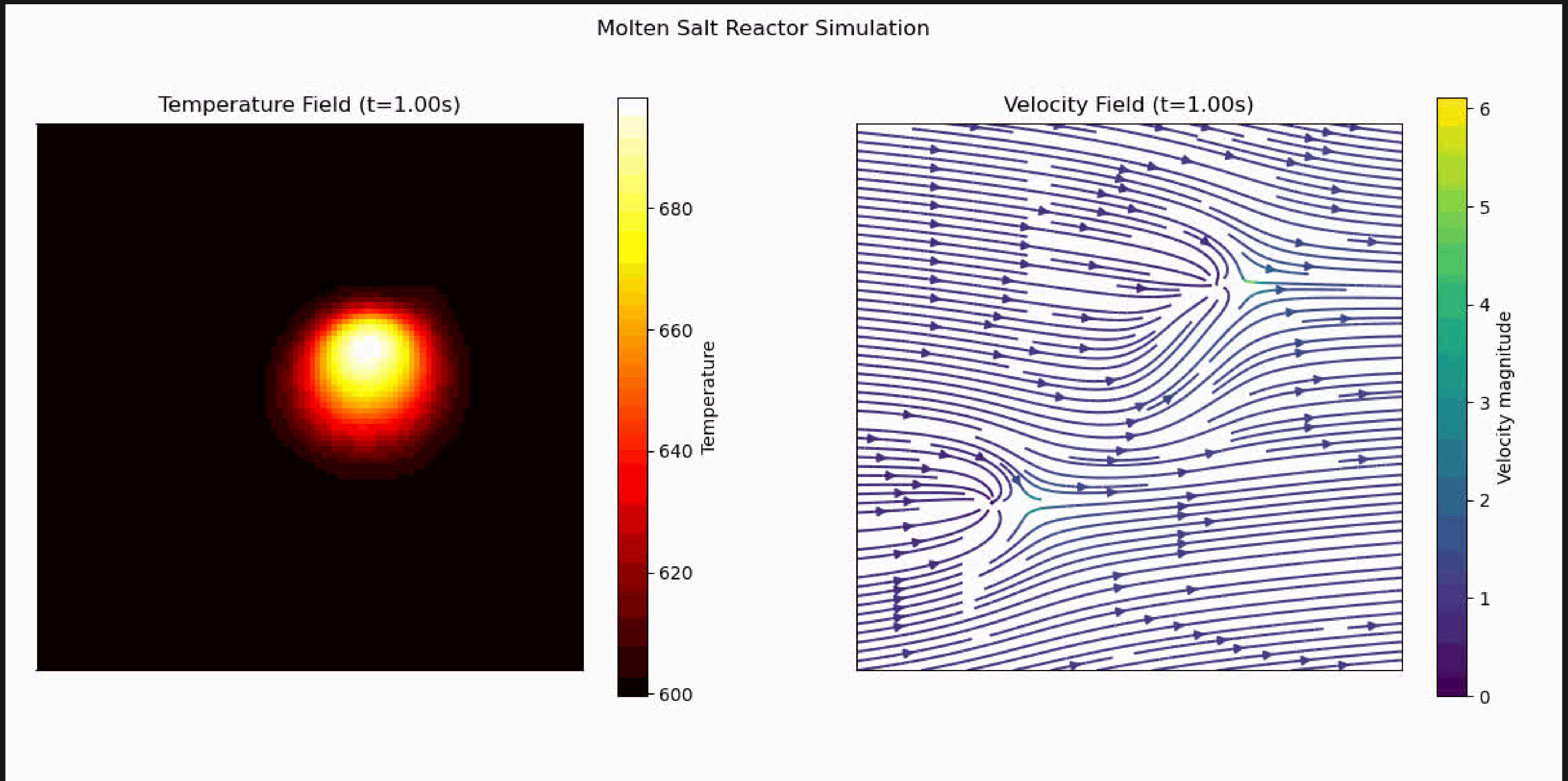
WHAT COULD GO WRONG?



A MORE MODERN EXAMPLE?

$$W(z,T) = (1 + \alpha \cdot (T - T_0))z + \sum_i \frac{1}{z - z_i}$$

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T + S_0 \Theta(|z| < R)$$



THANK YOU FOR LISTENING

Any questions?