Problem Set 7: QTM 200 Applied Regression Analysis

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Due: May 6, 2020

Instructions

- Please show your work! You may lose points by simply writing in the answer. If the problem requires you to execute commands in `R`, please include the code you used to get your answers. Please also include the `.R` file that contains your code. If you are not sure if work needs to be shown for a particular problem, please ask.
- Your homework should be submitted electronically on the course GitHub page in `.pdf` form.
- This problem set is due before midnight on Wednesday, May 6, 2020. No late assignments will be accepted.
- Total available points for this homework is 100.

Question 1 (50 points): Political Science

Consider the data set `MexicoMuniData.csv`, which includes municipal-level information from Mexico. The outcome of interest is the number of times the winning PAN presidential candidate in 2006 (`PAN.visits.06`) visited a district leading up to the 2009 federal elections, which is a count. Our main predictor of interest is whether the district was highly contested, or whether it was not (the PAN or their opponents have electoral security) in the previous federal elections during 2000 (`competitive.district`), which is binary (1=close/swing district, 0="safe seat"). We also include `marginality.06` (a measure of poverty) and `PAN.governor.06` (a dummy for whether the state has a PAN-affiliated governor) as additional control variables.

```
mexico_elections <- read.csv("MexicoMuniData.csv")
```

(a) Run a Poisson regression because the outcome is a count variable. Is there evidence that PAN presidential candidates visit swing districts more? Provide a test statistic and p-value.

```
poisson_model1 <- glm(PAN.visits.06 ~ competitive.district + marginality
    .06 + PAN.governor.06, data=mexico_elections, family = poisson)
summary(poisson_model1)
#Deviance Residuals:
#    Min       1Q   Median       3Q      Max
# -2.1441  -0.3596  -0.1742  -0.0783  15.2935
#Coefficients:
#                       Estimate Std. Error z value Pr(>|z|)
# (Intercept)            -3.9304     0.1747 -22.503   <2e-16 ***
#  competitive.district  -0.4594     0.3276  -1.402    0.161
# marginality.06         -2.0981     0.1210 -17.343   <2e-16 ***
#  PAN.governor.06       -0.2073     0.1660  -1.249    0.212
#  Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .
    0.1          1
#(Dispersion parameter for poisson family taken to be 1)
#Null deviance: 1433.83  on 2392  degrees of freedom
#Residual deviance:  963.57  on 2389  degrees of freedom
#(4 observations deleted due to missingness)
#AIC: 1255.9
#Number of Fisher Scoring iterations: 7
```

There is no statistically reliable evidence that PAN presidential candidates visit swing districts more or less. The z-statistic from the Poisson regression's estimate of -0.4594 is -1.402, which has an associated p-value of 0.161. This is not statistically significant and would lead to failing to reject the null hypothesis that PAN presidential candidates visit swing districts more.

(b) Interpret the `marginality.06` and `PAN.governor.06` coefficients.

```
exp(coef(poisson_model1))
#(Intercept) competitive.district       marginality.06
#0.0196349            0.6316508          0.1226841
#PAN.governor.06
#0.8127638
```

The marginality.06 coefficient of -2.0981 means that the average number of times the winning PAN presidential candidate in 2006 visited a district before 2009 federal elections decreases as poverty increases. Specifically, as poverty increases by a unit of one on this scale, holding all else equal, the average number of visits decreases by a multiplicative factor of 0.1226841. Also, the PAN.governor.06 coefficient of -0.2073 means that the average number of times the winning PAN presidential candidate in 2006 visited a district before 2009 federal elections decreases when switching from a state with a PAN-affiliated governor to a state without one. Specifically, holding all else equal, switching from a state without a PAN_affiliated governor to a state with one means the average number of visits decreases by a multiplicative factor of 0.8127638.

(c) Provide the estimated mean number of visits from the winning PAN presidential candidate for a hypothetical district that was competitive (`competitive.district`=1), had an average poverty level (`marginality.06` = 0), and a PAN governor (`PAN.governor.06`=1).

```
lambda1 <- exp(-3.9304 - 0.4594*1 - 2.0981*0 - 0.2073*1)
lambda1
```

Therefore, the estimated mean number of visits is 0.01, which rounds to zero or no visits on average.

Question 2 (50 points): Biology

We'll be using data from a longitudinal sleep study of under 20 undergraduate students (n=18), which took place over the course of 10 days to see if sleep deprivation has any effect on participants' reaction time. Load the data through the `lmer` package.

```
sleepstudy <- sleepstudy
```

1. Create a "pooled" linear model where you regress `Days` on the outcome `Reaction`. Make sure to run regression diagnostics to check if the variance around the regression line is equal for every year.

```
## Part a: Create a pooled linear model where you regress Days on
    the outcome Reaction. Make sure to run regression diagnostics to check
     if the variance around the regression line is equal for every year.
complete_poolingLM <- lm(Reaction ~ Days, data=sleepstudy)
summary(complete_poolingLM)
#Residuals:
#  Min       1Q   Median       3Q      Max
#-110.848  -27.483    1.546   26.142  139.953
#Coefficients:
#              Estimate Std. Error t value Pr(>|t|)
#  (Intercept)  251.405     6.610    38.033  < 2e-16 ***
#  Days          10.467      1.238    8.454    9.89e-15 ***
#  Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .
    0.1         1
#Residual standard error: 47.71 on 178 degrees of freedom
#Multiple R-squared:  0.2865, Adjusted R-squared:  0.2825
#F-statistic: 71.46 on 1 and 178 DF,  p-value: 9.894e-15
plot(complete_poolingLM, 3)
plot(complete_poolingLM, 1)
```

The diagnostic plots are shown in Figure 1. The scale location plot helps us verify that the variance is relatively constant with a straight horizontal line. The residuals vs. fitted values plot also helps us see equal spread across most fitted values.

2. Fit an "un-pooled" regression model with varying intercepts for patient (include an additive factor for patient) and save the fitted values.

```
no_poolingLM1 <- lm(Reaction ~ Days + factor(Subject) - 1, data=
    sleepstudy)
summary(no_poolingLM1)
#Residuals:
#  Min       1Q   Median       3Q      Max
```

```
#-100.540  -16.389   -0.341   15.215  131.159
#Coefficients:
#                     Estimate Std. Error t value Pr(>|t|)
#Days                 10.4673     0.8042   13.02   <2e-16 ***
#  factor(Subject)308 295.0310    10.4471   28.24   <2e-16 ***
#  factor(Subject)309 168.1302    10.4471   16.09   <2e-16 ***
#  factor(Subject)310 183.8985    10.4471   17.60   <2e-16 ***
#  factor(Subject)330 256.1186    10.4471   24.52   <2e-16 ***
#  factor(Subject)331 262.3333    10.4471   25.11   <2e-16 ***
#  factor(Subject)332 260.1993    10.4471   24.91   <2e-16 ***
#  factor(Subject)333 269.0555    10.4471   25.75   <2e-16 ***
#  factor(Subject)334 248.1993    10.4471   23.76   <2e-16 ***
#  factor(Subject)335 202.9673    10.4471   19.43   <2e-16 ***
#  factor(Subject)337 328.6182    10.4471   31.45   <2e-16 ***
#  factor(Subject)349 228.7317    10.4471   21.89   <2e-16 ***
#  factor(Subject)350 266.4999    10.4471   25.51   <2e-16 ***
#  factor(Subject)351 242.9950    10.4471   23.26   <2e-16 ***
#  factor(Subject)352 290.3188    10.4471   27.79   <2e-16 ***
#  factor(Subject)369 258.9319    10.4471   24.79   <2e-16 ***
#  factor(Subject)370 244.5990    10.4471   23.41   <2e-16 ***
#  factor(Subject)371 247.8813    10.4471   23.73   <2e-16 ***
#  factor(Subject)372 270.7833    10.4471   25.92   <2e-16 ***
#  Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .
   0.1         1
#Residual standard error: 30.99 on 161 degrees of freedom
#Multiple R-squared:  0.9907, Adjusted R-squared:  0.9896
#F-statistic: 901.6 on 19 and 161 DF,  p-value: < 2.2e-16
sleepstudy$npfittedb <- fitted(no_poolingLM1) #saving fitted values
```

3. Fit a "un-pooled" regression model with varying slopes of time (days) for patients (include only the interaction `Days:Subject`) and save the fitted values.

```
no_poolingLM2 <- lm(Reaction ~ Days:factor(Subject) - 1, data=sleepstudy)
summary(no_poolingLM2)
#Residuals:
#  Min       1Q   Median       3Q      Max
#-207.75  -25.20    71.24   169.32   321.54
#Coefficients:
#                              Estimate Std. Error t value Pr(>|t|)
#  Days:factor(Subject)308     60.321      8.618   7.000 6.45e-11 ***
#  Days:factor(Subject)309     34.639      8.618   4.019 8.92e-05 ***
#  Days:factor(Subject)310     38.244      8.618   4.438 1.67e-05 ***
#  Days:factor(Subject)330     48.748      8.618   5.657 6.83e-08 ***
#  Days:factor(Subject)331     50.383      8.618   5.846 2.69e-08 ***
#  Days:factor(Subject)332     51.291      8.618   5.952 1.59e-08 ***
#  Days:factor(Subject)333     52.566      8.618   6.100 7.53e-09 ***
#  Days:factor(Subject)334     50.174      8.618   5.822 3.03e-08 ***
#  Days:factor(Subject)335     38.651      8.618   4.485 1.38e-05 ***
#  Days:factor(Subject)337     64.832      8.618   7.523 3.49e-12 ***
#  Days:factor(Subject)349     47.459      8.618   5.507 1.41e-07 ***
#  Days:factor(Subject)350     55.162      8.618   6.401 1.59e-09 ***
```

```
#  Days:factor(Subject)351   47.667      8.618   5.531 1.25e-07 ***
#  Days:factor(Subject)352   57.204      8.618   6.638 4.56e-10 ***
#  Days:factor(Subject)369   51.606      8.618   5.988 1.32e-08 ***
#  Days:factor(Subject)370   51.285      8.618   5.951 1.60e-08 ***
#  Days:factor(Subject)371   49.236      8.618   5.713 5.18e-08 ***
#  Days:factor(Subject)372   53.463      8.618   6.204 4.43e-09 ***
#  Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .
    0.1         1
#Residual standard error: 145.5 on 162 degrees of freedom
#Multiple R-squared:  0.7935, Adjusted R-squared:  0.7706
#F-statistic: 34.59 on 18 and 162 DF,  p-value: < 2.2e-16
sleepstudy$npfittedc <- fitted(no_poolingLM2) #saving fitted values
```

4. Fit an "un-pooled" regression model with varying intercepts for patients with varying slopes of time (days) by patient (include the interaction and constituent terms of `Days` and `Subject`, `Days + Subject + Days:Subject`) and save the fitted values.

```
no_poolingLM3 <- lm(Reaction ~ Days + Subject + Days:factor(Subject) - 1,
    data=sleepstudy)
summary(no_poolingLM3)
#Residuals:
#  Min       1Q   Median       3Q      Max
#-106.397  -10.692   -0.177   11.417  132.510
#Coefficients:
#                          Estimate Std. Error t value Pr(>|t|)
#  Days                     21.765      2.818   7.725 1.74e-12 ***
#  Subject308              244.193     15.042  16.234  < 2e-16 ***
#  Subject309              205.055     15.042  13.632  < 2e-16 ***
#  Subject310              203.484     15.042  13.528  < 2e-16 ***
#  Subject330              289.685     15.042  19.259  < 2e-16 ***
#  Subject331              285.739     15.042  18.996  < 2e-16 ***
#  Subject332              264.252     15.042  17.568  < 2e-16 ***
#  Subject333              275.019     15.042  18.284  < 2e-16 ***
#  Subject334              240.163     15.042  15.966  < 2e-16 ***
#  Subject335              263.035     15.042  17.487  < 2e-16 ***
#  Subject337              290.104     15.042  19.287  < 2e-16 ***
#  Subject349              215.112     15.042  14.301  < 2e-16 ***
#  Subject350              225.835     15.042  15.014  < 2e-16 ***
#  Subject351              261.147     15.042  17.362  < 2e-16 ***
#  Subject352              276.372     15.042  18.374  < 2e-16 ***
#  Subject369              254.968     15.042  16.951  < 2e-16 ***
#  Subject370              210.449     15.042  13.991  < 2e-16 ***
#  Subject371              253.636     15.042  16.862  < 2e-16 ***
#  Subject372              267.045     15.042  17.754  < 2e-16 ***
#  Days:factor(Subject)309 -19.503      3.985  -4.895 2.61e-06 ***
#  Days:factor(Subject)310 -15.650      3.985  -3.928 0.000133 ***
#  Days:factor(Subject)330 -18.757      3.985  -4.707 5.84e-06 ***
#  Days:factor(Subject)331 -16.499      3.985  -4.141 5.88e-05 ***
#  Days:factor(Subject)332 -12.198      3.985  -3.061 0.002630 **
#  Days:factor(Subject)333 -12.623      3.985  -3.168 0.001876 **
#  Days:factor(Subject)334  -9.512      3.985  -2.387 0.018282 *
```

```
#  Days:factor(Subject)335  -24.646      3.985  -6.185 6.07e-09 ***
#  Days:factor(Subject)337   -2.739      3.985  -0.687 0.492986
#  Days:factor(Subject)349   -8.271      3.985  -2.076 0.039704 *
#  Days:factor(Subject)350   -2.261      3.985  -0.567 0.571360
#  Days:factor(Subject)351  -15.331      3.985  -3.848 0.000179 ***
#  Days:factor(Subject)352   -8.198      3.985  -2.057 0.041448 *
#  Days:factor(Subject)369  -10.417      3.985  -2.614 0.009895 **
#  Days:factor(Subject)370   -3.709      3.985  -0.931 0.353560
#  Days:factor(Subject)371  -12.576      3.985  -3.156 0.001947 **
#  Days:factor(Subject)372  -10.467      3.985  -2.627 0.009554 **
#  Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .
    0.1        1
#Residual standard error: 25.59 on 144 degrees of freedom
#Multiple R-squared:  0.9943, Adjusted R-squared:  0.9929
#F-statistic: 700.4 on 36 and 144 DF,  p-value: < 2.2e-16
sleepstudy$npfittedd <- fitted(no_poolingLM3) #saving fitted values
```

5. Fit a "semi-pooled" multi-level model with varying-intercept for subject and varying-slope of day by subject. Is it worthwhile for us to run a multi-level model with varying effects of time by subject? Why? Compare your model from part 5 to the other completely "pooled" or "un-pooled models".

```
semi_poolingLM <- lmer(Reaction ~ Days + (1 + Days | Subject), sleepstudy
    )
summary(semi_poolingLM)
#REML criterion at convergence: 1743.6
#Scaled residuals:
#  Min      1Q  Median      3Q     Max
#-3.9536 -0.4634  0.0231  0.4633  5.1793
#Random effects:
#  Groups   Name        Variance Std.Dev. Corr
#Subject  (Intercept) 611.90   24.737
#Days         35.08    5.923   0.07
#Residual             654.94   25.592
#Number of obs: 180, groups:  Subject, 18
#Fixed effects:
#            Estimate Std. Error t value
#(Intercept)  251.405      6.824  36.843
#Days          10.467      1.546   6.771
#Correlation of Fixed Effects:
#  (Intr)
#Days -0.138
sleepstudy$npfittede <- fitted(semi_poolingLM)
sleepstudy$npfitteda <- fitted(complete_poolingLM)
plot(sleepstudy$Days, sleepstudy$Reaction, main="Original Data")
plot(sleepstudy$Days, sleepstudy$npfittedb, main="Semi-Pooled Model")
plot(sleepstudy$Days, sleepstudy$npfitteda, main="Completely Pooled Model
    ")
plot(sleepstudy$Days, sleepstudy$npfittedd, main="Un-Pooled Model")
```

According to Figure 2, the multi-level model seems really similar to the un-pooled

model except that pooling is invovled. The multi-level model truly falls in between the completely and unpooled model in that it has pooling, but the chances are not equal in between days, like it is for the completely pooled model. This may provide the benefit of being able to manipulate levels in a structured way compared to unpooled model, but also providing some information as each unit has a different chance of success.

Figure 1: Diagnostic Plots

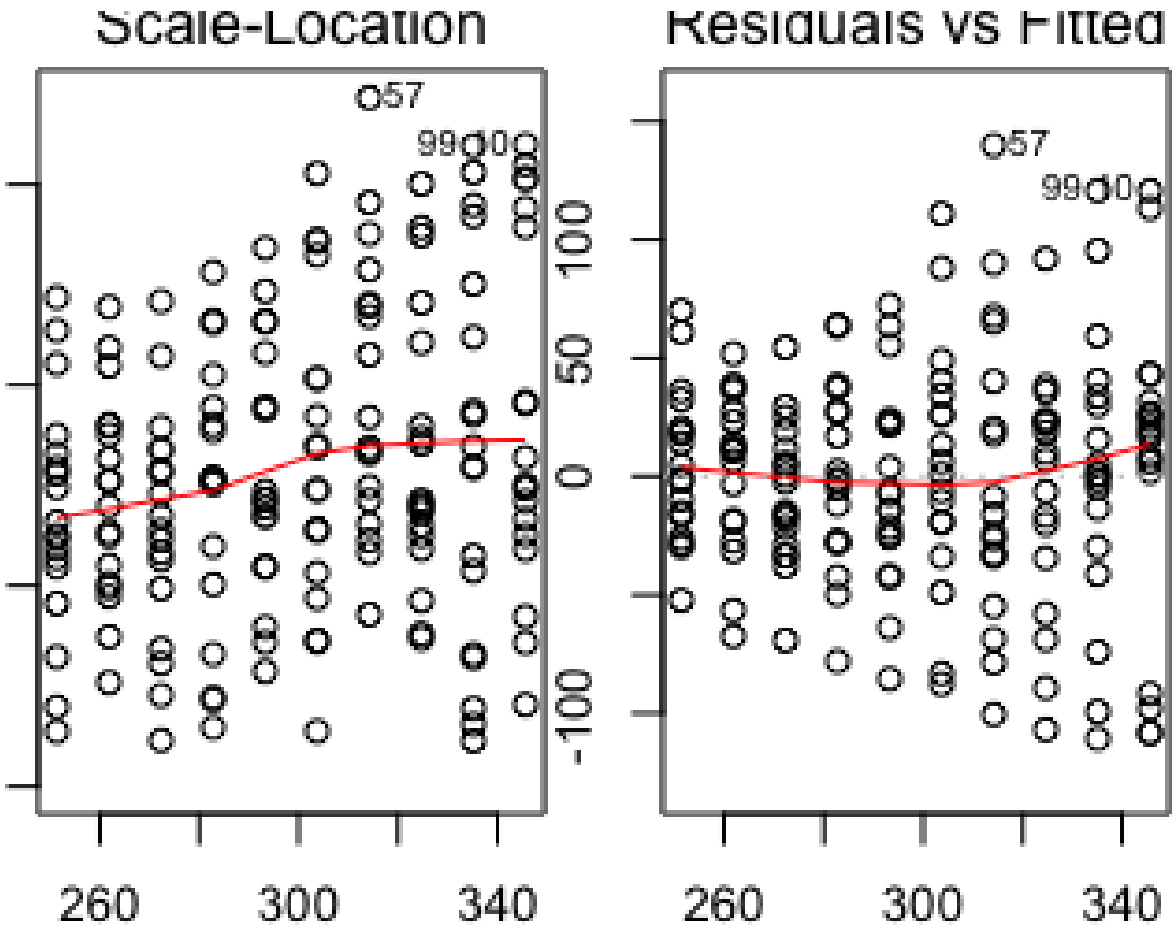

Figure 2: Comparing Models