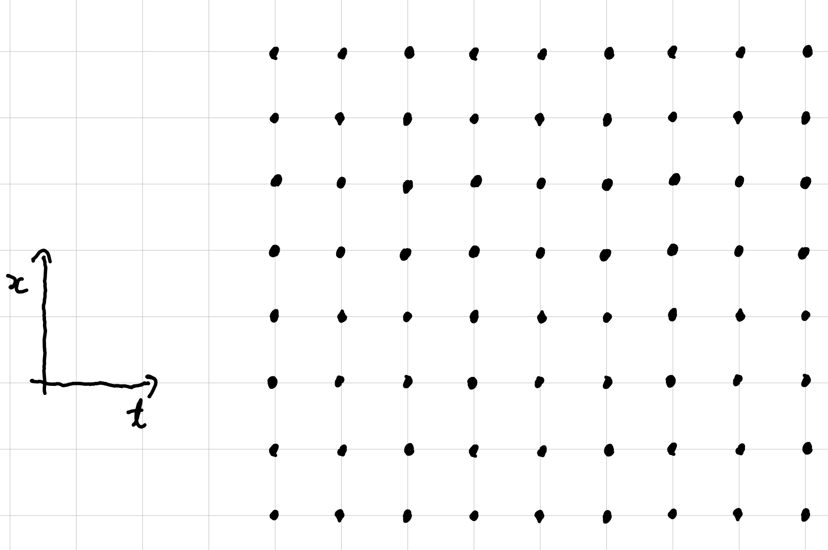
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u^2}{\partial x^2}$$



$$f'(x) = \frac{f(x) - f(x-h)}{h} = \frac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{h}$$

$$f''(x) = \frac{f'(x+\frac{h}{2}) - f'(x-\frac{h}{2})}{h} = \frac{f(x-h) - 2f(x) + f(x+h)}{h}$$

$$\frac{\frac{f(x+h)-f(x)}{h} - \frac{f(x)-f(x-h)}{h}}{h} = \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

$$\frac{u_{x,t-1} - 2u_{x,t} + u_{x,t+1}}{(\Delta t)^2} = \frac{c^2}{(\Delta x)^2} u_{x-1,t} - 2u_{x,t} + u_{x+1,t}$$

$$\Rightarrow u_{x,t-1} - 2u_{x,t} + u_{x,t+1} = \overbrace{c^2\left(\frac{\Delta t}{\Delta x}\right)^2}^{r} u_{x-1,t} - 2u_{x,t} + u_{x+1,t}$$

Para resolver a eq para $t+1$: u_x^{t+1}

$$u_{x,t+1} = r(u_{x-1,t} - 2u_{x,t} + u_{x+1,t}) + 2u_{x,t} - u_{x,t-1}$$

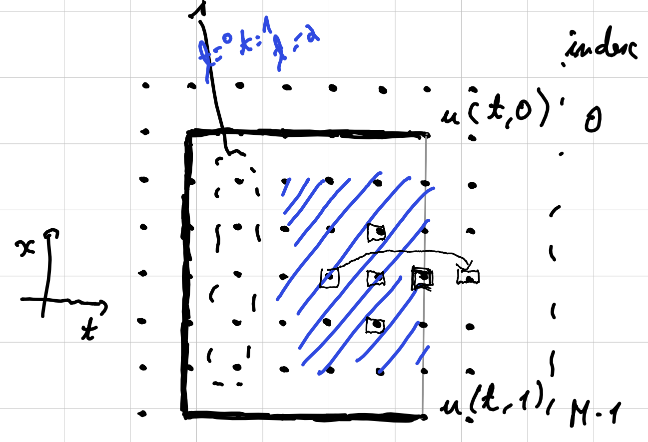
$$\frac{\partial f}{\partial t} = \frac{u_x(t+\Delta t) - u_x(t-\Delta t)}{\Delta t} = \frac{u_{x,t+1} - u_{x,t-1}}{\Delta t}$$

$$\Rightarrow u_{x,t-1} = u_{x,t+1} - \Delta t \frac{\partial f}{\partial t}$$

Para Boundary Condition $\frac{\partial f}{\partial t} = 0 \Rightarrow u_{x,t-1} = u_{x,t+1}$

$$\Rightarrow u_{x,t+1} = r(u_{x-1,t} - 2u_{x,t} + u_{x+1,t}) + 2u_{x,t} - u_{x,t+1}$$

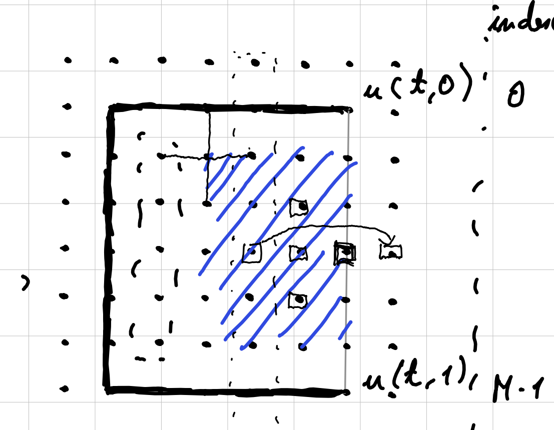
$$u_{x,t+1} = \frac{r}{2}(u_{x-1,t} - 2u_{x,t} + u_{x+1,t}) + u_{x,t}$$



$u(0,x) = f(x)$

/// Algoritmo Genético

Estabilidade $|\Delta t| \leq \frac{\Delta x}{|c|}$



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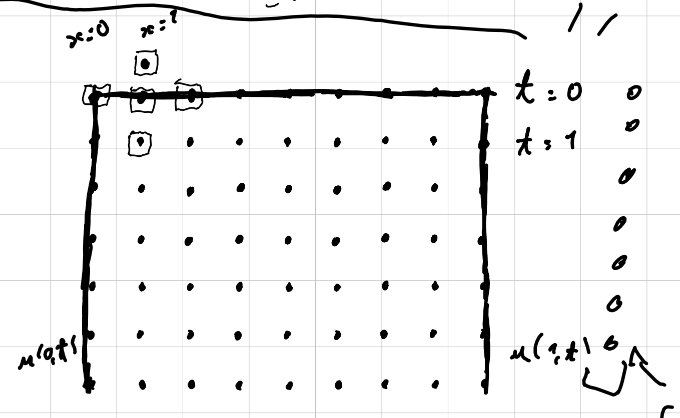


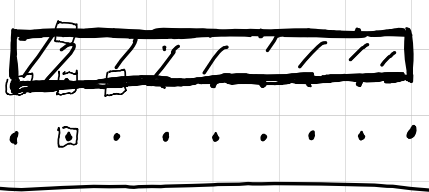
Coluna correspondente ao Tempo

Matriz: $u[t, x]$

$$u_{x,t+1} = r(u_{x-1,t} - 2u_{x,t} + u_{x+1,t}) + 2u_{x,t} - u_{x,t-1}$$ } Formula Geral

Para $t=0$ se $\frac{\partial f}{\partial t} = 0 \Rightarrow u_{x,-1} = u_{x,1}$

$f = u(x,0)$



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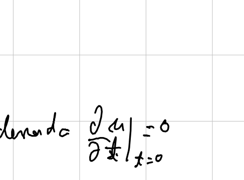
Free ends:

$$\left.\frac{\partial u}{\partial x}(x,t)\right|_{x=l} = \left.\frac{\partial u}{\partial x}(x,t)\right|_{x=0} = 0$$



$$\Rightarrow u_{-1,t} = u_{1,t}$$

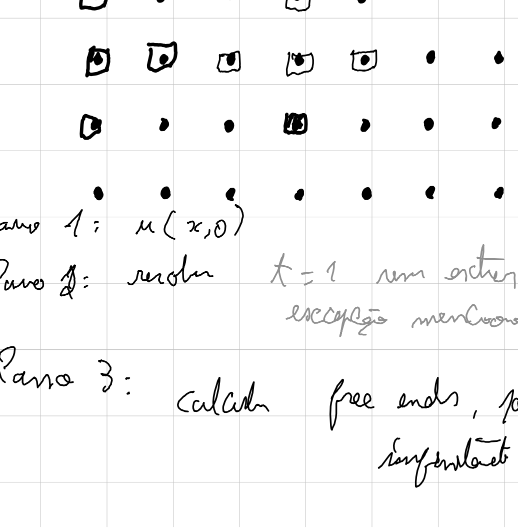
em $x=0$:

$$u_{0,t+1} = r(\underbrace{u_{-1,t}}_{u_{1,t}} - 2u_{0,t} + u_{1,t}) + 2u_{0,t} - u_{0,t-1}$$

$$= 2r(u_{1,t} - u_{0,t}) + 2u_{0,t} - \underbrace{u_{0,t-1}}_{u_{0,t+1}}$$ $\rightarrow$ pela derivada $\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$

$$\Rightarrow u_{0,t+1} = r(u_{1,t} - u_{0,t}) + u_{0,t} - \frac{1}{2}u_{0,t-1}$$

$$u_{L,t+1} = r(u_{L-1,t} - u_{L,t}) + u_{L,t} - \frac{1}{2}u_{L,t-1}$$



Passo 1: $u(x,0)$

Passo 2: resolver $t=1$ nos outros com formula geral com excepção mencionada antes nos free ends

Passo 3: calcular free ends, partir do meio simplesmente até ao final

2D

$\Delta q = \Delta x = \Delta y$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$r = c\frac{\Delta t}{\Delta x}$

$$\frac{u^{t-1}_{x,y} - 2u^{t}_{x,y} + u^{t+1}_{x,y}}{(\Delta t)^2} = \frac{c^2}{(\Delta q)^2}\left(u^t_{x-1,y} - 2u^t_{x,y} + u^t_{x+1,y} + u^t_{x,y-1} - 2u^t_{x,y} + u^t_{x,y+1}\right)$$

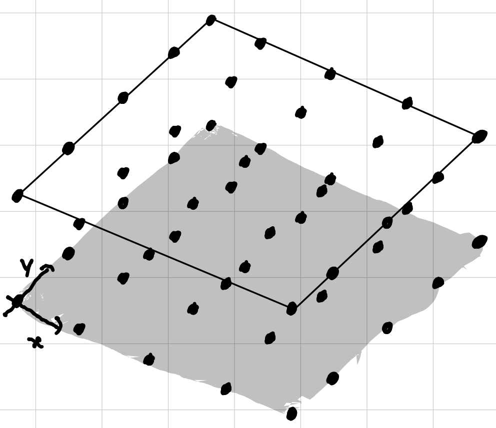
$$u^{t+1}_{x,y} = 2u^t_{x,y} - u^{t-1}_{x,y} + r^2\left(u^t_{x-1,y} - 2u^t_{x,y} + u^t_{x+1,y} + u^t_{x,y-1} - 2u^t_{x,y} + u^t_{x,y+1}\right)$$

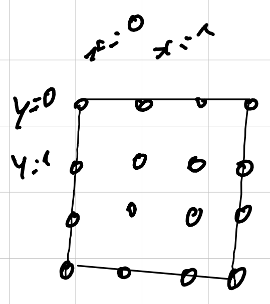
$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0 \Rightarrow u^{-1}_{x,y} = u^{1}_{x,y}$$

$u^0_{x,y}$

Fixed Boundaries: $u^t_{x,0}$ $u^t_{x,L}$ $u^t_{0,y}$ $u^t_{L,y}$
(Faces do Cubo)

$$S = \begin{bmatrix} u^0 \\ u^1 \\ \vdots \\ u^t \end{bmatrix}$$



$u^t_{x,y} =$ 

$x=1\ y=1$
$u_{1,1}$

Free Boundaries:

$$u^{t+1}_{x,y} = 2u^t_{x,y} - u^{t-1}_{x,y} + r^2\left(u^t_{x-1,y} - 2u^t_{x,y} + u^t_{x+1,y} + u^t_{x,y-1} - 2u^t_{x,y} + u^t_{x,y+1}\right)$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0 \Rightarrow u^{-1}_{x,y} = u^{1}_{x,y}$$