

Name: \_\_\_\_\_

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**Q1.** Following is a description of the 'Maximum Subarray Sum' problem. You are given an array  $A[0..n-1]$  of  $n$  integers. These integers may be positive or negative. Let  $\text{sum}(i, j)$  denote the sum of integers  $A[i \dots j]$ , where  $i \leq j$ .

$$\text{(i.e. } \text{sum}(i, j) = \sum_{k=i}^j A[k] \text{)}$$

You have to find indices  $\text{max\_i}$  and  $\text{max\_j}$  such that:

$$\text{maxSum} = \text{sum}(\text{max\_i}, \text{max\_j}) = \max(\text{sum}(i, j) \mid 0 \leq i \leq j < n)$$

Following is an **incomplete** divide and conquer pseudocode for finding maximum subarray. **You have to fill in the missing statements in the code. You are not allowed to write more than one statement in each blank.** [8 marks]

FIND-MAXIMUM-SUBARRAY( $A, \text{low}, \text{high}$ )

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1  if high == low
2      return (low, high, A[low])           // base case: only one element
3  else mid =  $\lfloor (\text{low} + \text{high}) / 2 \rfloor$ 
4      (left-low, left-high, left-sum) =
        FIND-MAXIMUM-SUBARRAY( $A, \text{low}, \text{mid}$ )
5      (right-low, right-high, right-sum) =
        FIND-MAXIMUM-SUBARRAY( $A, \text{mid} + 1, \text{high}$ )
6      (cross-low, cross-high, cross-sum) =
        FIND-MAX-CROSSING-SUBARRAY( $A, \text{low}, \text{mid}, \text{high}$ )
7      if left-sum  $\geq$  right-sum and left-sum  $\geq$  cross-sum
8          return (left-low, left-high, left-sum)
9      elseif right-sum  $\geq$  left-sum and right-sum  $\geq$  cross-sum
10         return (right-low, right-high, right-sum)
11     else return (cross-low, cross-high, cross-sum)
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FIND-MAX-CROSSING-SUBARRAY( $A, \text{low}, \text{mid}, \text{high}$ )

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1  left-sum =  $-\infty$ 
2  sum = 0
3  for i = mid downto low
4      sum = sum + A[i]
5      if sum > left-sum
6          left-sum = sum
7          max-left = i
8  right-sum =  $-\infty$ 
9  sum = 0
10 for j = mid + 1 to high
11     sum = sum + A[j]
12     if sum > right-sum
13         right-sum = sum
14         max-right = j
15 return (max-left, max-right, left-sum + right-sum)
```

What is the recurrence relation for Find-Maximum-Subarray?

$T(1) = O(1)$  and  $T(n) = 2T(n/2) + O(n)$  if  $n > 1$