Flow Networks

Given a flow network G(V, E), with one source node 'S' and one sink node 'T'. The edge capacities that are positive integers. Consider the state when the Ford Fulkerson algorithm has run on this network and terminated with the max flow. Can we say all that the edges coming out of 'S' are saturated in this state?

If YES, then prove it. If NO, then give a counterexample.
YES / NO ___**NO**___________

S: Q;3, P;3
Q: T;1
P: T;1

Describe the condition that ensures that the Ford-Fulkerson algorithm will terminate.

All flows and capacities are integer values, which means that Δ is also an integer. Δ cannot be smaller than 1. In each iteration the flow is being augmented by Δ. Since there is only finite amount for flow that can come out of the source, Ford Fulkerson algorithm will eventually terminate.

Let G be a directed weighted graph with a source vertex s and a sink vertex t. Let f be a flow in G. Let G_f be the residual graph of the flow network. **Suppose G_f has no s-t path**.

What conclusion can you draw about the flow f in G? _____Max flow_________________

Let cut (A,B) be the min cut. In graph G_f, how do we find out which vertices are in set A?

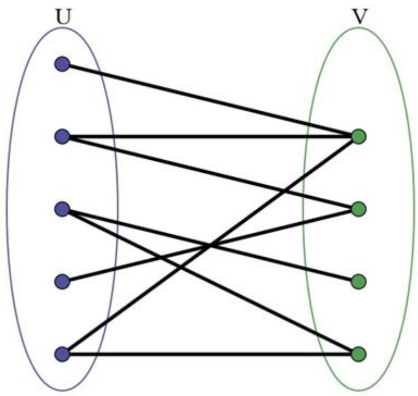
The set of vertices reachable by source.

Let cut (A,B) be the min cut. In graph G, what conclusion can you draw about the flow in edges going across the cut from set B to set A? Explain.

The flow is zero.

You are given the following bipartite graph G. Convert it into a flow network G’ with one source ‘s’ and one sink ‘t’ such that the value of maximum s-t flow in G’ is equal to the size of maximum matching in bipartite graph G.

Add a source vertex S and a sink vertex T. Add directed edges from source S to all vertices in U and add directed edges from all vertices in V to sink T. Direct all edges in the graph from vertices in U towards vertices in V. All edge weights are 1.



Given a graph G(V,E). Let f be the value of maximum s-t flow in G. If we increase the capacity of every edge in G by 1, then the maximum s-t flow in the modified graph will have a value of at most f+1.

Is the above statement TRUE or FALSE? ___FALSE_________

Brief explanation of your answer.

Easy to construct a counterexample. Think about it.

Let G be a directed weighted graph with a source vertex s and a sink vertex t. Let f be a flow in G. Let G_f be the residual graph of the flow network.

Suppose there is a s-t path in G_f. Let (A,B) be a cut of graph G (Note: here we are referring to (A,B) as any cut, not the min cut).

Value of f = $\sum_{e \in \delta^+(A)} (f_e) - \sum_{e \in \delta^-(A)} (f_e)$

What is the relationship between value of f and the capacity of (A,B) cut. Explain.

Value of f <= capacity (A,B) cut

Suppose G_f has no s-t path. Let cut (A,B) be the **min** cut. In graph G, is the flow in edges, going across the cut, from set A to set B equal to the flow going from set B to set A? Explain.

No. The edges from A to B are saturated while those from B to A have zero flow.

Consider the Maximum-Flow problem and the Ford-Fulkerson algorithm, where the input was a flow network with a single source and a single sink. Now, suppose you are given a flow network (F) with multiple sources and multiple sinks. Is it possible for you to convert F to a flow network with a single source and a single sink and then run Ford-Fulkerson algorithm on it?

If YES, then explain how you will transform F. If NO, then explain why not.

Yes. Add a super source and a super sink. Think about the capacities of edges attached to the super source and super sink.

Let G be a directed weighted graph with a source vertex s and a sink vertex t. Let f be a flow in G. Let G_f be the residual graph of the flow network. **Suppose G_f has no s-t path**.

What conclusion can you draw about the flow f in G? _____**Max flow**_______________

Let cut (A,B) be the min cut. In graph G_f, how do we find out which vertices are in set A?

The set of vertices reachable by S (source).

Let cut (A,B) be the min cut. In graph G, what conclusion can you draw about the flow in edges going across the cut from set B to set A? Explain.

The flow is zero.

Given a graph G(V,E). Let f be the value of maximum s-t flow in G. If we increase the capacity of every edge in G by 1, then the maximum s-t flow in the modified graph will have a value of at most f+1.

Is the above statement TRUE or FALSE? ____**FALSE**________ Brief explanation of your answer. **Think of a counter example.**

Consider the bipartite matching problem of finding a matching in G of largest possible size. We learned in class that we can use maximum flow problem to find a maximum matching. We can use Ford-Fulkerson algorithm to find the maximum matching in the bipartite graph in O(mn) time. How?

Suppose our bipartite graph G(V,E) is partitioned as V=X ∪ Y. The Ford Fulkerson time complexity is O(mC), however in bipartite graph case C = n, where |X| = n. **Note: You should be able to reason along the same lines to show that the time complexity of the 'Edge-disjoint paths in a graph' problem also has O(mn) time complexity.**