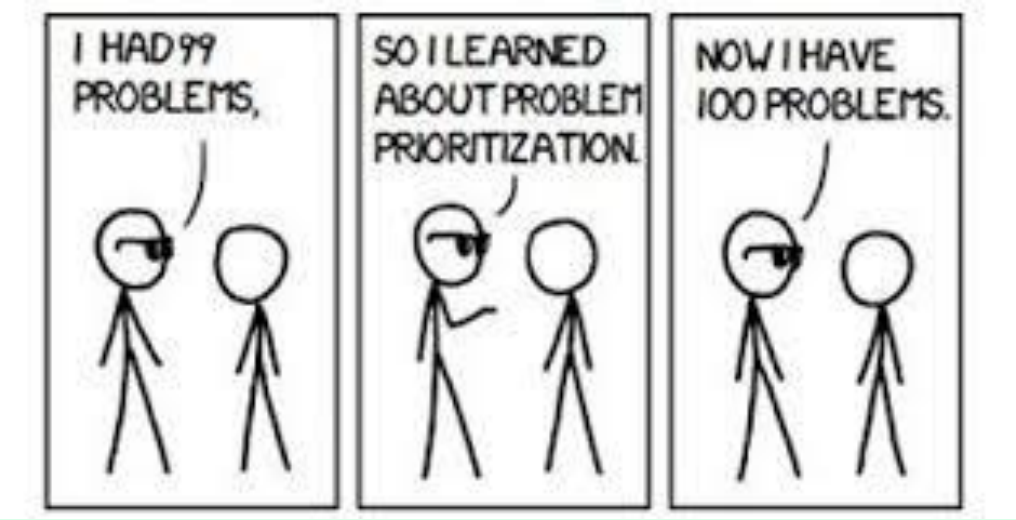
CS 310


I HAD 99 PROBLEMS,
SO I LEARNED ABOUT PROBLEM PRIORITIZATION.
NOW I HAVE 100 PROBLEMS.

Announcements

Assignment-1 will be up this week

- **Tutorial** on input parsing this week

Office hours are posted on LMS.

Notation

Graph G=(V,E)

| V | = n

| E | = m

In an undirected graph G:

$$\sum_{v \in V} \text{degree}(v) = 2\,|E|$$

Trees

Root, parent, child, ancestor, descendant, leaf.

A node w is a descendant of v (or v is an ancestor of w) if v lies on the **path from the root to w**.

How many edges in an n-node tree?

How many edges in an n-node tree?

Number of edges = n-1

Suppose you are given a **connected undirected weighted graph** G(V,E) with **no cycles**.

How will you find the shortest path between two vertices ‘u’ and ‘v’ in G?

Trees

Undirected graph is a tree if it is connected and does not contain a cycle.

Only **one** simple path exists in an n-node tree from any vertex u to vertex v

Deleting **any** edge in a tree will disconnect it.

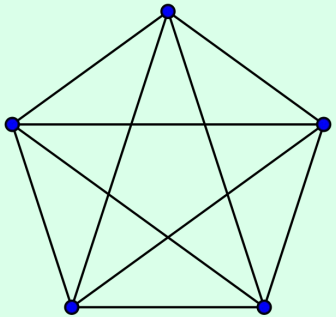
Complete graph

Complete graph

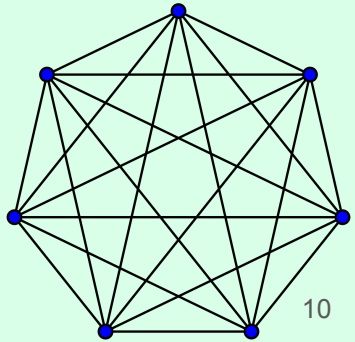
A complete graph is a graph in which each pair of graph vertices is connected by an edge.

The complete graph with n graph vertices is denoted by K_n and has $\binom{n}{2} = n(n-1)/2$ undirected edges.

K_5



K_7



Definitions

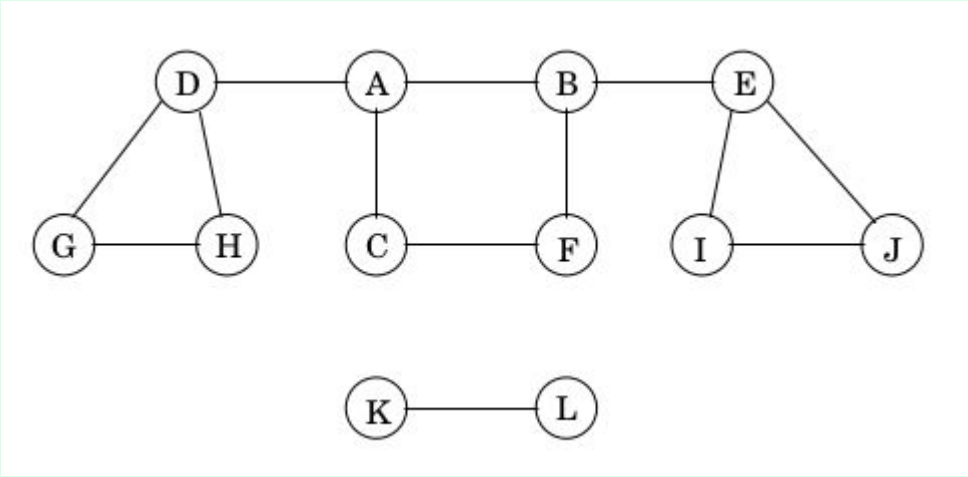
Sparse graph:

- If number of edges |E| is close to |V|, the graph is sparse. (e.g. a tree).

Dense graph

- Number of edges can be close to $|V|^2$ (e.g. a complete graph, when all possible edges are present).
- When |E| is close to the upper limit of this range, we call the graph dense.

How do we find connected components in an **undirected** graph?



How do we find connected components in an **undirected** graph?

In undirected graph, which parts of the graph are reachable from a given vertex.

- Can use BFS or DFS.

You are given ‘n’ chemicals c_1, c_2, … c_n.

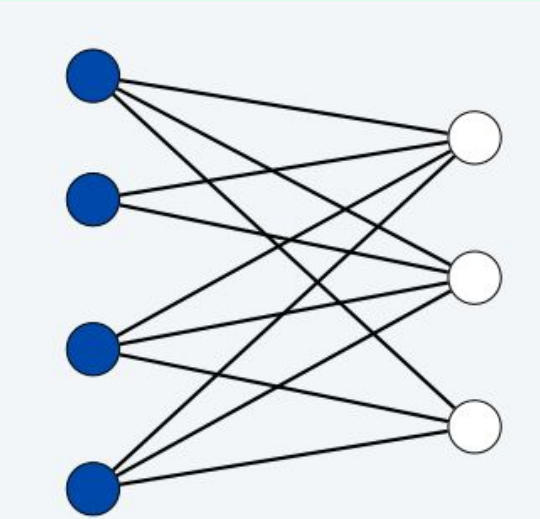
Some chemicals may react with other chemicals.

Can we safely pack all chemicals in **two** boxes so that none of the chemicals in each box react with each other?

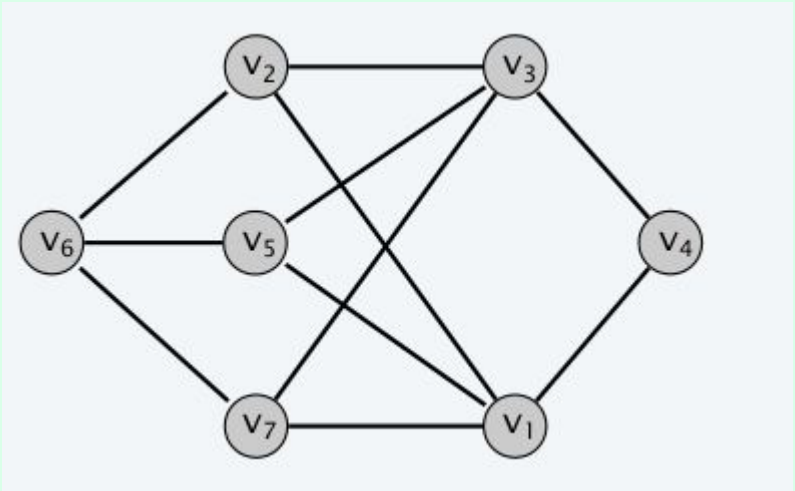
What is the time complexity of the chemicals problem?

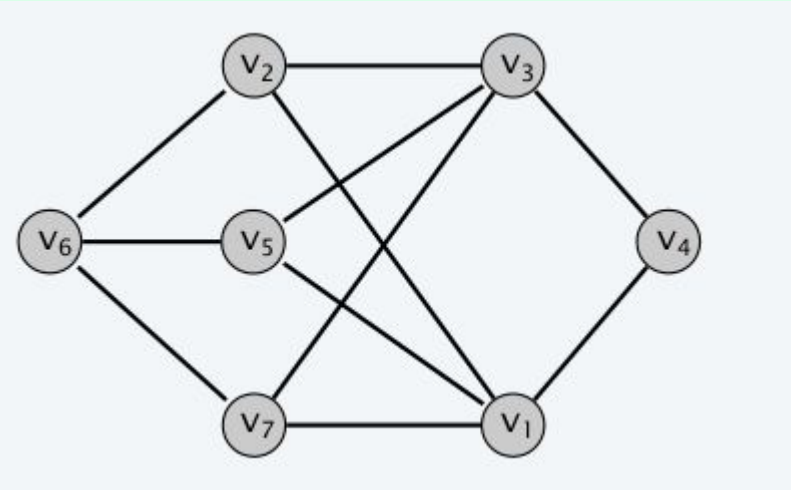
Bipartite graph

Partition the set of vertices V into sets X and Y such that every edge has one end in X and the other in Y.



a bipartite graph


v2
v3
v6
v5
v4
v7
v1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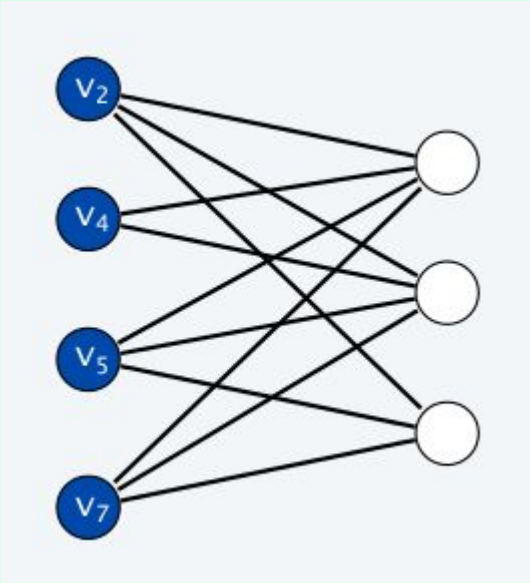
v2
v4
v5
v7

Which graphs can’t be bipartite?

Bipartite graphs

Lemma. Let G be a connected graph, and let $L_0, \ldots, L_k$ be the layers produced by BFS starting at node s. Exactly one of the following holds.

(i) No edge of G joins two nodes of the same layer, and G is bipartite.

(ii) An edge of G joins two nodes of the same layer, and G contains an odd-length cycle (and hence is not bipartite).