Stable Matching

Suppose there are N companies where each company has a job vacancy for one intern. We are given the preference lists of N companies and N interns. In an instance of the Stable Matching Problem, Company C_1 has ranked intern I_3 as first in its preference list and intern I_3 has also ranked C_1 as first in his preference list. Will the (C1, I3) pair be present in every stable matching for this instance? You have to prove it or give a counterexample.

(C1, I3) pair will be present in every stable matching for this instance **TRUE** / FALSE

Proof or Counterexample:

Proof by Contradiction.

Assume that a perfect matching 'S' does not contain the (C1, I3) pair, but instead contains pairs (C1, Ix) and (Cy, I3). Since C1 and I3 had ranked each other as first, pair (C1, I3) is an instability with respect to 'S' (meaning pair (C1, I3) does not belong to S, but both C1 and I3 prefer each other to their partners in S). Hence matching S cannot be stable.

While executing the Gale-Shapley algorithm, we want to find if company C_x prefers I_y over I_z in constant time. Describe the data structure you will use to perform this operation in O(1) time.

Suppose the preference list of a company is as follows.

1st	2nd	3rd	4th	5th	6th	7th	8th
8	3	7	1	4	5	6	2

For constant time access, construct an array which stores the ranking of each intern as follows. The indices of this array are interns.

1	2	3	4	5	6	7	8
4th	8th	2nd	5th	6th	7th	3rd	1st

You are given the following intern and employer preference lists. Apply the Gale-Shapley Matching algorithm to these lists and give the stable matching produced by the algorithm. Assume that the communication is initiated by the interns. You also have to mention the order in which employers are crossed out by the interns.

Intern Preference Matrix

Intern 1 (I_1)	E_3	E_1	E_5	E_6	E_4	E_2
Intern 2 (I_2)	E_5	E_2	E_1	E_6	E_4	E_3
Intern 3 (I_3)	E_2	E_4	E_3	E_5	E_1	E_6
Intern 4 (I_4)	E_5	E_2	E_1	E_6	E_4	E_3
Intern 5 (I_5)	E_1	E_2	E_3	E_4	E_5	E_6
Intern 6 (I_6)	E_5	E_2	E_6	E_1	E_3	E_4

Employer Preference Matrix

Employer 1 (E_1)	I_2	I_4	I_5	I_1	I_6	I_3
Employer 2 (E_2)	I_5	I_6	I_3	I_1	I_2	I_4
Employer 3 (E_3)	I_4	I_2	I_3	I_1	I_6	I_5
Employer 4 (E_4)	I_6	I_2	I_3	I_4	I_5	I_1
Employer 5 (E_5)	I_2	I_4	I_5	I_1	I_6	I_3
Employer 6 (E_6)	I_1	I_2	I_3	I_4	I_5	I_6

EMPLOYERS	Day1	Day2	Day3	Day 4	Day5	Final Matching
E_1	I_5	I_5	I_5, I_4	I_4	I_4	(E_1, I_4)
E_2	I_3	I_3, I_4, I_6	I_6	I_6, I_5	I_5	(E_2, I_5)
E_3	I_1	I_1	I_1	I_1	I_1	(E_3, I_1)
E_4			I_3	I_3	I_3	(E_4, I_3)
E_5	I_2, I_4, I_6	I_2	I_2	I_2	I_2	(E_5, I_2)
E_6					I_6	(E_6, I_6)