

Given the preference lists for five interns and five hospitals, suppose the Gale-Shapley algorithm produces the following matching 'M'.

$M = \{ (I_1, H_1), (I_2, H_3), (I_3, H_2), (I_4, H_5), (I_5, H_4) \}$

Someone claims that  $(I_1, H_3)$  is an instability in M. Without looking at the preference lists, how will you **prove** them wrong?

**You had to show that the output produced by Gale-Shapley algorithm is stable. The proof was done in class.**

Consider the maximum subarray sum problem given to you in a home activity.

**The description of the maximum subarray sum problem is as follows:**

You are given an array  $P[0 \dots n-1]$  of  $n$  integers. These integers can be both positive and negative. Let  $\text{sum}(i, j)$  denote the sum of integers  $P[i \dots j]$ , where  $i \leq j$ .

(i.e.  $\text{sum}(i, j) = \sum_{k=i}^j P[k]$  )

**You have to find indices  $\text{max\_i}$  and  $\text{max\_j}$  such that:**

$\text{maxSum} = \text{sum}(\text{max\_i}, \text{max\_j}) = \max(\text{sum}(i, j) \mid 0 \leq i \leq j < n)$

You may assume that the 'n' integers are a combination of both positive and negative integers (i.e., they will not be all negative nor will they be all positive integers).

Your friend has written the following code to find  $\text{maxSum}$  in  $O(n)$  time, however, the statements for determining  $\text{max\_i}$  and  $\text{max\_j}$  are missing. **You have to fill in the missing statements in the code.** You can assume that initializing of 'n' and array  $P[]$  is already done. You may declare additional variables, if needed.

Write your answer **clearly** inside the following box.

```

int currentSum = 0, maxSum = 0;
int index, max_i = 0, max_j = 0;
int curr_i = 0;

for (index=0; index<n; index++) {
    currentSum += P[index];

    if (currentSum < 0) {
        currentSum = 0;
        curr_i = index + 1;
    }
    if (maxSum < currentSum ) {
        maxSum = currentSum;
        max_j = index;
        max_i = curr_i;
    }
}

```

Suppose you have a single processor and there are a set of ‘n’ jobs that need to run on it. The jobs can only run one at a time. Each job has a weight associated with it that indicates its priority. Let the jobs be  $J_1, J_2, \dots, J_n$ , which require time  $t_1, t_2 \dots t_n$  respectively to run. Their weights are  $w_1, w_2, \dots, w_n$ . Suppose job  $J_a$  runs first, followed by job  $J_b$ , then completion time for  $J_a$  is  $C_a = t_a$  and completion time for  $J_b$  is  $C_b = t_a + t_b$ . We want to find how to schedule the ‘n’ jobs such that their weighted sum of completion times is minimized. Suppose the optimal schedule of jobs is given by  $S_1, S_2, \dots, S_n$  then you have to **minimize**  $\sum_{i=1}^{i=n} w_{s_i} \cdot C_{s_i}$

**Write your algorithm in clear descriptive steps as in Q1 above (do NOT write C code).**

**The optimal schedule is obtained by sorting the jobs in ascending order of the ratios  $t_i/w_i$**

**Running time of your algorithm in Big-Oh notation**  **$O(n \log(n))$**

Consider the assignment question on boomerangs.

What is its recurrence relation \_\_\_\_\_  **$4T(n/2) + c$**  \_\_\_\_\_

What is the time complexity of this recurrence in Big-Oh notation \_\_\_\_\_  **$O(n^2)$**  \_\_\_\_\_

Suppose you have coded 'n' programs in the last year and you want to take backup of your hard work on a USB drive. Let's call the programs  $P_1, P_2, \dots, P_n$ . Program  $P_i$  requires  $k_i$  bytes of space. The capacity of the USB drive is C bytes, where

$$C < \sum_{i=1}^n k_i$$

Notice C is less than the total space required, so you can't save all the 'n' programs.

Can you design an efficient and optimal greedy algorithm to **maximize the space utilization** of the drive?

If YES, then **describe** your algorithm. If NO, then explain why not.

YES / NO \_\_\_\_ **NO** \_\_\_\_\_

**This is similar to the knapsack problem.**