

Name: \_\_\_\_\_

Roll No: \_\_\_\_\_

**Q1.** Following is a description of the 'Counting Inversions' problem. You are given a sequence of  $n$  numbers  $a_1, a_2, \dots, a_n$ . Assume that all the numbers are distinct. Two indices  $i < j$  form an inversion if  $a_i > a_j$ . Determine the total number of inversions in the input sequence.

Following is an **incomplete** divide and conquer pseudocode for counting inversions. **You have to fill in the missing statements in the code. You are not allowed to write more than one statement in each blank.** [8 marks]

**Sort-and-Count( $L$ )**

    If the list has one element then

        there are no inversions

    Else

        Divide the list into two halves:

$A$  contains the first  $\lceil n/2 \rceil$  elements

$B$  contains the remaining  $\lfloor n/2 \rfloor$  elements

$(r_A, A) = \text{Sort-and-Count}(A)$

$(r_B, B) = \text{Sort-and-Count}(B)$

$(r, L) = \text{Merge-and-Count}(A, B)$

    Endif

    Return  $r = r_A + r_B + r$ , and the sorted list  $L$

<b>Merge-and-Count(<math>A, B</math>)</b>	
1	Maintain a Current pointer into each list, initialized to point to the <b>front elements</b>
2	Maintain a variable Count for the number of inversions, initialized to 0
3	<b>While</b> both lists are <b>nonempty</b>
4	Let $a_i$ and $b_j$ be the elements pointed to by the Current pointer
5	Append to the output list: <b>the smaller of these two elements</b>
6	<b>If ( <math>b_j</math> is the smaller element )</b> then
7	<b>Increment count by the number of elements remaining in A</b>
9	<b>Endif</b>
10	<b>Advance the current pointer in the list from which the smaller element was selected</b>
11	<b>EndWhile</b>
12	Once one list is empty, <b>append the remainder of the other list to the output</b>
13	Return Count and the merged list

**What is the recurrence relation for Sort-and-Count?  $T(1) = O(1)$  and  $T(n) = 2T(n/2) + O(n)$  if  $n > 1$**