Rod Cutting Problem

Given a rod of length n inches and a table of prices p_i for i = 1, 2, ... n, determine the maximum revenue r_n obtainable by cutting up the rod and selling the pieces.

| length i	1	2	3	4	5	6	7	8	9	10
price p_i	1	5	8	9	10	17	17	20	24	30

Rod Cutting Problem

Given a rod of length n inches and a table of prices p_i for $i = 1, 2, ...$ n, determine the maximum revenue r_n obtainable by cutting up the rod and selling the pieces.
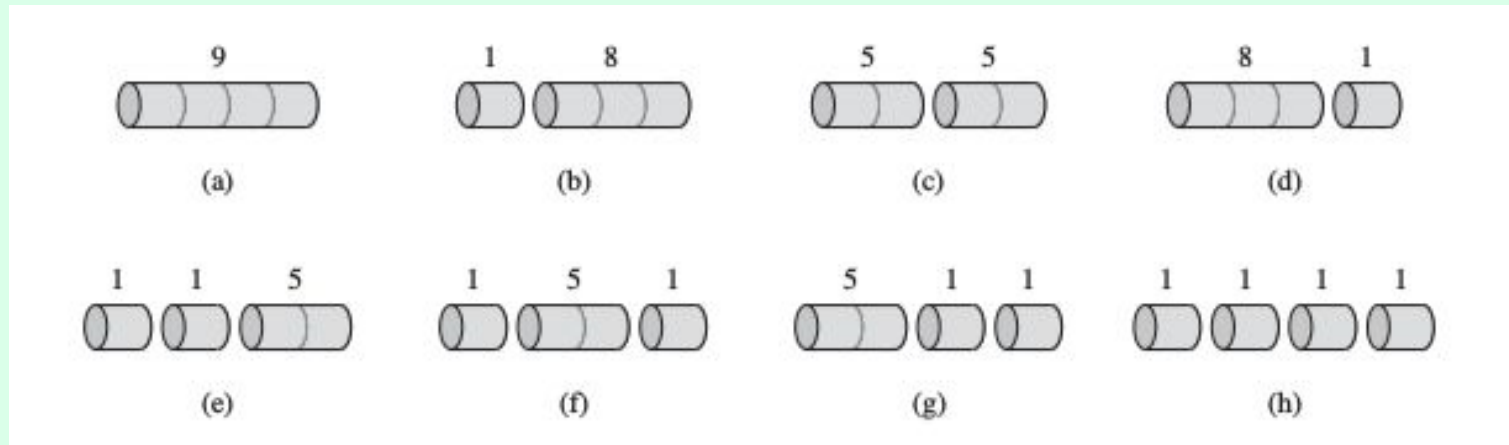
Brute Force: How many different ways to cut the rod?

Rod Cutting Problem

The 8 possible ways of cutting up a rod of length 4. Above each piece is the value of that piece.
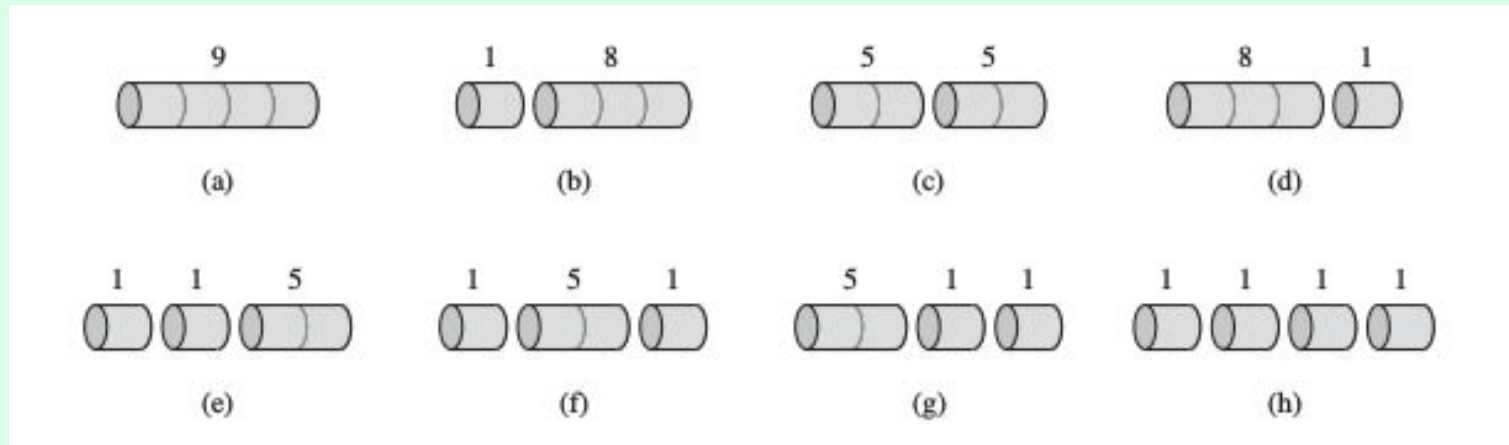
Brute Force: 2^{n-1} different ways to cut the rod.

Rod Cutting example

CUT-ROD(p, n)

1   **if** $n == 0$
2   **return** 0
3    $q = -\infty$
4   **for** $i = 1$ **to** n
5        $q = \max(q, p[i] + \text{CUT-ROD}(p, n - i))$
6   **return** q

Rod Cutting example

MEMOIZED-CUT-ROD-AUX(p, n, r)

1   **if** $r[n] \geq 0$
2   **return** $r[n]$
3   **if** $n == 0$
4       $q = 0$
5   **else** $q = -\infty$
6   **for** $i = 1$ **to** n
7           $q = \max(q, p[i] + \text{MEMOIZED-CUT-ROD-AUX}(p, n - i, r))$
8   $r[n] = q$
9   **return** q

Rod Cutting example

BOTTOM-UP-CUT-ROD (p, n)

1   let $r[0 \mathinner{.\,.} n]$ be a new array
2   $r[0] = 0$
3   **for** $j = 1$ **to** n
4        $q = -\infty$
5   **for** $i = 1$ **to** j
6            $q = \max(q, \, p[i] + r[j - i])$
7        $r[j] = q$
8   **return** $r[n]$

Reference Reading

Algorithm Design by Tardos et. al. 2006
    Chapter 6: **§6.1 - §6.2** Weighted Interval Scheduling

Introductions to Algorithms, 3rd Edition, by Cormen, et. al.
    Chapter 5: **§5.1** Rod cutting problem

Algorithms by Dasgupta, Papadimitriou, and Vazirani
    Chapter 6: **§6.3** Edit distance