

*The best preparation for tomorrow is
doing your best today.*

- H. Jackson Brown, Jr.

Time complexity of finding topological ordering.

Ordering is powerful!

DAGs will show up several times in this course.

Applications of DAGs

- Temporal dependencies
e.g. course prerequisite graph
- Pipe lines
e.g. factory assembly lines
- Hierarchies
- Causalities

Directed reachability. Given a node s , find all nodes reachable from s .

Directed s-t shortest path problem. Given two node s and t , what is the length of the shortest path from s and t ?

Graph search. BFS extends naturally to directed graphs.

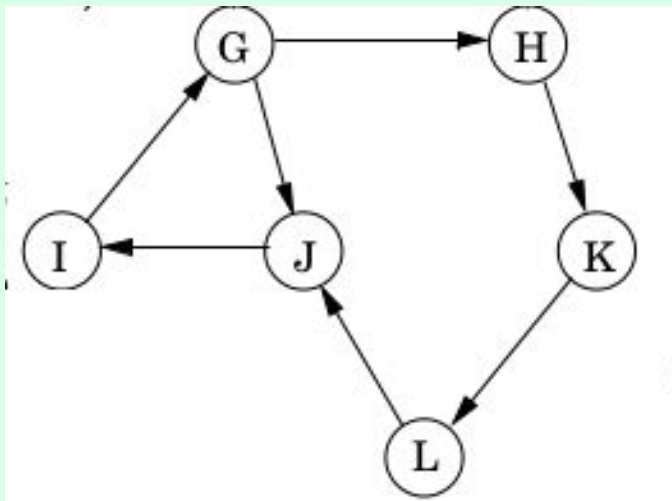
Strong connectivity

Def. Nodes u and v are **mutually reachable** if there is a both path from u to v and also a path from v to u .

Def. A graph is **strongly connected** if every pair of nodes is mutually reachable.



How do we find if a **directed** graph G is strongly connected?

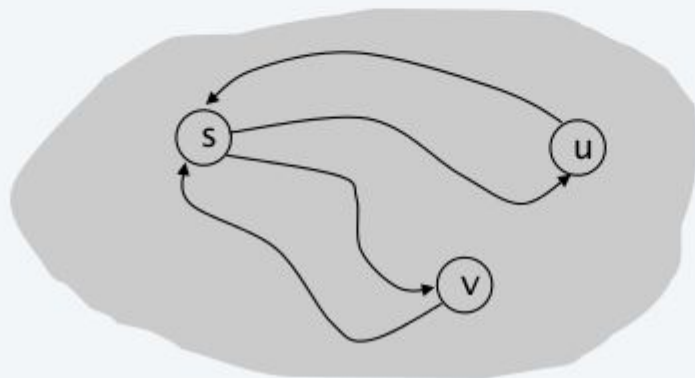


Lemma. Let s be any node. G is strongly connected iff every node is reachable from s , and s is reachable from every node.

Pf. \Rightarrow Follows from definition.

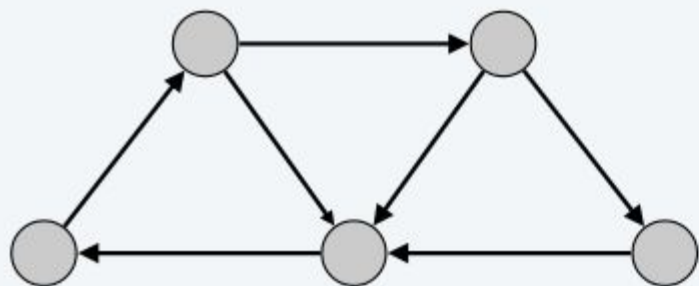
Pf. \Leftarrow Path from u to v : concatenate $u \rightarrow s$ path with $s \rightarrow v$ path.

Path from v to u : concatenate $v \rightarrow s$ path with $s \rightarrow u$ path. ■

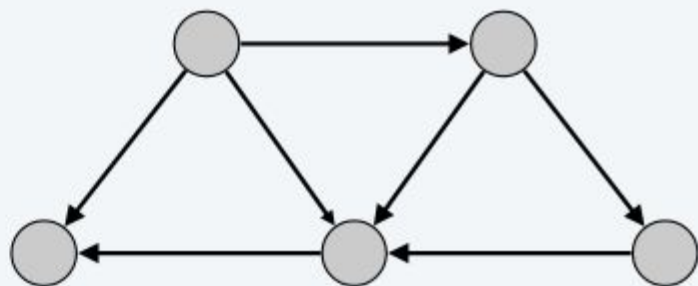


ok if paths overlap

- Pick any node s .
- Run BFS from s in G .
- Run BFS from s in $G^{reverse}$. reverse orientation of every edge in G
- Return true iff all nodes reached in both BFS executions.
- Correctness follows immediately from previous lemma. ■



strongly connected



not strongly connected