Poisson Arrival, Pure and Slotted Aloha

Lecture slides by

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K events arrival probability

$$\Pr[k\ events\ arrival\ from\ n\ events\ with\ an\ arrival\ rate\ \lambda\ ]$$

$$= \underbrace{{}^{n}C_{k}}_{\text{K selected among n}} \underbrace{\left(\frac{\lambda}{n}\right)^{k}}_{\text{K transmission}} \underbrace{\left(1-\frac{\lambda}{n}\right)^{n-k}}_{\text{n-K no transmission}}$$

Assume ,

$n \rightarrow \infty$

$$\lim_{n\rightarrow\infty} {}^{n}C_{k}\left(\frac{\lambda}{n}\right)^{k}\left(1-\frac{\lambda}{n}\right)^{n-k}$$

$$\lim_{n\to\infty} {}^{n}C_{k} \left(\frac{\lambda}{n}\right)^{k} \left(1-\frac{\lambda}{n}\right)^{n-k}$$

$$= \lim_{n\to\infty} \frac{n!}{k!\,(n-k)!} \left(\frac{\lambda}{n}\right)^{k} \left(1-\frac{\lambda}{n}\right)^{n-k}$$

$$= \lim_{n\to\infty} \frac{n!}{k!\,(n-k)!} \left(\frac{\lambda}{n}\right)^{k} \left(1-\frac{\lambda}{n}\right)^{n} \left(1-\frac{\lambda}{n}\right)^{-k}$$

$$= \lim_{n\to\infty} \frac{\lambda^{k}}{k!} \underbrace{\left(1-\frac{\lambda}{n}\right)^{n}}_{1} \underbrace{\left(1-\frac{\lambda}{n}\right)^{-k}}_{2} \underbrace{\frac{n!}{n^{k}(n-k)!}}_{3}$$

1) $\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n}$=?

If x= - $\frac{\lambda}{n}$, $n\to\infty$ means

x→ 0

1. $\lim_{x\to 0}(1+x)^{-\frac{\lambda}{x}}$=?

$= \lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^{-\lambda}$

= $e^{-\lambda}$

$\lim_{x\to 0}(1+x)^{1/x}$=2.788

=e=napier`s constant (Euler number)

2. $\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-k}$ = 1,

Because, n tends to infinite

3. $\lim_{n\to\infty}\frac{n!}{n^k(n-k)!}$

$$= \lim_{n\to\infty}\frac{n!}{(n-k)!} * \frac{1}{n^k}$$

$$= \lim_{n\to\infty} n(n-1)(n-2)\ldots.(n-k+1) * \frac{1}{n^k}$$

$$= \lim_{n\to\infty}\left(n^k + an^{k-1} + bn^{k-2} + \cdots\right) * \frac{1}{n^k}$$

$$= \lim_{n\to\infty}\left(1 + \frac{a}{n} + \frac{b}{n^2} + \cdots\right) * \frac{n^k}{n^k}$$

= 1

Here, a,b and c are some undefined number

So,

$$= \lim_{n\to\infty} \frac{\lambda^k}{k!}\left(1-\frac{\lambda}{n}\right)^n \left(1-\frac{\lambda}{n}\right)^{-k} \frac{n!}{n^k(n-k)!}$$

$$= \frac{\lambda^k}{k!} * e^{-\lambda} * 1 * 1$$

$$= \frac{\lambda^k e^{-\lambda}}{k!}$$

$$= \text{Poisson Distribution}$$

Pure Aloha

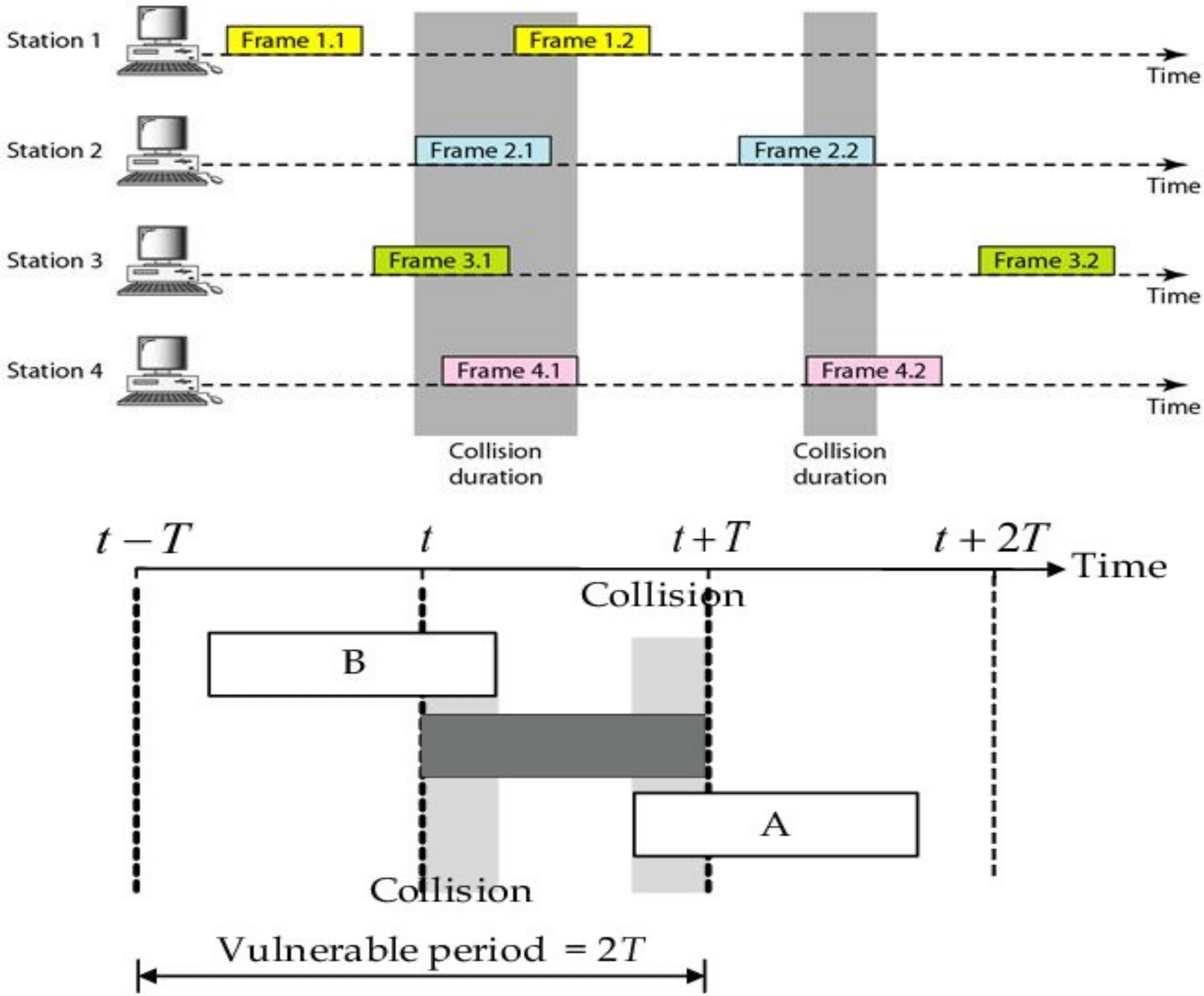


Fig. 1: Pure Aloha protocol and it`s vulnerable time

Throughput of pure aloha

Pr{k arrivals with arrival rate λ Per time unit} $\frac{\lambda^k e^{-\lambda}}{k!}$

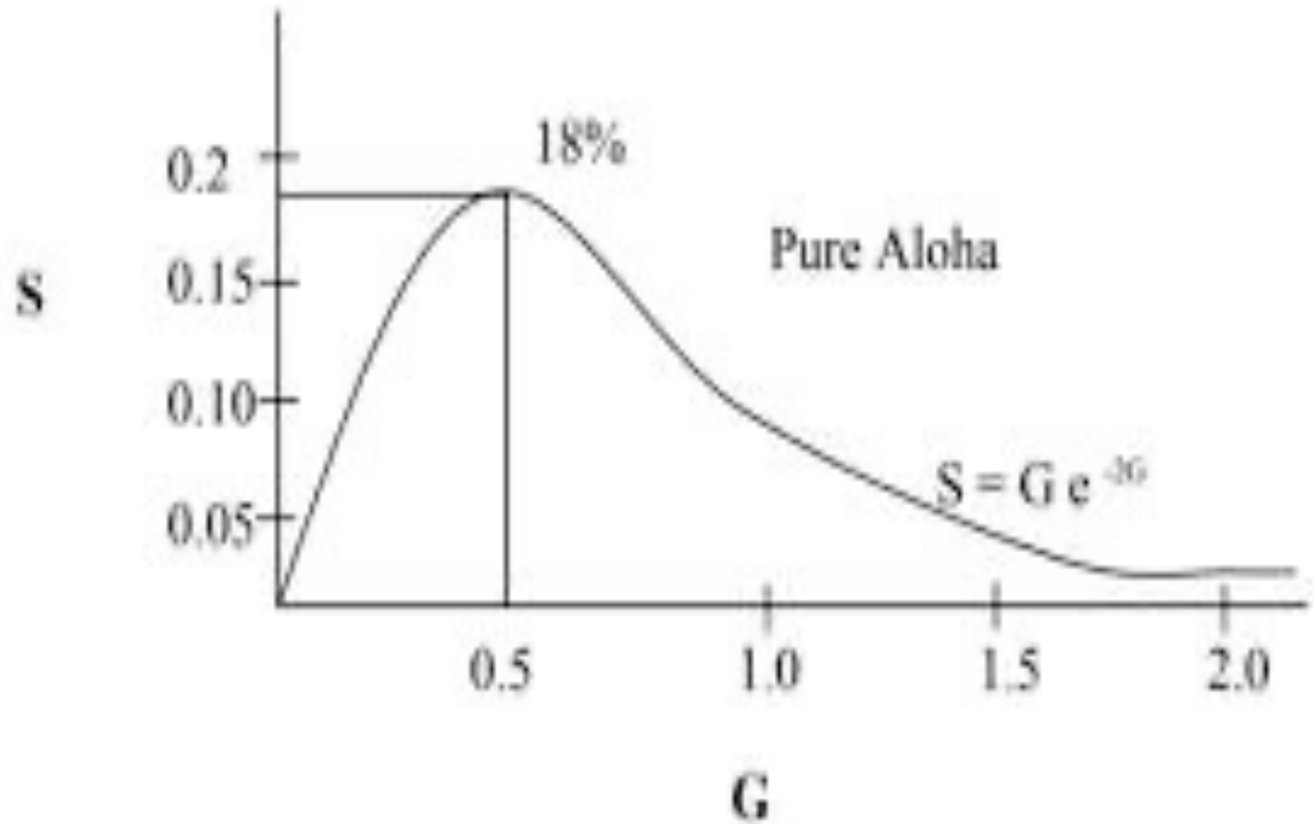
Pr{k arrivals with arrival rate λ during time t} $\frac{(\lambda t)^k e^{-\lambda t}}{k!}$

Pr{k arrivals with λ during time 2 unit}$\frac{(2\lambda)^k e^{-2\lambda}}{k!}$

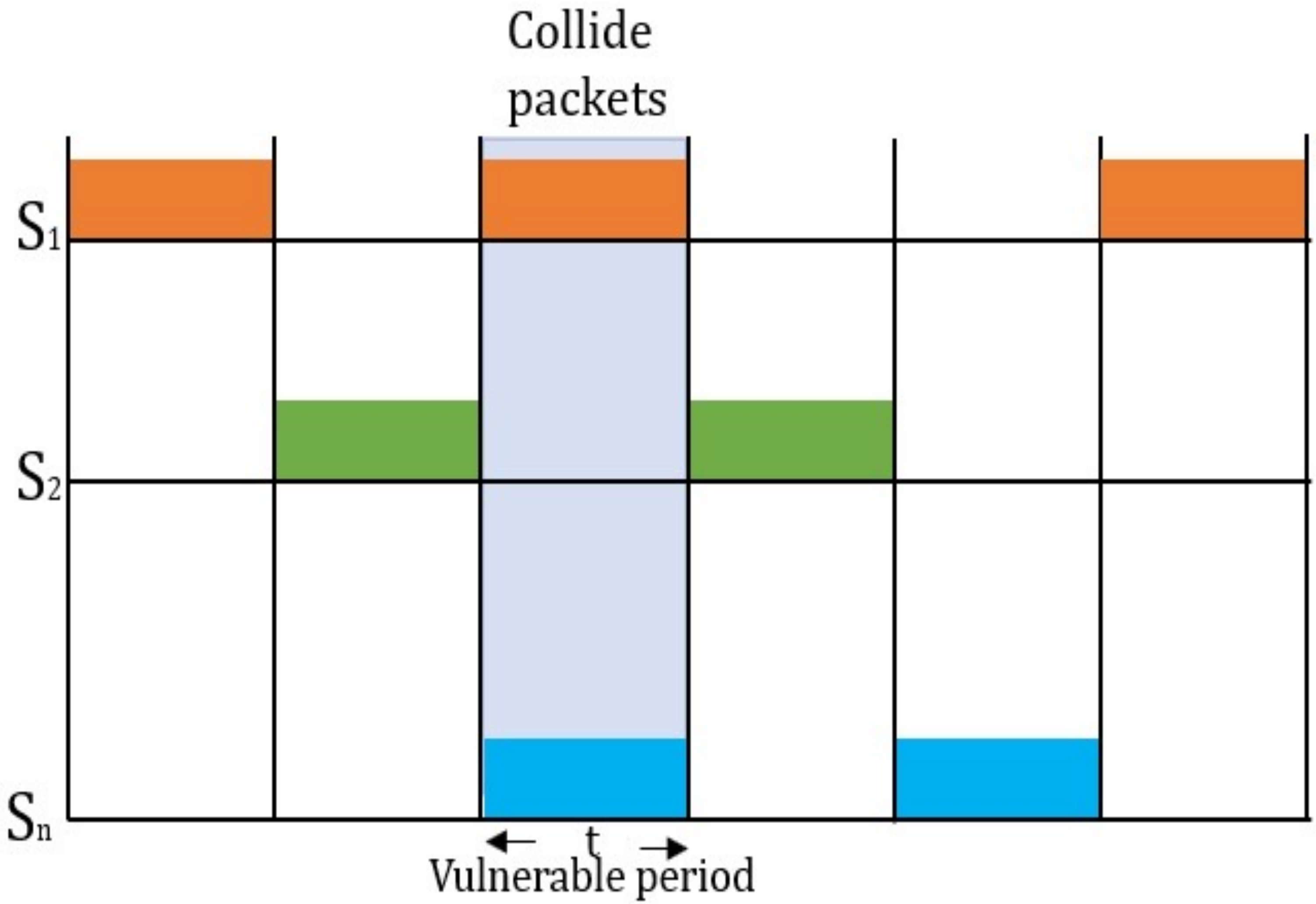
Pr{0 arrivals with λ during time 2} $= e^{-2\lambda}$

Throughput for arrival rate λ for pure aloha
$\lambda e^{-2\lambda}$

Throughput Graph of pure Aloha



Slotted Aloha Protocol



Throughput of Slotted Aloha

- Collision period is same as data period(1 data packet duration)
- Throughput of slotted aloha:

Pr{k arrivals with arrival rate λ during time} $\frac{\lambda^k e^{-\lambda}}{k!}$

Pr{k arrivals with arrival rate λ during time t}$\frac{(\lambda t)^k e^{-\lambda t}}{k!}$

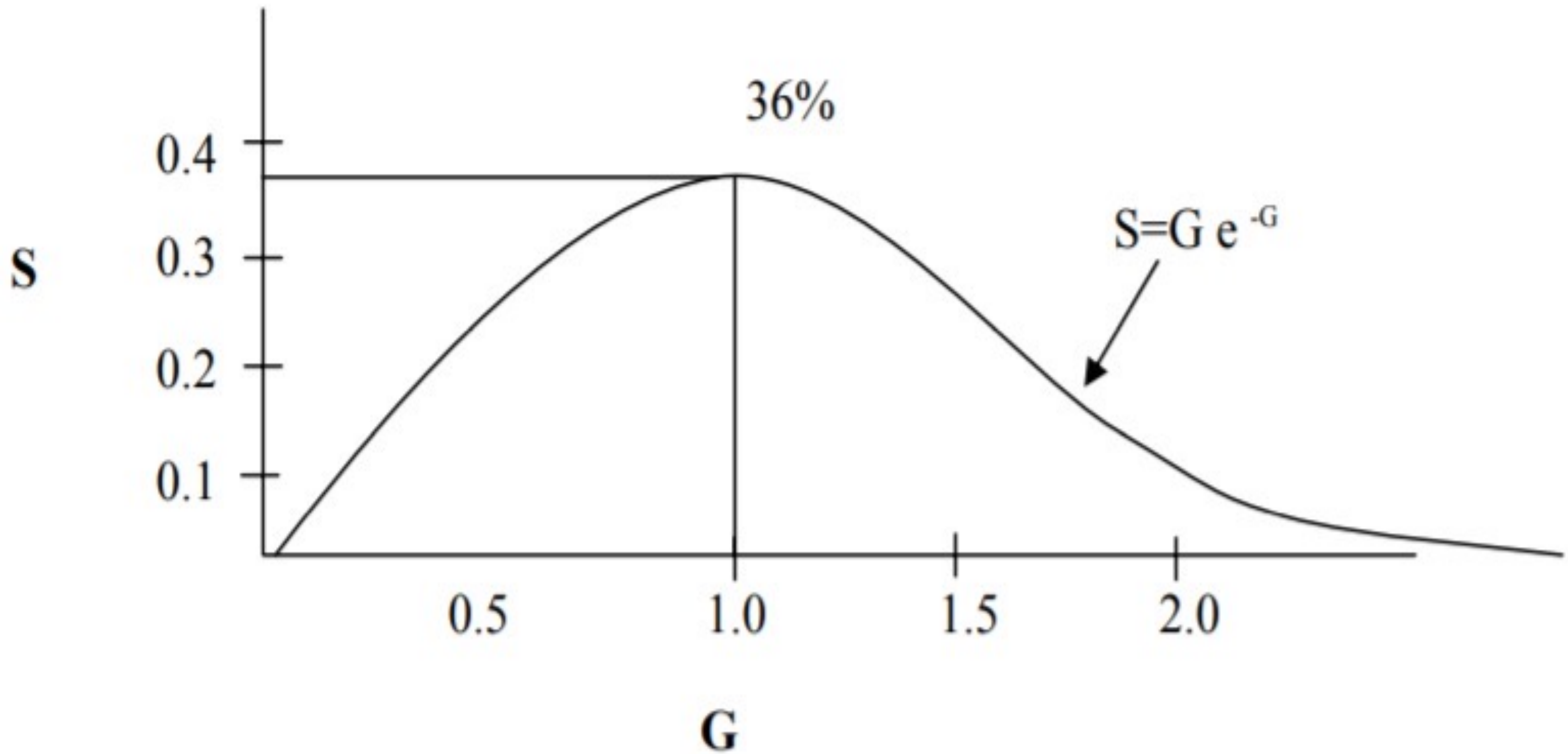
Pr{k arrivals with arrival rate λ during time 1}$\frac{(\lambda)^k e^{-\lambda}}{k!}$

Pr{0 arrivals with λ during time 1} $= e^{-\lambda}$

Throughput with arrival rate λ for Slotted Aloha Protocol is

$\lambda e^{-\lambda}$

Throughput Graph of Slotted Aloha



Throughput Comparison of pure Aloha and slotted Aloha