To find the volume of a sphere using integration, we can use the method of slicing the sphere into small disks and summing up their volumes. In this explanation, we will consider a sphere with radius R and volume V.

First, let's set up a coordinate system with the center of the sphere at the origin (0, 0, 0). We can express the radius r of a disk at a given height z as a function of z. To do this, we will use the Pythagorean theorem:

$$r^2 + z^2 = R^2$$

Solving for r as a function of z, we get:

$$r(z) = \sqrt{R^2 - z^2}$$

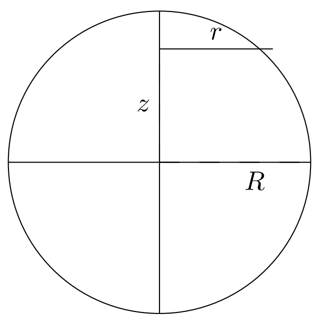

Figure 1: A sphere with radius R and a disk at height z with radius r.

Now, let's consider a small disk of thickness dz at height z. The volume of this disk is given by:

$$dV = \pi \cdot r(z)^2 \cdot dz$$

Substitute the expression for $r(z)$:

$$dV = \pi \cdot \left(\sqrt{R^2 - z^2}\right)^2 \cdot dz$$

$$dV = \pi \cdot (R^2 - z^2) \cdot dz$$

Now we integrate dV with respect to z from the bottom hemisphere (z = -R) to the top hemisphere (z = R):

$$V = \int_{-R}^{R} (\pi \cdot (R^2 - z^2) \cdot dz)$$

Solve the integral:

$$V = \pi \int_{-R}^{R} (R^2 - z^2) \cdot dz$$

$$V = \pi \left[R^2 \left[z \right]_{-R}^{R} - \left[\frac{z^3}{3} \right]_{-R}^{R} \right]$$

$$V = \pi \left[R^2 (R - (-R)) - \left(\frac{R^3}{3} - \frac{-R^3}{3} \right) \right]$$

$$V = \pi \left[\frac{4R^3}{3} \right]$$

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$$V = \frac{4}{3} \cdot \pi \cdot R^3$$

Thus, the volume of a sphere with radius R is $\frac{4}{3}\pi R^3$, which is the familiar formula.