Volume of a Cone Using Integrals

Consider a right circular cone with radius r and height h. We will calculate its volume using the method of integration. Let's take a cross-sectional disk with thickness Δx at a distance x from the apex of the cone.

The radius of the disk, $R(x)$, is proportional to its distance x from the apex. Therefore, we can write $R(x) = kx$, where k is the proportionality constant. Since $R(h) = r$, we get $k = \frac{r}{h}$.

Now, we can express the volume of the disk as $\Delta V = \pi R(x)^2 \Delta x = \pi k^2 x^2 \Delta x$. To find the volume of the entire cone, we integrate ΔV from 0 to h.

$$\begin{aligned} V &= \int_0^h \pi k^2 x^2 dx \\ &= \pi \left(\frac{r}{h}\right)^2 \int_0^h x^2 dx \\ &= \pi \left(\frac{r}{h}\right)^2 \left[\frac{1}{3}x^3\right]_0^h \\ &= \pi \left(\frac{r}{h}\right)^2 \left[\frac{1}{3}h^3 - 0\right] \\ &= \frac{1}{3}\pi r^2 h \end{aligned}$$

Thus, the volume of the cone is given by:

$$V = \frac{1}{3}\pi r^2 h \tag{1}$$

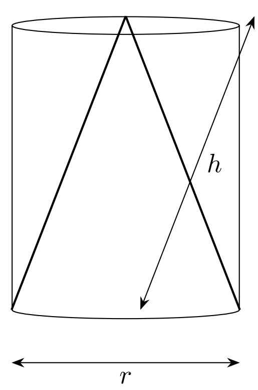

Figure 1: A cone with radius r and height h.