Bilkent Mathematics Journal (2023), **xx**:xx 1–11

RESEARCH ARTICLE

Nagel–Schreckenberg Model

O.Faruk Beygo, Yusuf Nazım Pehlivan and Olgu Okur

Bilkent, Mathematics, Ankara, Turkey.

Keywords: Traffic Flow, Traffic Dynamics at Intersections, Simulation, Automaton Model

Abstract
This paper examines the Nagel-Schreckenberg model, a cellular automaton model used for simulating traffic flow, focusing on understanding traffic dynamics at intersections. By extending the model to incorporate traffic signals, we explore the effects of different signal patterns and control strategies on overall traffic flow efficiency. The study aims to provide insights into the optimal management of intersection traffic signals, leading to improved road safety and reduced travel times. Through simulation scenarios and analysis, we evaluate various traffic signal control strategies and assess their impact on critical metrics such as vehicle throughput, average travel time, queue lengths, and congestion levels. The findings contribute to understanding traffic flow behavior at intersections and offer implications for traffic management and planning. The limitations of the cellular automaton model are discussed, and suggestions for future research in traffic flow modeling and intersection management are provided. This research provides a foundation for developing efficient traffic management techniques and informing sustainable urban mobility planning.

A Study on Nagel–Schreckenberg Model

Contents

1. Introduction

The Nagel–Schreckenberg model is a fundamental mathematical framework that sheds light on the intricate dynamics of traffic flow. Developed by Kai Nagel and Michael Schreckenberg, this model offers valuable insights into the collective behavior of vehicles on a roadway, unraveling the factors that contribute to congestion, traffic jams, and the overall efficiency of transportation networks. By capturing the essential features of vehicular movement and interactions, the Nagel–Schreckenberg model has become a cornerstone in traffic research, providing a platform for analyzing real-world scenarios, testing traffic management strategies, and exploring the potential of emerging technologies in tackling congestion-related challenges. This article delves into the inner workings of the Nagel–Schreckenberg model, its key components, and its implications for understanding and optimizing traffic systems.

1.1. Background on cellular automata and traffic flow modeling

Cellular automata (CA) have emerged as a powerful tool for modeling complex systems, including traffic flow. Initially conceived by John von Neumann and Stanislaw Ulam in the 1940s, CA is a computational model composed of a grid of cells, each of which can exist in a finite number of states. These states evolve over discrete time steps based on predefined rules that govern the system's behavior.

In traffic flow modeling, cellular automata have proved to be particularly effective in capturing the spatiotemporal dynamics of vehicles on a road network. The discrete nature of CA allows for modeling individual vehicles and their interactions, enabling researchers to study the emergence of collective phenomena such as congestion, traffic waves, and phase transitions.

One influential cellular automaton model in traffic research is the Nagel–Schreckenberg model, proposed in 1992 and inspired by real-world observations. Nagel and Schreckenberg sought to simulate the intricate dynamics of the traffic flow by considering a few essential elements: vehicle movement, speed adjustment, randomization, and local interactions.

In the Nagel–Schreckenberg model, each vehicle is represented as a single cell on a one-dimensional lattice, with periodic boundary conditions to simulate a circular road. Vehicles move forward one cell at a time, limiting their speeds by a maximum velocity parameter. The model incorporates a simple traffic rule that governs speed adjustment: if a vehicle is too close to the one ahead, it slows down to avoid collisions. A stochastic component is also introduced to model driver behavior, allowing for random speed reduction even without obstacles.

Through simulations of the Nagel–Schreckenberg model, researchers have been able to reproduce and study various traffic phenomena. The model exhibits fundamental characteristics observed in real traffic, including the formation and dissipation of traffic jams, the propagation of congestion, and the relationship between traffic density and flow. Furthermore, the model has been extended and modified to incorporate additional factors such as lane-changing behavior, traffic signals, and the effects of driver anticipation.

The cellular automaton approach to traffic modeling, exemplified by the Nagel–Schreckenberg model, provides valuable insights into the underlying dynamics of transportation systems. By capturing the complex interactions between individual vehicles, these models help researchers understand and predict traffic flow behavior, laying the groundwork for developing effective traffic management strategies, intelligent transportation systems, and the optimization of road networks.

In the following sections of this article, we will delve deeper into the intricacies of the Nagel–Schreckenberg model, examining its key components, limitations, and relevance in contemporary traffic research.

1.2. Importance of understanding traffic dynamics at intersections

Intersections are critical nodes in transportation networks where multiple streams of traffic converge. Understanding intersectional traffic dynamics is vital for several reasons, including safety, efficiency, and urban planning. Here, we delve into the significance of comprehending the intricacies of traffic dynamics at intersections.

1. **Safety**: Intersections are often hotspots for traffic accidents and collisions. By understanding traffic dynamics at intersections, traffic engineers can identify potential safety hazards and design effective measures to mitigate them. This includes optimizing signal timings, implementing dedicated turning lanes, and improving visibility through signage and traffic control devices. A thorough understanding of traffic behavior at intersections allows for identifying and implementing safety enhancements, ultimately reducing the risk of accidents and improving overall road safety.

2. **Efficiency**: Efficient traffic flow at intersections is vital for minimizing congestion and maximizing the capacity of road networks. Understanding how traffic interacts and moves through intersections enables the development of optimized signal timings, phasing plans, and traffic management strategies. By utilizing models and simulations that capture the complex dynamics of vehicles at intersections, transportation planners can evaluate different scenarios, test alternative intersection configurations, and identify measures to enhance traffic flow efficiency. This knowledge facilitates the design of more innovative transportation systems that can handle increased traffic volumes while minimizing delays and maximizing throughput.

3. **Traffic Management**: Effective traffic management is crucial for urban planning and development. Transportation authorities can make informed decisions about traffic signal coordination, lane configurations, and intersection geometries by understanding traffic dynamics at intersections. This knowledge allows for implementing intelligent transportation systems that can adapt to changing traffic conditions, prioritize specific traffic movements, and optimize the overall operation of intersections. Moreover, understanding traffic dynamics at intersections enables implementation of adaptive signal control systems, traffic flow monitoring, and real-time traffic management strategies to respond to incidents and optimize traffic distribution.

4. **Sustainable and Livable Cities**: Understanding intersectional traffic dynamics is essential for building sustainable and livable cities. By improving traffic flow efficiency and reducing congestion, the environmental impact of transportation can be minimized. Efficient intersections facilitate smoother traffic flow, reducing fuel consumption and emissions. Moreover, optimized intersections can improve pedestrian and cyclist safety, encouraging active transportation modes and enhancing the overall livability of urban areas.

In conclusion, comprehending traffic dynamics at intersections is vital for ensuring safety, improving efficiency, and managing traffic in urban areas. By leveraging models, simulations, and data-driven approaches, transportation experts can design and implement intersection improvements that enhance overall traffic flow, reduce congestion, and contribute to developing sustainable and livable cities.

2. The Nagel-Schreckenberg Model

2.1. Description of the model and its application to traffic flow

Our computational model is defined on a one-dimensional array of $\boldsymbol{L}$ *sites with open or periodic boundary conditions. Each site may either be occupied by one vehicle or remain empty. Each vehicle has an integer velocity with values between zero and* $\boldsymbol{v}_{max}$*. For an arbitrary configuration, one update of the system consists of the following four consecutive steps, which are performed in parallel for all vehicles:*

1. **Acceleration**: If the velocity v of a vehicle is lower than $v_{\max}$ and the distance to the next car ahead is larger than $v + 1$, the speed is advanced by one ($v \to v + 1$).

2. **Slowing down (due to other cars)**: If a vehicle at site i sees the following vehicle at site $i + j$ (with $j < v$), it reduces its speed to j ($v \rightarrow j$).
3. **Randomization**: With probability p, the velocity of each vehicle (if greater than zero) is decreased by one ($v \rightarrow v - 1$).
4. **Car motion**: Each vehicle is advanced v sites.

Through these steps, very general properties of single-lane traffic are modeled using integer-valued probabilistic cellular automaton rules. The introduction of randomness in step 3 is essential for simulating realistic traffic flow since it accounts for natural velocity fluctuations due to human behavior or varying external conditions. Without this randomness, every initial configuration of vehicles and velocities quickly reaches a fixed pattern shifted backward (i.e., opposite to the vehicle motion) by one site per time step.

The described model exhibits nontrivial and realistic behavior, demonstrating its capability to capture essential traffic flow features. To simulate the model, Monte Carlo simulations were primarily carried out with a choice of $\boldsymbol{v_{max} = 5}$. *The model was implemented in FORTRAN, utilizing a logical array for the positions of the cars and an integer array for the velocities. For a relatively low density of occupied sites (usually around one-fifth), implementing IF statements were approximately five times faster than one using only boolean variables, which was necessary for multispin coding. Although multispin coding could potentially provide a factor of about six improvements, it was postponed for future work.*

The model's simplicity and ability to produce realistic traffic behavior make it valuable for studying and analyzing traffic flow in single-lane scenarios. By simulating the interactions, acceleration, and deceleration of vehicles, this model provides insights into traffic patterns, aids in optimizing traffic management strategies, and facilitates the assessment of various factors on traffic flow efficiency.

3. Incorporating Traffic Signals into the Model

This section presents results from systems with periodic boundary conditions, simulating traffic in a closed loop (similar to car races) but restricted to a single lane. The total number $\boldsymbol{N}$ *of cars in the circle remains constant during the dynamics, allowing us to define a constant system density* $\boldsymbol{p}$, *representing the ratio of the number of cars* $\boldsymbol{N}$ *to the number of sites* $\boldsymbol{L}$ *in the circular loop.*

We consider a fixed site $\boldsymbol{i}$ *and average the occupancy over a while* $\boldsymbol{T}$ *to measure the density or occupancy. The occupancy of site* $\boldsymbol{i}$ *is denoted by* $\boldsymbol{n_i(t)}$, *where* $\boldsymbol{n_i(t) = 1}$ *if site* $\boldsymbol{i}$ *is occupied by a car at time step* $\boldsymbol{t}$ *and* $\boldsymbol{n_I(t) = 0}$ *if it is empty. Thus, the density* $\boldsymbol{\tilde{p}}$ *at site* $\boldsymbol{I}$ *can be calculated as the average occupancy over time:*

$$\tilde{p}^T = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_i(t) \quad (1)$$

where $n_i(t) = O(1)$ if site I is empty (occupied) at time step t. For large T we have.

$$\lim_{T \to \infty} \tilde{p}^T = p \quad (2)$$

The time-averaged flow $\tilde{p}$ between i and $i + 1$ is defined by

$$\tilde{p}^T = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_{i,i+1}(t) \quad (3)$$

where $n_{i,i+1}(t) = 1$ if a car motion is detected between sites i and $i + 1$ at time step t.

Using these definitions, we perform simulations with different densities $\boldsymbol{p}$ *and observe the fundamental diagrams, which plot the vehicle flow* $\tilde{\boldsymbol{p}}$ *against the density* $\boldsymbol{p}$*. We start with a random initial configuration of cars with density* $\boldsymbol{p}$ *and velocity 0. Data collection begins after the system has relaxed to equilibrium, discarding the initial transient behavior (the first* $\mathbf{10 \times \boldsymbol{L}}$ *time steps).*

Figure 1 illustrates a simulated traffic situation at a low density of 0.03 cars per site. Each line represents the traffic lane after one complete velocity update, just before the car's motion. Dots represent empty sites, and occupied sites are represented by the integer number indicating the car's velocity. At low densities, the traffic exhibits undisturbed motion.

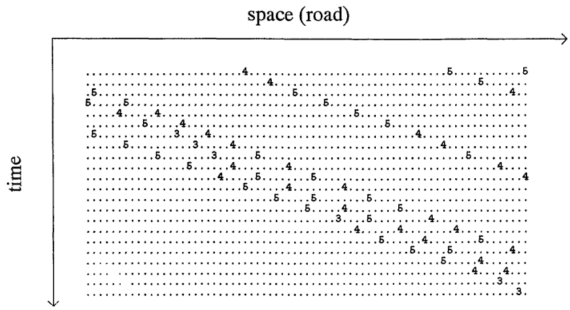

Figure 1. *Simulated traffic at a low density of 0.03 cars per site. Each line represents the traffic lane after one complete velocity update. Empty sites are represented by dots, and occupied sites show the car's velocity..*

Figure 1 illustrates a simulation at a higher density of 0.1 cars per site. In this case, congestion clusters or small traffic jams randomly form due to velocity fluctuations of the cars. Observing an individual car entering from the left in Figure 2, we can see that the car initially comes in with speed varying between four and five but eventually stops due to the congestion cluster. The car remains stuck in the queue for a particular time, making slow advances, and then accelerates to its full speed after leaving the cluster at the end. This cluster represents a typical start-stop wave similar to that observed in freeway traffic.

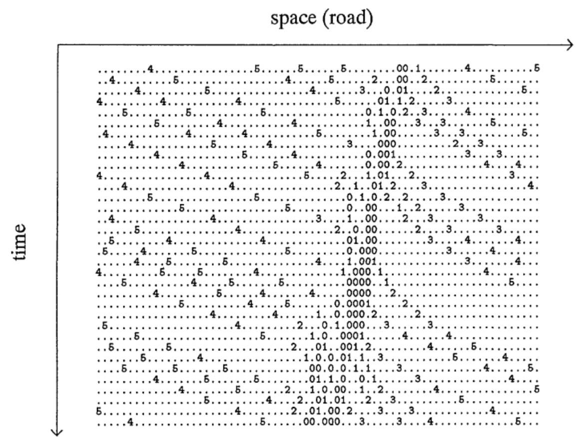

Figure 2. *Simulated traffic at a higher density of 0.1 cars per site. Note the backward motion of the traffic jam..*

We present the fundamental diagram of our model in Figure 2. The line represents the results averaged over $\mathbf{10^6}$ *time steps, while the dots represent averages over only* $\mathbf{10^4}$ *time steps. These results can be compared with actual data (Figure 4). A changeover occurs near* $\boldsymbol{p} = \mathbf{0.08}$*. Further simulations reveal that the position and shape of the maximum of* $\boldsymbol{q(p)}$ *depending on the system size, with simulations without randomization showing no dependence on the system size. However, the precise location of the transition point remains to be determined based on these figures.*

For an analytical treatment of circular traffic, we consider the case where the maximum velocity $\boldsymbol{v}_{max}$ *is set to 1. In this case, the update procedure simplifies significantly, as every car can accelerate to its maximum speed in a single time step. An analytical approach can explore the model's behavior in this specific case, focusing on site occupation and speed updates.*

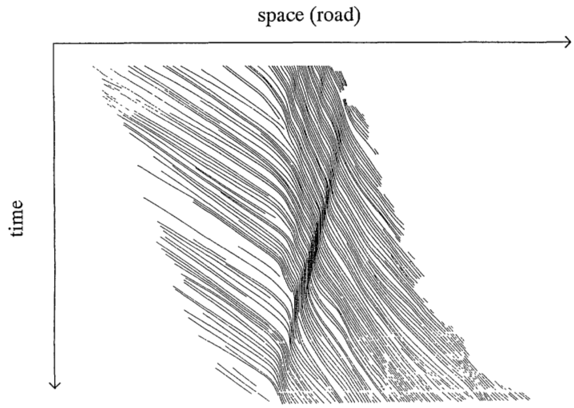

Figure 3. *Fundamental diagram of the modified Nagel-Schreckenberg model. The line represents results averaged over* 10^6 *time steps, and the dots represent averages over* 10^4 *time steps..*

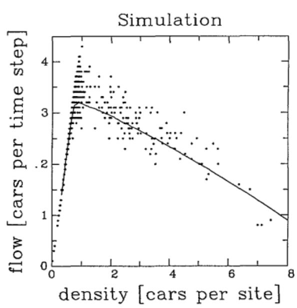

Figure 4. *Comparison of simulated fundamental diagram (dots) with real data (line).*

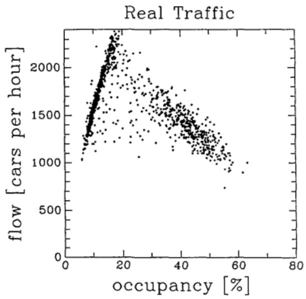

Figure 5. *Traffic flow q (in cars per hour) vs. occupancy (in cars per hour) from measurements in reality. Occupancy is the percentage of the road which is covered by vehicles..*

For speeds larger than v_{max}, an additional parameter for the current speed is needed, which presents serious difficulties for analytical treatments. However, even in this case, a kind of mean-field approximation is possible, and the results will be reported elsewhere.

The easiest way to formulate the dynamics for direct calculations is based on a master equation with continuous time and random sequential updates. This differs from parallel updating (which can be seen by simulating the two other updates). However, the results should be qualitatively similar, and the randomization parameter p plays a specific role in random sequential updates.

Using the more familiar spin variables $\sigma_i = +1$ for occupied and -1 for empty sites, the transition probability $W(-\sigma_i, -\sigma_{i+1}|\sigma_i, \sigma_{i+1})$ (i.e., a car moves from i to $i+1$) simply reads:

$$W(-\sigma_i, -\sigma_{i+1}|\sigma_i, \sigma_{i+1}) = (1-p)\frac{1+\sigma_i}{2} \tag{4}$$

With periodic boundary conditions, this transition probability ensures the conservation of the total magnetization M in the system. The master equation for the probability $P((\sigma_i), t)$ to find configuration (σ_i) at time t is given by:

$$\frac{dP((\sigma_i), t)}{dt} = \sum_i W(-\sigma_i, -a_{i+1}|\sigma_i, \sigma_{i+1})P(\sigma_i, \sigma_{i+1}, t) + \dots \tag{5}$$

From this formula, the factor $(1-p)$ only gives a simple scaling factor of the time axis. Therefore, the systems with different $p < 1$ are equivalent in continuous time because they have the same equilibrium distribution. Analyzing this equilibrium distribution, one finds that every spin configuration is equally probable given a fixed total magnetization. Therefore, in this simple situation, one has:

$$q = p \cdot (1-p) \tag{6}$$

Which is just the probability that a car has free site in front of it. The property that every configuration of the cars is equally probable is undoubtedly not actual for $v_{max} > 0$. The parallel update gives a different function $q(p)$, but it is also symmetric to $p = \frac{1}{2}$.

4. Simulation and Analysis of Traffic Flow

4.1. The base simulation

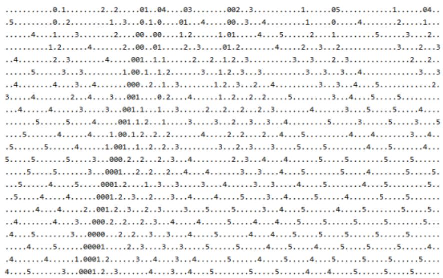

Figure 6. *Base simulation code.*

Figure 6 represents the output from my written code. We begin by explaining the format of what is being produced. Each line represents the road; in this case, we emulate that of a closed loop by wrapping the end of the line with the beginning of the following line. Each new line represents a new iteration; effectively, it is the next time step. The road, as previously mentioned, consists of sites. These sites are either occupied by a vehicle or empty. Empty sites are displayed as a '.', and sites occupied by a vehicle are displayed with their corresponding velocity. We should note that each vehicle obeys the rules the model sets. These rules include when to accelerate and when to slow down. For this specific base case, we have set the number of iterations to 22, the number of sites to 100, the number of vehicles/cars to 20, the probability of a car randomly reducing its speed by 1 to 20, and the maximum velocity of a vehicle to 5. These specific base parameters follow the original output produced by the Nagel-Schreckenberg model.

```
...........0.1........2..2.....01..04....03........002..3...........1......05............1......04..
.5.........0..2.........1..3...0.1.0....01...4.....00..3...4.........1.....0.....4........2.....1...
```

Figure 7. *First two iterations from base code.*

In Figure 7, we analyze the first two lines of output. The first 0 on the very first line represents a vehicle at current velocity 0. We notice a vehicle in front, so this vehicle decides not to increase its speed. In the following line, the exact vehicle still has not moved. It is important to note that the rule for accelerating relies on current velocity and the number of free sites ahead. On the second line, we see that this car should be ready to accelerate since two spaces are free, which is more significant than 0+1 (current velocity+1). However, an element of human nature has been accounted for with a probability. This probability randomly reduces the speed of a vehicle by 1 to account for human nature; this could include a late reaction time or become distracted while stopped.

We notice very clearly where traffic begins to build up. This is displayed by clusters of 0's, representing vehicles that have stopped. Seeing the backward movement of traffic as we go from one iteration to the next is apparent. The very purpose of this project is to display such a phenomenon. In the early iterations, we see most vehicles to the right of the traffic build with speeds between 0-3. As we proceed to later iterations, we notice that most vehicles to the right of the traffic build-up have speeds of 2-5. This shows us that since more cars are becoming jammed, more sites on the road are empty, so by obeying the rules of acceleration; the vehicles increase speed.

4.2. Simulations with varying parameters

Within this section we discuss the results of simulations from varying parameters. These variable parameters include the number of vehicles, number of sites and the maximum velocity of a vehicle.

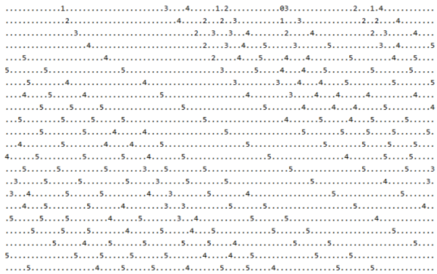

Figure 8. *Simulation with vehicles reduced to 10 (Undisturbed motion).*

This simulation in Figure 8 was done with half the number of vehicles occupying the road at any time (10). As we can see, there is no traffic build-up or any backward movement of traffic. Within the Nagel-Schreckenberg model, this is described as "undisturbed motion." This is due to low, low overall density. Regarding the fundamental diagram, when the number of cars is less than the number of sites, the flow of vehicles has yet to reach its maximum state. In this case, since we have halved the number of vehicles, we have reduced the overall density, thus reducing the flow.

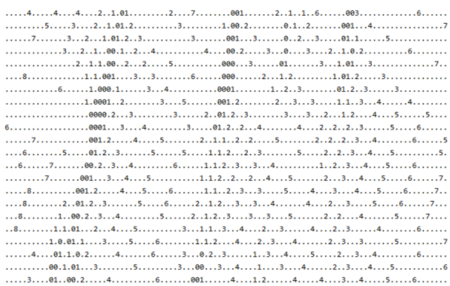

Figure 9. *Simulation with maximum velocity 10.*

In Figure 9, we see the results from a simulation where the maximum velocity of vehicles has been increased to 10, and the number of vehicles remains at 20. We notice a few things. Firstly, the velocity of 10 is never achieved because the road is not large enough to allow any vehicle to accelerate this much; the vehicle will encounter another vehicle ahead before it has time to increase its speed to 10. Secondly, two different traffic jams are building up, although the second is less noticeable. The first traffic jam also evidently shows the backward movement of traffic. Secondly, although traffic jams do not frequently build-up, vehicles are much closer together with lower speeds compared to the base simulation. This is due to vehicles catching up to the next much faster than in the base simulation.

```
....1..2...0.000003........3....1.01.0.00.1.1.03..
4....1...1.0.00001...4........1..01.01.01..1.00...
....1.2...01.0000.2......5.....0.1.01.00.1..001...
.....2..1.1.00000...3.........01..01.001..1.00.2..
.......1.0.000001......4......0.1.1.001.2..001...3
..4.....01.00000.2.........2..1..1.001.2..001.2...
......1.1.000001...3.........1.1..001.1..000.2..3.
.4.....1.000001.1.....4.......1.1.01.1.1.001...3..
4....2..000001.1.2........3....0.01.1.1.001.2.....
....2..000001.1.2..3.........1.1.1.1.1.001.1..3...
......000000.1.2..3...4.......1.1.1.1.000.1.1....4
...2..000000..1..3...4....4....1.1.1.0000..1.2....
.....0000001...2....4....4....1.1.1.00000...2..3..
4....000001.2....2......4....1.1.1.000001.....3...
....000001.2..3....3........0.1.1.000001.2.......4
...000000.2..3...4....4.....1..1.000001.2..3......
...000001...3...4....4....2..1..000001.2..3...4...
2..00001.2.....4....4....2..1.1.00001.2..2...4....
..00001.2..3.......4....2..1.1.00001.2..2..3.....2
.00001.2..3...4........2..1.1.00001.2..2..2...4...
00001.2..3...4....5......1.1.00001.2..1..1..3.....
0001.2..2...4....4.....2..1.00001.2..1.2..2....2..
```

Figure 10. *Simulation with sites reduced to 50.*

In Figure 10, we see the results from a simulation where the only varied parameter is the number of sites, which has been reduced from 100 to 50. We see that not only are there two traffic jams, but both traffic jams display a backward movement. Compared to all other cases, the traffic jams also have more vehicles stationary. This is due to the size of the road being reduced by half; since there are fewer sites on the road, there is less space for cars to move. This shows that the length of the road can impact the probability of a traffic jam building.

5. Discussion

The Nagel-Schreckenberg Model provides a valuable framework for understanding traffic flow dynamics and simulating real-world traffic scenarios. This model offers insights into various traffic phenomena by incorporating essential aspects of individual driver behavior and traffic interactions.

One notable feature of the Nagel-Schreckenberg Model is its ability to capture the transition from laminar flow to start-stop traffic, closely resembling real-world traffic patterns. This transition occurs naturally within the model, allowing for a realistic representation of traffic congestion and propagation.

Furthermore, the Nagel-Schreckenberg Model demonstrates the influence of different parameters on traffic behavior. By varying parameters such as the number of vehicles, maximum velocity, and road length, simulations can provide valuable insights into how these factors impact traffic flow. For example, reducing the number of vehicles or increasing the maximum velocity may result in smoother traffic flow, while increasing the road length can lead to traffic jams.

It is important to note that while the Nagel-Schreckenberg Model provides valuable insights into traffic dynamics, it also makes certain simplifying assumptions. For instance, it assumes homogeneous vehicles and uniform road conditions, disregarding driver heterogeneity, road topology, and traffic lights. These simplifications allow for computational efficiency but may limit the model's accuracy in specific real-world scenarios.

Despite these limitations, the Nagel-Schreckenberg Model is a foundation for further research and development of traffic simulation and optimization techniques. By refining the model's assumptions and incorporating more complex factors, it can be adapted to study specific traffic scenarios and assist in traffic management and planning.

In conclusion, the Nagel-Schreckenberg Model offers a valuable tool for studying traffic flow dynamics. This model contributes to our understanding of traffic behavior through its ability to capture the transition from laminar to start-stop traffic and its insights into parameter dependencies. Further advancements and refinements of the model can enhance its applicability in real-world traffic simulations and assist in developing effective traffic management strategies.

6. Conclusions

It has been demonstrated that utilizing a discrete model for traffic flow offers computational advantages while incorporating crucial aspects of the fluid-dynamical approach to traffic flow. As mentioned in the previous section, one of these aspects is the natural transition from laminar to start-stop traffic. This discrete model also retains more elements of individual driver behavior, albeit in a statistical manner, which can be advantageous for traffic simulations involving individual behavior, such as dynamic routing.

An interesting observation is that the behavior of sand falling a long and narrow tube exhibits similar characteristics, namely "start-stop waves," which arise from fluctuations in the velocity of the particles. In the case of sand, these fluctuations are caused by dissipation at the boundary.

References

[1] **Nagel, K.** and **Schreckenberg, M.** (1992) A cellular automaton model for freeway traffic. *Journal de Physique I*, *2*(12), 2221–2229. doi:10.1051/jp1:1992277.

[2] **Gazis, D. C.**, **Herman, R.**, and **Potts, R. B.** (1961) Nonlinear Follow-The-Leader Models of Traffic Flow. *Operations Research*, *9*(4), 545–567. Retrieved from http://www.jstor.org/stable/167126.

[3] **Rose, M.** (2004) Modeling of Freeway Traffic. Institute of Computer Science in Civil Engineering, University of Hannover, Germany. Retrieved from https://e-pub.uni-weimar.de/opus4/frontdoor/deliver/index/docId/263/file/icccbe-x_045_pdfa.pdf.

[4] **Chiang, Y.-H.** and **Juang, J.-C.** (2008) Control of Freeway Traffic Flow in Unstable Phase by H∞ Theory. *Journal of Transportation Engineering*, *137*(5), 427–436. doi:10.1061/(ASCE)0733-947X(2011)137:5(427).

[5] **Nagel, K.** and **Schreckenberg, M.** (1992) A cellular automaton model for freeway traffic. *Journal de Physique I*, December 1992. Retrieved from https://www.researchgate.net/publication/263504490.