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- You can form a group of up to 2 people in this homework. It is sufficient to have one submission from one of the members within each group.
- Plagiarism is strictly prohibited and punishable. You are expected to give your own answer and report for each question.
- You are only allowed to use array manipulation functions (element-wise operations, indexing, reshaping, concatenation, and permutation) in NumPy and no other third-party libraries unless permission is given. It is important to do your own implementation to have a firm grasp of the concepts covered in this class.
- Do not hesitate to ask your questions related to the homework to the teaching assistant via email.
- The submission includes a report in which you are expected to explain your answers and write your insights and a Jupyter notebook that includes your Python code.
- Prepare your report in **LaTeX** (You can use an Overleaf template).

Question 1

We find working with gray-scaled images simple whenever having a single channel simplifies the application of an operation. Hence it is important to know how to convert a colored image into a gray-scaled one. **Apply** gray-level scaling to Figure 1 (You can find the template code in the Jupyter notebook provided within the homework files).

(i) $(R + G + B)/3$

(ii) $(0.299R + 0.587G + 0.114B)$

Comment about the output of the two gray-scaling methods in your report.

Figure 1: Colored image of Webb James Space Telescope.

Question 2

In this question, we will explore the **Histogram Equalization** algorithm. The main idea behind it is to transform the input image I to obtain an output image I' such that all grey levels appear equally often in the transformed image I'.
The motivation of the algorithm is to consider the gray-level relative frequencies as probability density functions (PDF). Let p_X denote the density of the input image I and p_Y denote the density of the output image I' with the corresponding random variables X and Y. We define a transformation $g(.)$ such that $Y = g(X)$. We want $p_Y(y)$ to be a Uniform distribution defined over gray levels $\mathcal{U}(0, G_{max} - 1)$.

1. Find the transformation $g(.)$ that can transform its input distribution to a uniform distribution by following the steps below (We use a capital letter to denote cumulative density function (CDF). For example, the CDF of a random variable X is denoted by $F_X(x)$).

   (i) Given that $F_Y(y) = P(Y \leq y)$. Show that $F_Y(y) = F_X(g^{-1}(y))$. Assume that g^{-1} exists.

   (ii) We know that $p_Y(y) = \frac{\partial}{\partial y} F_Y(y)$. Show that $p_Y(y) = \frac{\partial}{\partial y} F_X(g^{-1}(y))$. Assume that g^{-1} is differentiable.

   (iii) In order to obtain a uniform $p_Y(y)$, we need to show that $p_Y(y) = c$ where c is a positive constant. Show that if we choose the CDF of X, namely $F_X(x)$, as our transformation $g(.)$, we can obtain a uniform $p_Y(y)$.

   (iv) Since we want to satisfy $y \sim \mathcal{U}(0, G_{max} - 1)$, the density function of Y must be $p_Y(y) = \frac{1}{G_{max}-1}$. Finally, find a function $g(.)$ that satisfies the aforementioned condition for $p_Y(y)$.

2. Implement the **Histogram Equalization** algorithm you showed above and apply it to the given grayscaled image that you can find in the Jupyter notebook. You can transform the input image I by applying the function $g(.)$ to each pixel of the image. You can find the template code in the Jupyter notebook. Compare and discuss the histograms of the input and the output images.

3. In the second part of the question, you applied the transformation function $g(.)$ to the input image to obtain an image with a uniform histogram over the gray levels. What if you apply the function $g^{-1}(.)$ to a random image where each pixel is randomly sampled from $\mathcal{U}(0, G_{max} - 1)$? Discuss the process in the report.

Question 3

Apply conditional scaling to image J based on the statistics of the image I in the related section of the Jupyter notebook. Compare the histograms and statistics of the output image J_{new} and image I. Briefly explain the process in your report.

Question 4

Apply local mean and max operations to images in the related section of the Jupyter notebook. In your report, explain in which case you would prefer one over the other.