Ahmet Faruk Ulutaş - 21803717 – CS202 – 1

Question-1

T(n) = 3T(n/3) + n, where T(1) = 1 and n is exact power of 3.

= 3T(n/3) + n

= 3(3 (T(n/9) + n/3)) + n

= 9 T (n/9) + 2n

= 9 T (3T (n/27) + n/9) + 2n

= …

= 3^i T (n / (3^i)) + i*n

= 3 ^(log3(n)) T(1) + log3(n) * n

= n + log3(n) * n

T(n) = O(n*log(n))

T(n) = 2T(n-1) + n^2, where T(1) = 1.

T(n) = 2T(n-1) + n^2

= 2 (2T (n – 2) + (n – 1)^2) + n ^ 2

= 4 T (n – 2) + 2 (n – 1) ^ 2

= 4 (2 T (n – 3) + (n – 1)^2) + (2 (n-1) ^2 + n^2

= 8T (n – 3) + 6 (n-1)^2 + n^2

= …

= 2^k T (n-k) + 2 (2^k – 1) (n-1)^2 + n^2 , n – k = 1, k = n – 1;

= 2 ^(n-1) T(1) + 2 (2^(n-1) - 1) (n-1) ^2 + n^2

= 2^(n-1) + (2^n –2) (n^2 –2n +1) + n^2

= …

= O (2^n)

T(n) = 3T(n/2) + 1, where T(1) = 1 and n is exact power of 2.

T(n) = 3T (n/2) + 1

= 3 (3T (n/4) + 1) + 1

= 9T (n/4) + 3 + 1

= 9T (3T (n/8) + 1) + 3 + 1

= 27T(n/8) + 9 + 3 + 1

= …

= 3^k T (n/2^k) + (3^k – 1) / 2

= 3^(log2(n)) T(1) + (3^(log2(n)) - 1) /2

= O (n^(log(3)/log(2))

= O (n^c)

Bubble Sort: move left to right swapping adjacent elements as needed.

5,6,8,4,10,2,9,1,3,7

5,6,4,,8,10,2,9,1,3,7

5,6,4,8,2,10,9,1,3,7

5,6,4,8,2,9,10,1,3,7

5,6,4,8,2,9,1,10,3,7

5,6,4,8,2,9,1,10,3,7

5,6,4,8,2,9,1,3,10,7

5,6,4,8,2,9,1,3,10,7

5,4,6,8,2,9,1,3,7,10

5,4,6,2,8,9,1,3,7,10

5,4,6,2,8,1,9,3,7,10

5,4,6,2,8,1,3,9,7,10

5,4,6,2,8,1,3,7,9,10

4,5,6,2,8,1,3,7,9,10

4,5,2,6,8,1,3,7,9,10

4,5,2,6,1,8,3,7,9,10

4,5,2,6,1,3,8,7,9,10

4,5,2,6,1,3,7,8,9,10

4,2,5,6,1,3,7,8,9,10

4,2,5,1,6,3,7,8,9,10

4,2,5,1,3,6,7,8,9,10

2,4,5,1,3,6,7,8,9,10

2,4,1,5,3,6,7,8,9,10

2,4,1,3,5,6,7,8,9,10

2,1,4,3,5,6,7,8,9,10

2,1,3,4,5,6,7,8,9,10

1,2,3,4,5,6,7,8,9,10

1,2,3,4,5,6,7,8,9,10

1,2,3,4,5,6,7,8,9,10

Selection sort; Find the biggest one and swap with the end of unsorted elements.

5,6,8,4,10,2,9,1,3,7

5,6,8,4,7,2,9,1,3,10

5,6,8,4,7,2,3,1,9,10

5,3,1,4,7,2,3,8,9,10

5,6,1,4,3,2,7,8,9,10

5,2,1,4,3,6,7,8,9,10

3,2,1,4,5,6,7,8,9,10

3,2,1,4,5,6,7,8,9,10

1,2,3,4,5,6,7,8,9,10

1,2,3,4,5,6,7,8,9,10

Quick Sort's Worst Case

$T(0) = T(1) = 0;$

$T(n) = n + T(n-1)$

$T(n-1) = n - 1 + T(n-2)$

$T(n-2) = n - 2 + T(n-3)$

…

$T(3) = 3 + T(2)$

$T(2) = 2 + T(1)$

$T(1) = 0$

Hence, $T(n) = n + (n-1) + (n-2) + \ldots + 3 + 2 + 0$ which is approximately $(N^2) / 2$

$T(n) = O(N^2)$