

Q1)

- a) $A \rightarrow B$ does not hold (violated by tuples 1 and 2).
- b) $B \rightarrow C$ does not hold (violated by tuples 1, 5, 4, and 6).
- c) $D \rightarrow A$ holds.
- d) $AD \rightarrow C$ holds.
- e) $AC \rightarrow B$ does not hold (violated by tuples 1 and 2).
- f) $BC \rightarrow A$ holds.

Q2)

- a) R into $R1(A,B,C)$ and $R2(C,D,E)$

Intersection: $R1 \cap R2 = C$

Functional dependencies:

- i) $AB \rightarrow C$,
- ii) $AD \rightarrow E$,
- iii) $A \rightarrow C$,
- iv) $D \rightarrow C$,

The decomposition is **lossless** since $C \rightarrow A$ and $C \rightarrow D$, and both A and D are in R1 and R2, respectively. Both R1 and R2's characteristics are functionally determined by the common attribute C in the intersection.

- b) R into $R1(A,C,D)$ and $R2(B,D,E)$

Intersection: $R1 \cap R2 = D$

Functional dependencies:

- i) $AB \rightarrow C$,
- ii) $AD \rightarrow E$,
- iii) $C \rightarrow A$,
- iv) $C \rightarrow D$,

Based on the specified functional dependencies, D by itself is unable to functionally identify any other property in either R1 or R2. Since there is no functional dependence that permits us to recover the original relation R without losing any information, this decomposition may be **lossy**.

Q3) $R(A, B, C, D, E)$, $F = \{A \rightarrow B, A \rightarrow D, E \rightarrow D, BD \rightarrow C\}$

- a) $A \rightarrow C$

We need to check if A can functionally identify C using the provided set F in order to deduce $A \rightarrow C$. We may infer that A is able to predict B and D from $A \rightarrow B$ and $A \rightarrow D$. As $BD \rightarrow C$ and A are known to be able to predict B and D, it stands to reason that A is likewise able to determine C. Consequently, $A \rightarrow C$ can be derived from F.

b) $CE \rightarrow B$

We must investigate if B can be determined by C and E together in order to deduce $CE \rightarrow B$. $E \rightarrow D$ suggests that E ascertains D. However, there aren't any functional linkages that would enable C and E to decide B together, either directly or indirectly. There is no functional dependency chain connecting CE and B, nor does C show up on the right side of any functional dependency. Thus, $CE \rightarrow B$ cannot be derived from F.