# IE 400 - Principles of Engineering Management

## Term Project

## Group 46

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# Part A. Win the Election

**Decision Variables:**

$x_i$: $1\ if\ candidate\ invests\ in\ City\ i,\ 0\ otherwise$

$y_i$: $1\ if\ candidate\ receives\ vote\ from\ City\ i,\ 0\ otherwise$

$I_i$: $Amount\ of\ investment\ made\ to\ City\ i$

$i\ =\ 0,\ 1,\ \dots\ ,\ 34$

The decision variables in this part are binary variables $x_i$ and $y_i$, and integer variables $I_i$, where i is an index representing a city in a grid. Variables $x_i$ and $y_i$ represent whether or not the candidate will invest in and receive a vote from city i, respectively. The integer variable $I_i$ represent the amount invested in city i.

**The Objective Function:**

The objective function is to minimize the total investment.

$$min \sum_{i=0}^{34} I_i$$

**Constraints:**

$x_i\ >=\ y_i\ (i\ =\ 0,\ 1,\ \dots\ ,\ 34)$

$\sum_{i=0}^{34} y_i >= 18\ \ (i\ =\ 0,\ 1,\ \dots\ ,\ 34)$

$I_i\ >=\ E_i \times y_i\ \ (i\ =\ 0,\ 1,\ \dots\ ,\ 34)$

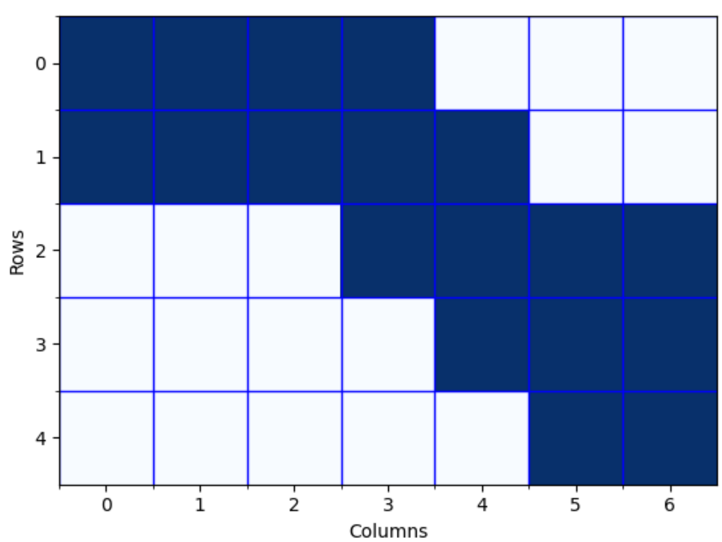
$$\sum_{i=0}^{4} I_i >= 15 \times y_i \quad (i = 0, 1, \ldots, 34)$$

$$\sum_{n=0}^{4} y_n >= y_i \ (i = 0, 1, \ldots, 34)$$

$$x_i \times M >= I_i \ where\ M\ is\ an\ arbitrarily\ large\ number\ (i = 0, 1, \ldots, 34)$$

$$x_i <= I_i \ (i = 0, 1, \ldots, 34)$$

There are several constraints in this part. The first set of constraints ensure that the investment and vote decisions are consistent, i.e., if the candidate invests in a city, they must also receive a vote from that city. The second constraint ensures that the candidate wins the election by requiring that the number of votes received is greater than or equal to the minimum number of votes needed to win. The third set of constraints ensure that the investment in each city is greater than or equal to the expected return on investment, and that the average investment in the neighboring cities is greater than a threshold value T. The fourth constraint ensures that at least one neighboring city must also vote for the candidate. The final set of constraints ensure that the binary variable $x_i$ is equal to the integer variable $I_i$ when $y_i$ is equal to 1, and is equal to 0 when $y_i$ is equal to 0.



# Part B. Region Partitioning

**Parameters:**

$V: Voted\ Cities\ =\ 0,\ 1,\ 2,\ 3,\ 7,\ 8,\ 9,\ 10,\ 11,\ 14,\ 17,\ 18,\ 19,\ 20,\ 25,\ 26,\ 27,\ 33,\ 34$

**Decision Variables:**

$S_i$: $1\ if\ Region\ i\ has\ a\ seat\ from\ out\ representative,\ 0\ otherwise$

$SC_j$: $Number\ of\ seats\ a\ region\ will\ have,\ j\ =\ 0,\ 1,\ \dots\ ,\ 6$

$A_i,\ B_i,\ C_i,\ D_i,\ E_i,\ F_i,\ G_i$: $Binary\ variables\ representing\ whether\ a\ city\ belongs\ to\ a\ region$

$i\ =\ 0,\ 1,\ \dots\ ,\ 34$

**The Objective Function:**

To maximize the number of seats.

$$max\ \sum_{i=0}^{6} S_i$$

**Constraints:**

These constraints ensure that every region must have exactly 5 cities.

$$\sum_{i=0}^{34} A_i\ =\ 5,\ \sum_{i=0}^{34} B_{i_i}\ =\ 5,\ \sum_{i=0}^{34} C_i\ =\ 5,\ \sum_{i=0}^{34} D_i\ =\ 5,\ \sum_{i=0}^{34} E_i\ =\ 5,\ \sum_{i=0}^{34} F_i\ =\ 5,\ \sum_{i=0}^{34} G_i\ =\ 5$$

This constraint limits city so that it belongs only to one region.

$A_i + B_i + C_i + D_i + E_i + F_i + G_i\ <=\ 1\ (i\ =\ 0,\ 1,\ \dots\ ,\ 34)$

Below are the constraints for the predetermined cities given in the question definition.

$A_{23} = 1,\ B_9 = 1,\ B_8 = 1,\ B_{10} = 1,\ B_{16} = 1,\ C_{11} = 1,\ D_{25} = 1$

$E_1 = 1,\ E_2 = 1,\ E_3 = 1,\ E_4 = 1,\ E_5 = 1$

$F_0 = 1,\ F_7 = 1,\ F_{14} = 1,\ F_{21} = 1,\ F_{28} = 1$

$G_6 = 1,\ G_{13} = 1,\ G_{20} = 1,\ G_{27} = 1,\ G_{34} = 1$

$B_{29} = 0,\ B_{30} = 0,\ B_{31} = 0,\ B_{32} = 0,\ B_{33} = 0$

$C_{29} = 0,\ C_{30} = 0,\ C_{31} = 0,\ C_{32} = 0,\ C_{33} = 0$

Region A should have at least 2 cities from the South side.

$A_{29} + A_{30} + A_{31} + A_{32} + A_{33} >= 2$

Counting the cities in the regions that have voted for us.

$SC_0 = \sum A_V$

$SC_1 = \sum B_V$

$SC_2 = \sum C_V$

$SC_3 = \sum D_V$

$SC_4 = \sum E_V$

$SC_5 = \sum F_V$

$SC_6 = \sum G_V$

S is a binary variable so if $SC_i - 2$ is less than or equal to 0 then $S$ is 0.

$S_0 = SC_0 - 2$

$S_1 = SC_1 - 2$

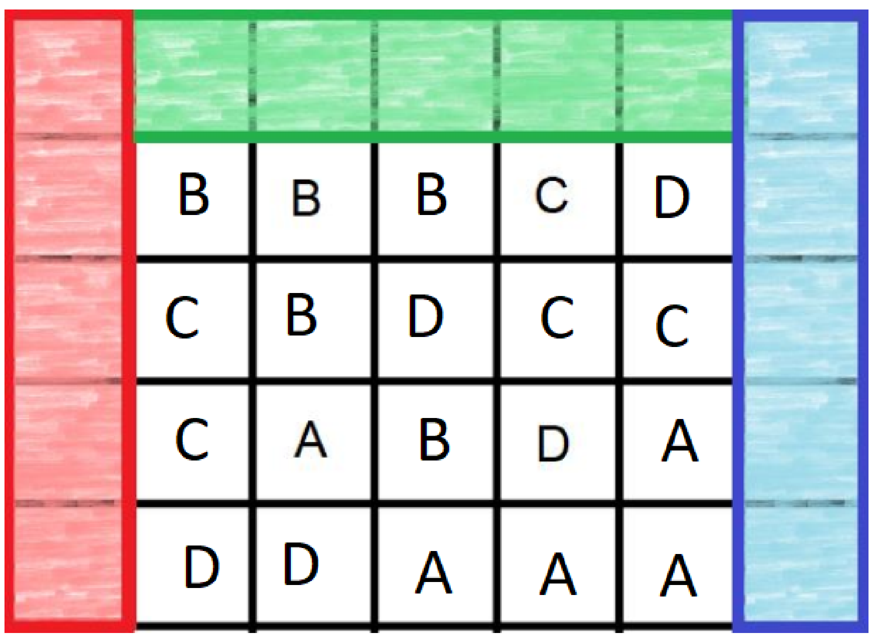
$S_2 = SC_2 - 2$

$S_3 = SC_3 - 2$

$S_4 = SC_4 - 2$

$S_5 = SC_5 - 2$

$S_6 = SC_6 - 2$




## Part C. Region Partitioning II

**Parameters:**

$V: Voted\ Cities = 0, 1, 2, 3, 7, 8, 9, 10, 11, 14, 17, 18, 19, 20, 25, 26, 27, 33, 34$

**Decision Variables:**

$S_i$: $1\ if\ Region\ i\ has\ a\ seat\ from\ out\ representative,\ 0\ otherwise$

$SC_j$: $Number\ of\ seats\ a\ region\ will\ have,\ j = 0, 1, \dots, 6$

$A_i, B_i, C_i, D_i, E_i, F_i, G_i$: $Binary\ variables\ representing\ whether\ a\ city\ belongs\ to\ a\ region$

$i = 0, 1, \dots, 34$

**The Objective Function:**

To maximize the number of seats.

$$max \sum_{i=0}^{6} S_i$$

**Constraints:**

These constraints ensure that every region must have exactly 5 cities.

$$\sum_{i=0}^{34} A_i = 5, \sum_{i=0}^{34} B_{i_i} = 5, \sum_{i=0}^{34} C_i = 5, \sum_{i=0}^{34} D_i = 5, \sum_{i=0}^{34} E_i = 5, \sum_{i=0}^{34} F_i = 5, \sum_{i=0}^{34} G_i = 5$$

This constraint limits the city so that it belongs only to one region.

$A_i + B_i + C_i + D_i + E_i + F_i + G_i <= 1$

Predetermined regions for west and east.

$F_0 = 1, F_7 = 1, F_{14} = 1, F_{21} = 1, F_{28} = 1$

$G_6 = 1, G_{13} = 1, G_{20} = 1, G_{27} = 1, G_{34} = 1$

Ensuring that every city in the region has at least 1 neighbor from the same region.

$A_{i-1} + A_{i+1} + A_{i+7} + A_{i-7} >= A_i \ (i = 0, 1, \dots, 34)$

$B_{i-1} + B_{i+1} + B_{i+7} + B_{i-7} >= B_i \ (i = 0, 1, \dots, 34)$

$C_{i-1} + C_{i+1} + C_{i+7} + C_{i-7} >= C_i \ (i = 0, 1, \dots, 34)$

$D_{i-1} + D_{i+1} + D_{i+7} + D_{i-7} >= D_i \ \ (i = 0, 1, \dots, 34)$

$E_{i-1} + E_{i+1} + E_{i+7} + E_{i-7} >= E_i \ \ (i = 0, 1, \dots, 34)$

Adding constraints to ensure that horizontal dominoes have at least one direct neighbor.

$A_{i-7} + A_{i-6} + A_{i+7} + A_{i+8} + A_{i-1} + A_{i+2} >= A_i \ \ (i = 0, 1, \dots, 34)$

$B_{i-7} + B_{i-6} + B_{i+7} + B_{i+8} + B_{i-1} + B_{i+2} >= B_i \ \ (i = 0, 1, \dots, 34)$

$C_{i-7} + C_{i-6} + C_{i+7} + C_{i+8} + C_{i-1} + C_{i+2} >= C_i \ \ (i = 0, 1, \dots, 34)$

$D_{i-7} + D_{i-6} + D_{i+7} + D_{i+8} + D_{i-1} + D_{i+2} >= D_i \ \ (i = 0, 1, \dots, 34)$

$E_{i-7} + E_{i-6} + E_{i+7} + E_{i+8} + E_{i-1} + E_{i+2} >= E_i \ \ (i = 0, 1, \dots, 34)$

Adding constraints to ensure that vertical dominoes have at least one direct neighbor.

$A_{i-7} + A_{i+14} + A_{i-1} + A_{i+6} + A_{i+1} + A_{i+8} >= A_i \ \ (i = 0, 1, \dots, 34)$

$B_{i-7} + B_{i+14} + B_{i-1} + B_{i+6} + B_{i+1} + B_{i+8} >= B_i \ \ (i = 0, 1, \dots, 34)$

$C_{i-7} + C_{i+14} + C_{i-1} + C_{i+6} + C_{i+1} + C_{i+8} >= A_i \ \ (i = 0, 1, \dots, 34)$

$D_{i-7} + D_{i+14} + D_{i-1} + D_{i+6} + D_{i+1} + D_{i+8} >= A_i \ \ (i = 0, 1, \dots, 34)$

$E_{i-7} + E_{i+14} + E_{i-1} + E_{i+6} + E_{i+1} + E_{i+8} >= A_i \ \ (i = 0, 1, \dots, 34)$

Counting the cities in the region that have voted for us.

$SC_0 = \sum A_V$

$SC_1 = \sum B_V$

$SC_2 = \sum C_V$

$SC_3 = \sum D_V$

$SC_4 = \sum E_V$

$SC_5 = \sum F_V$

$SC_6 = \sum G_V$

S is a binary variable so if $SC_i - 2$ is less than or equal to 0 then S is 0.

$S_0 = SC_0 - 2$

$S_1 = SC_1 - 2$

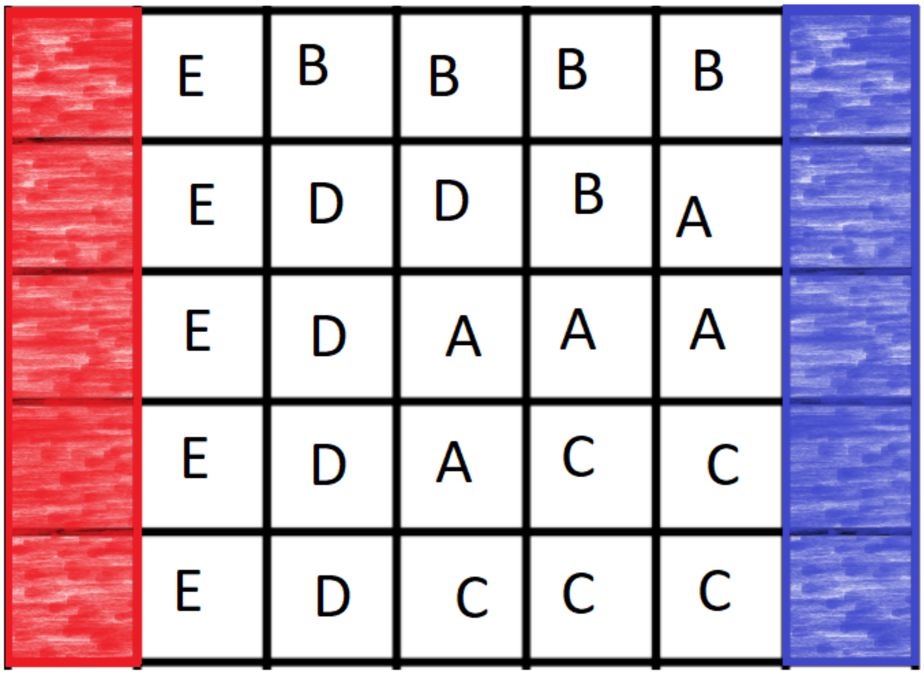
$S_2 = SC_2 - 2$

$S_3 = SC_3 - 2$

$S_4 = SC_4 - 2$

$S_5 = SC_5 - 2$

$S_6 = SC_6 - 2$




## Results of Each Part

**Part A:**

**Investment amount**

| | | | | | | |
|---|---|---|---|---|---|---|
| 9 | 19 | 26 | 6 | 0 | 0 | 0 |
| 22 | 29 | 26 | 29 | 10 | 0 | 0 |
| 7 | 0 | 0 | 29 | 31 | 15 | 10 |
| 0 | 0 | 0 | 0 | 6 | 29 | 33 |
| 0 | 0 | 0 | 0 | 0 | 6 | 16 |

Optimal total investment amount: 358

**Part B:**

The green cells in the next tables show cities that voted for us.

| | | | | | | |
|---|---|---|---|---|---|---|
| F | E | E | E | E | E | G |
| F | B | B | B | C | D | G |
| F | C | B | D | C | C | G |
| F | C | A | B | D | A | G |
| F | D | D | A | A | A | G |

We have 5 seats from B, C, E, F, G.

**Part C:**

| | | | | | | |
|---|---|---|---|---|---|---|
| F | E | B | B | B | B | G |
| F | E | D | D | B | A | G |
| F | E | D | A | A | A | G |
| F | E | D | A | C | C | G |
| F | E | D | C | C | C | G |

We have 5 seats from A, B, C, F, G.

## Contribution of Each Member

**Ahmet Faruk Ulutaş:** Member played an active role in writing the code and report of part a, part b, and part c.

**Seymur Abdulla:** Member played an active role in writing the code and report of part a, part b, and part c.

**Emin Berke Ay:** Member played an active role in writing the code and report of part a, part b, and part c.