The Impact of Connectivity on the Production and Diffusion of Knowledge

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Abstract

We study a social bandit problem featuring production and diffusion of knowledge. While higher connectivity enhances knowledge diffusion, it may reduce knowledge production as agents shy away from experimentation with new ideas and free-ride on the observation of other agents. As a result, under some conditions, greater connectivity can lead to homogeneity and lower social welfare.

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1 Introduction

Advances in travel and communication technologies have cleared the way for more connected organizations and societies. In well-connected structures, new ideas spread quickly leading to rapid innovation adoption.

While such enhanced knowledge diffusion is beneficial, it may come at the cost of reduced knowledge production. When an organization or society is well-connected, agents may shy away from experimentation with new ideas, since they can easily see the results of the experimentation efforts of other agents and adapt their actions accordingly. Because of this free-riding, more connected organizations or societies may become homogeneous, converging on an inferior technology, and having lower overall welfare than less connected organizations or societies.

We study this tension between knowledge diffusion and knowledge production in a simple two-period social bandit model. In each period, each agent has the choice between exploiting a safe well-known action or exploring a risky novel action. At the end of the first period, each agent observes the outcome of a randomly selected group of agents. We show that in equilibrium social welfare is not necessarily increasing in connectivity between agents. That is, in a better connected society or organization, in which each agent is likely to meet with a greater number of agents, the costs of free-riding on knowledge production may dominate the benefits of connectivity on knowledge diffusion, leading to lower social surplus.

We begin our analysis in Section 2 with a two-player economy. In this economy, equilibrium features three different regions based on initial beliefs: (i) both agents exploit; (ii) one agent exploits, while the other explores; (iii) both agents explore (Propositions 1 and 2). Due to free-riding, there is over-exploitation and under-exploration relative to the social optimum (Proposition 3). Moreover, equilibrium social surplus is non-monotonic in the connection probability between the two agents. For some intermediate levels of connectivity, an increase in the connection probability leads to lower equilibrium social surplus.

In Section 3, we study the n-agent economy, where in the second period each agent observes the exploration outcomes of a random group of contacts (denoted by M). The equilibria mirror the two-agent case: for high (respectively, low) initial beliefs, all players explore (respectively, exploit) in equilibrium (Theorem 1). For intermediate beliefs, the pure-strategy equilibrium is asymmetric, with some players exploring and the rest exploiting (Theorem 2), and there is a unique symmetric mixed-strategy equilibrium in which agents explore independently with equal probability (Theorem 3).

As the distribution of M increases in the sense of first-order stochastic dominance, free-riding incentives strengthen and the exploration threshold rises. This mirrors the comparative statics in Keller et al. (2005), who study how increasing the *number* of agents in a fully connected bandit game affects exploration behavior. Our framework additionally allows for a richer comparative static: mean-preserving spreads in the distribution of M encourage more private exploration and thus lower the equilibrium exploration threshold.

The equilibrium results of Section 3 apply to any ensemble of random networks. In Section 4, we apply them to economies with *local* and *global* connections based on Erdős-Rényi (ER) random graphs (Erdős and Rényi, 1960), where every pair of agents is connected independently with the same probability of λ/n, where λ denotes the expected number of connections per agent.

In the local ER economy, each agent only observes her *immediate* neighbors' outcomes; in the global case, her observable circle extends to all agents reachable via any path. For local economies, the tractability of Binomial processes yields a closed-form asymptotic expression for the pure-strategy equilibrium fraction of explorers (Proposition 4). The mixed-strategy equilibrium exploration probability converges to the same limit—an instance of the *exact law of large numbers* (Sun, 2006) (Proposition 5). In the global case, the exploration region tightens sharply as λ crosses 1, driven by the phase transition in ER graphs from components of size $O(\log n)$ to $O(n)$. This effect is more pronounced for radical innovation. We derive the scaling limits of the exploration threshold and show it behaves asymmetrically around $\lambda = 1$ (Theorem 4).

In Section 5, we study the pure- and mixed-strategy equilibrium social surplus in the local ER economy. As in the two-player case, the pure-strategy equilibrium social surplus is not monotone in connectivity (Proposition 7), and the equilibrium exhibits over-exploitation and under-exploration relative to the social optimum (Theorem 6).

Higher connectivity exacerbates free-riding, as agents who observe others' experimentation reduce their own exploration. The number of explorers weakly decreases in λ: constant within each equilibrium region and dropping by one at equilibrium regime-change thresholds, where the social surplus falls discontinuously. As $n \to \infty$, these drops vanish and the limiting per-capita surplus becomes weakly increasing and continuous in λ (Proposition 8). For intermediate initial beliefs, it even becomes *constant* above a connectivity threshold, where the informational gain from one additional explorer precisely offsets the exploration cost.

In Section 5.3, we characterize the solution to the planner's problem under mixed strategies

via an intuitive first-order condition (Lemma 8 and Theorem 7), and use stochastic order techniques to show that the equilibrium mixing probability always falls below the social optimum. The paper concludes in Section 6.

Related Literature. In his seminal work, Rothschild (1974) studies the *single-agent* experimentation problem in the two-armed bandit environment and shows that the agent settles on the sub-optimal arm with positive probability. The literature on multi-agent strategic experimentation begins with Bolton and Harris (1999) and Keller et al. (2005),[1] where players are completely connected and each observes the outcomes of *all* others. Our paper interpolates between these two extremes by considering agents who are neither fully connected nor fully isolated, allowing us to explicitly study how connectivity shapes exploration incentives.

While Keller et al. (2005) show that the exploration threshold increases with the number of players under complete observability, we show that it increases with the connectivity parameter under partial observability. More importantly, by varying connectivity while holding n fixed, we uncover a non-monotonicity of equilibrium social surplus in connectivity—a relationship that cannot be examined under complete observability, where connectivity is mechanically determined by the number of agents. We further extend our analysis to mean-preserving spreads (second-order stochastic dominance) in the distribution of contacts, a type of comparative statics unavailable in settings with deterministic observability.

Our work is closely related to two papers studying experimentation in networks. Board and Meyer-ter Vehn (2024) shows that social surplus is single-peaked in network density. While we share their finding that welfare is non-monotonic in connectivity, we work with random Erdős-Rényi graphs ($p = \lambda/n$), which enables tractable asymptotics and a clean separation between local and global information regimes, and we establish additional results on discontinuous welfare drops at regime-change thresholds and conditions under which limiting surplus becomes constant in connectivity. Salish (2015) studies a multi-period version of our model with richer strategic dynamics. We opt for a two-period framework, which allows us to derive closed-form exploration thresholds, characterize the limiting fraction of explorers as $n \to \infty$, and provide complete equilibrium-versus-optimum welfare comparisons—while still capturing the core tensions between knowledge production and diffusion.

Bala and Goyal (1998), Gale and Kariv (2003) and Sadler (2020) study the social learning

[1] A non-exhaustive list of related papers in strategic bandits includes Heidhues et al. (2015), Keller and Rady (2015), Bonatti and Hörner (2017), and Pourbabaee (2020).

dynamics of *myopic* agents who are connected in networks and collect information from their neighbors to maximize their *short-run* payoff. Our two-period experimentation framework is a first stab to depart from these works by letting agents to have long-run incentives in their strategic interactions.

Topics such as long-run social conformity and information aggregation in multi-agent strategic experimentation—when agents observe actions rather than payoffs—are studied in Chamley and Gale (1994), Aoyagi (1998), Rosenberg et al. (2007), Rosenberg et al. (2009), and Camargo (2014). Beyond the difference in observability, our paper focuses on how connectivity shapes equilibrium strategies and social welfare, rather than on long-run conformity or social learning.

Our paper also relates to the broader literature on games with information sharing and externalities. Duffie et al. (2009) studies a continuum economy where individuals endowed with informative signals incur costly search to meet and share information. Wolitzky (2018) investigates social learning and innovation adoption where agents arrive continuously over time, learn from a random sample of past outcomes, and make once-and-for-all decisions. In a Poisson news setting, Frick and Ishii (2024) studies how the arrival rate of a public signal—which depends on the mass of current adopters—affects innovation adoption. Somewhat related to the tension between private experimentation and information sharing, Gordon et al. (2021) show that increasing the observation delay among team members working on a joint project raises individual effort. In collective search settings, Lomys (2023) studies the relationship between learning outcomes and the structure of the connection graph, and Cetemen et al. (2023) study how the outcomes depend on the members' complementarity and alliances. In moral hazard settings, Bonatti and Hörner (2011) show that collaboration among agents diminishes over time and that the expected time to success increases with the number of agents. Lastly, the analysis of our paper on how connectivity impacts exploration incentives has implications for designing optimal policies to motivate innovation and exploration in networked economies (see, Manso, 2011; Kerr et al., 2014).

2 Two-Player Economy

In this section, we propose a simple model that captures the essence of equilibrium forces and provides intuition for the general case of $n > 2$ agents. We study perfect and imperfect connections between the two agents, followed by social surplus analysis. Propositions 1-3

follow from the general case in Sections 3 and 5, so we omit their proofs here.

2.1 Perfect Connections

There are two agents i and j, and the game consists of two periods, $t \in \{0, 1\}$. Each agent faces a binary action choice in each period: a safe action ($a = 0$, exploiting the status quo) with normalized payoff 0, or a risky action ($a = 1$, exploring the alternative). For the risky action, the return is a binary random variable $y \in \{-\alpha, 1\}$ (with $\alpha \in (0, 1)$) conditioned on the hidden state $\theta \in \{0, 1\}$, with the following structure:

$$\mathsf{P}\left(y = 1 \mid \theta = 1\right) = \beta \in (0, 1), \text{ and } \mathsf{P}\left(y = 1 \mid \theta = 0\right) = 0\,.$$

Therefore, receiving a high payoff $y = 1$ perfectly reveals the underlying state θ, as the probability of a high payoff in the low state is zero. The realized payoff also reveals the action choice: observing a payoff of 0 (respectively, nonzero) implies the agent took the safe (respectively, risky) action.

Let $\pi = \mathsf{P}(\theta = 1)$ be the initial prior of both players. The following timeline elucidates the order of events in this two-period economy:

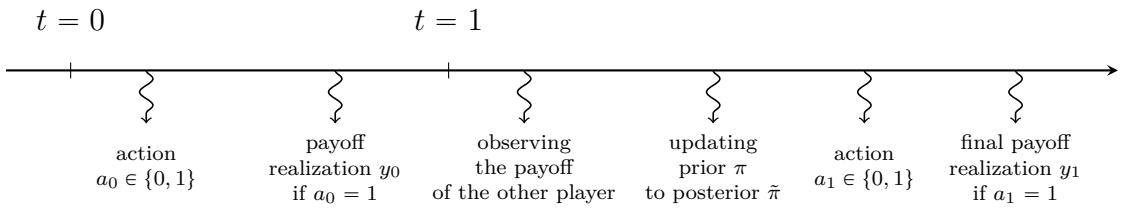


Figure 1: Timeline of the two-period bandit

At the beginning of the second period, agent i observes the outcome of agent j's first period experimentation if j chose the risky arm. This communication step is the main focus throughout the paper. Agent i then updates her prior about θ given $y_0(i)$ and $y_0(j)$, yielding posterior $\tilde{\pi}$. Let $\pi_{\ell,m}$ denote the posterior when agent i observes $m \in \{0, 1, 2\}$ payoff signals, of which $\ell \in \{0, 1, 2\}$ are high realizations ($y = 1$), where $\ell \leqslant m$. Then,

$$\pi_{\ell,m} = 1_{\{\ell \geqslant 1\}} + \frac{\pi(1-\beta)^m}{1-\pi+\pi(1-\beta)^m}\, 1_{\{\ell=0\}}\,.$$

Before observing $y_0(i)$ and $y_0(j)$, $\tilde{\pi}$ is a random variable with ex post values $\pi_{\ell,m}$.

The game ends with each agent making a second action choice. Since the safe option is always available, the expected payoff *after* Bayesian updating with posterior $\tilde{\pi}$ is

$$\left(\tilde{\pi}\beta - \alpha(1-\tilde{\pi}\beta)\right)^+ \coloneqq \max\{\tilde{\pi}\beta(1+\alpha) - \alpha, 0\}\,.$$

Thus, unlike the strategic first period choice a_0, the second period decision a_1 follows a simple threshold rule: choose the risky arm if and only if $\tilde{\pi} > \frac{\alpha}{\beta(1+\alpha)}$ (ties favor the safe arm).

Definition 1. Denote the second period's exploration threshold by $\tau \coloneqq \frac{\alpha}{\beta(1+\alpha)}$.

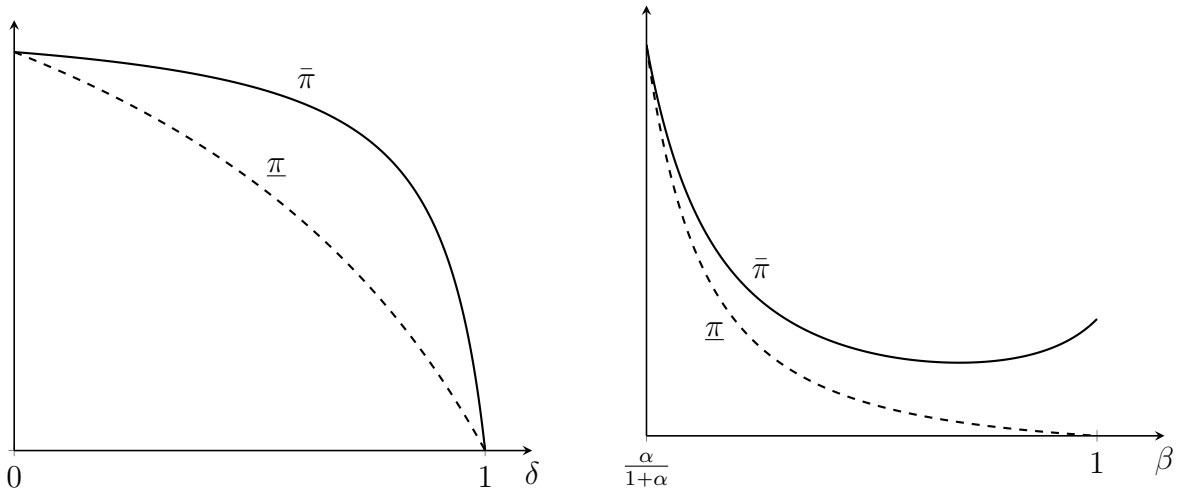
There will be two types of *symmetric* pure-strategy equilibrium: *exploration* equilibrium in which both agents choose the risky arm in the first period (i.e., $a_0(i) = a_0(j) = 1$), and *exploitation* equilibrium where both agents select the safe arm in the first period (i.e., $a_0(i) = a_0(j) = 0$). The equilibrium is called *asymmetric* when one agent explores and the other one exploits (thus $a_0(i) \neq a_0(j)$). Let $\delta \in [0,1]$ be the time discount factor, that is each agent values the payoffs in the first and second periods with the respective weights of $1-\delta$ and δ. This means that our agents are not myopic and they incorporate future gains from current exploration in their decision problem.[2]

Proposition 1. *There exist two thresholds $\underline{\pi} < \bar{\pi}$ such that the exploitation equilibrium appears on $[0, \underline{\pi}]$, and the exploration equilibrium appears on $(\bar{\pi}, 1]$. In the intermediate region $(\underline{\pi}, \bar{\pi}]$ the equilibrium is asymmetric with only one agent exploring. Closed form expressions for the cutoffs are*

$$\underline{\pi} = \frac{\alpha(1-\delta)}{\beta\left((1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta)\right)}\,, \quad \bar{\pi} = \frac{\alpha(1-\delta)}{\beta\left((1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta))(1-\beta)\right)}\,. \tag{2.1}$$

This result shows that the equilibrium number of explorers is weakly increasing in the initial belief. Two important comparative statics are the effects of patience (δ) and signal precision (β) on these thresholds. As it appears from Figure 2a higher patience (namely higher δ) is associated with smaller exploration thresholds, thereby increasing the incentives to sacrifice the present payoff to learn about the risky arm and gain the benefits in the next period. Specifically, higher patience enlarges the exploration equilibrium region and shrinks the exploitation region.

[2]This is in contrast to the social learning models of Bala and Goyal (1998) and Sadler (2020) in which players are myopic. In particular, they collect information from their neighbors just to maximize their *present* period payoff.



(a) Effect of δ

(b) Effect of β

Figure 2: Comparative statics of thresholds

Higher uncertainty about the risky arm (namely intermediate levels of β) is associated with higher gains from exploration, and hence lower exploration threshold. Figure 2b confirms this intuition. In addition, higher β increases the exploration gain upon receiving conclusive signals about θ more so than it raises the opportunity cost of exploration absent such signals. Therefore, it lowers the individual's incentive to exploit the safe arm, thereby shrinking the exploitation region (see $\underline{\pi}$ in Figure 2b).

2.2 Imperfect Connections

Suppose the connection between the two players is *imperfect*. That is, each agent observes the outcome of the other agent's first period experimentation, and thereby their action choice, with probability p. In the next proposition, we show that such imperfect communication does not impact the exploitation region and *expands* the exploration region.

Proposition 2. *In the presence of imperfect connections ($p < 1$), there exist two thresholds $\underline{\pi} < \bar{\pi}$ such that the exploitation equilibrium appears on $[0, \underline{\pi}]$, and the exploration equilibrium appears on $(\bar{\pi}, 1]$. In the intermediate region $(\underline{\pi}, \bar{\pi}]$ the equilibrium is asymmetric with only one agent exploring. Closed form expressions for the cutoffs are*

$$\underline{\pi} = \frac{\alpha(1-\delta)}{\beta\left((1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta)\right)}, \quad \bar{\pi} = \frac{\alpha(1-\delta)}{\beta\left((1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta)(1-p\beta))\right)}.$$

The key result is that $\mathrm{d}\bar{\pi}/\mathrm{d}p > 0$: weaker ties correspond to higher exploration in this two-player economy. Stronger connections increase free-riding incentives, reducing first period exploration and thus raising the belief threshold required for exploring the risky arm.

2.3 Social Surplus

So far we have analyzed the equilibrium response in the two-player bandit game with imperfect connections. We now investigate when a "benevolent" planner prescribes exploitation or exploration by both agents.

Proposition 3. *The socially optimal outcome is for both players to exploit the safe arm whenever $\pi \leqslant \underline{\pi}^*$, and to jointly explore the risky arm on $\pi \geqslant \bar{\pi}^*$, where*

$$\underline{\pi}^* = \frac{\alpha(1-\delta)}{\beta\left[(1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta))(1+p)\right]},$$
$$\bar{\pi}^* = \frac{\alpha(1-\delta)}{\beta\left[(1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta))\,(1+p(1-2\beta))\right]}.$$

The substantial lesson from this proposition is that the equilibrium outcome features over-exploitation ($\underline{\pi} > \underline{\pi}^*$) and under-exploration ($\bar{\pi} > \bar{\pi}^*$) relative to the social optimum.

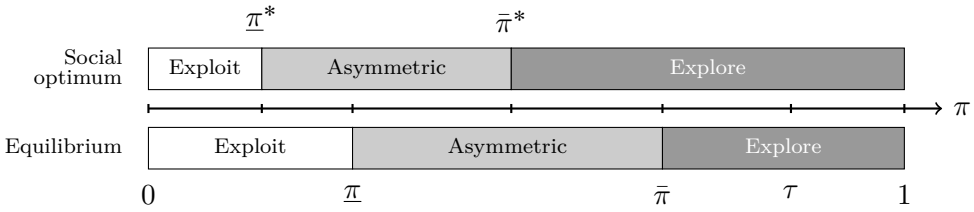
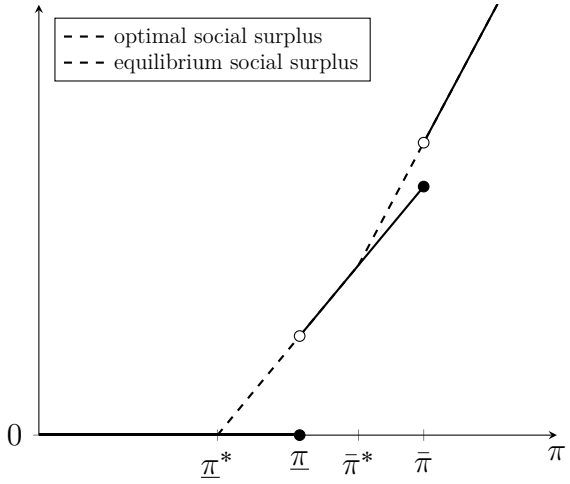


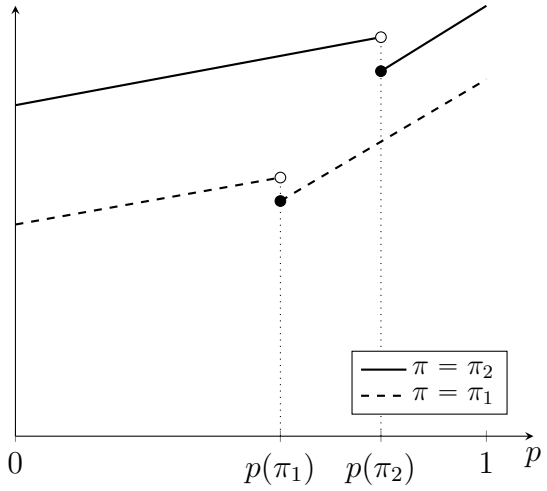
Figure 3: Equilibrium and optimum thresholds on the unit interval; $\underline{\pi}$ is the equilibrium exploitation cutoff; $\bar{\pi}$ is the equilibrium exploration cutoff; $\underline{\pi}^*$ is the optimal exploitation cutoff; $\bar{\pi}^*$ is the optimal exploration cutoff; τ is the second-period exploration cutoff; In equilibrium (bottom), over-exploitation ($\underline{\pi} > \underline{\pi}^*$) and under-exploration ($\bar{\pi} > \bar{\pi}^*$) shift the thresholds rightward relative to the social optimum.

We wrap up this section by investigating the effect of the connection probability p on equilibrium social surplus. Social surplus is *increasing* in p within each equilibrium region, but drops discontinuously at the regime change from full exploration to asymmetric equilibrium, as illustrated in Figure 4 for two levels of initial beliefs $\pi_2 > \pi_1$. Let $p(\pi)$ be the level at which $\bar{\pi}(p) = \pi$. When $p < p(\pi)$, full exploration prevails and social surplus increases with p. Once p surpasses $p(\pi)$, the number of explorers drops from two to one, causing the discontinuous

fall. Thereafter, raising p increases social surplus by enhancing information sharing without altering free-riding incentives.



(a) Optimal vs. equilibrium social surplus



(b) Equilibrium social surplus with respect to the connection probability p for levels of initial belief $\pi_1 < \pi_2$

Figure 4: Social surplus

In Section 5.1, we show that this pattern—increasing in connection probability within each regime, with discontinuous drops at regime changes—is robust in economies with many players.

3 Equilibria with Multiple Agents

We study an economy with n ex ante identical agents who share a common initial belief π. In the second period, each agent observes the exploration outcomes of a randomly selected group of size M. The realization of this group is unknown in the first period; agents know only the distribution of M, which is common across all players.

Aside from the above, we impose no further structure on the connection graph. Our results in Sections 3.1 and 3.2 depend only on the probability-generating function of M. We characterize pure-strategy equilibria as a function of π: for high (respectively, low) initial beliefs, the pure-strategy equilibrium is symmetric with full exploration (respectively, exploitation); at intermediate beliefs, the pure-strategy equilibrium is asymmetric, featuring both explorers and exploiters. On this intermediate region, we also identify the unique

symmetric mixed-strategy equilibrium and study its comparative statics.

In Section 3.3, we apply these results to Erdős-Rényi random graphs and analyze the limiting equilibrium properties as $n \to \infty$.

3.1 Symmetric Pure-Strategy Equilibria

We study two symmetric pure-strategy equilibria: exploitation and exploration. The exploitation equilibrium occurs when *all* players choose the safe arm in the first period, and prevails whenever the initial belief falls below the threshold $\underline{\pi}$ in equation (2.1). One can verify this by comparing an individual's exploitation payoff when everyone else exploits ($w_0(\pi)$) with her exploration payoff as the sole explorer ($v_1(\pi)$). This implies $w_0(\pi) \geqslant v_1(\pi)$ whenever $\pi \leqslant \underline{\pi}$. Thus, the exploitation equilibrium condition remains unchanged despite having more than two players and imperfect connections.

The more intriguing case is the examination of the existence of the exploration equilibrium in which *all* agents choose the risky arm in the first period.

Define $v_k(\pi)$ and $w_k(\pi)$ as an agent's expected payoff from exploration and exploitation in the first period when, in total, k agents explore. Thus the exploration equilibrium appears if and only if $v_n(\pi) > w_{n-1}(\pi)$. We reduce this inequality and prove the next theorem in the appendix.

Theorem 1 (Exploration equilibrium)**.** *Let M be the size of the random group of contacts in the second period. Then, the exploration equilibrium appears on $\pi > \bar{\pi}$, where*

$$\bar{\pi} = \frac{\alpha(1-\delta)}{\beta\left((1+\alpha)(1-\delta)+\delta\left(\beta-\alpha(1-\beta)\right)\mathsf{E}\left[(1-\beta)^M\right]\right)}. \tag{3.1}$$

Our key comparative static studies how connection sparsity shifts the exploration threshold. Below, we consider (i) first-order stochastic dominance (FOSD) shifts and (ii) mean-preserving spreads (SOSD) of the distribution of M.

FOSD shift. Since $x \mapsto (1-\beta)^x$ is decreasing, a FOSD increase in the distribution of M lowers $\mathsf{E}\left[(1-\beta)^M\right]$. Hence the exploration threshold rises (the exploration region shrinks): this means denser connections strengthen free-riding.

SOSD shift. Fix $\beta \in (0,1)$ and let $M'' \geq_2 M'$ (i.e., M'' second-order stochastically dominates M'). By definition of SOSD, for every nondecreasing and concave function f, one

has

$$\mathsf{E}\left[f(M'')\right] \geqslant \mathsf{E}\left[f(M')\right].$$

Take $f(x) \coloneqq -(1-\beta)^x$. Since $1-\beta \in (0,1)$,

$$f'(x) = -\ln(1-\beta)\,(1-\beta)^x > 0 \quad \text{and} \quad f''(x) = -\big(\ln(1-\beta)\big)^2(1-\beta)^x < 0,$$

so f is nondecreasing and concave, implying that,

$$\mathsf{E}\Big[(1-\beta)^{M''}\Big] \leqslant \mathsf{E}\Big[(1-\beta)^{M'}\Big].$$

Therefore, if $\widetilde{M}$ is a mean-preserving spread of M (so that $M \succeq_2 \widetilde{M}$), then $\bar{\pi}(\widetilde{M}) \leqslant \bar{\pi}(M)$. In other words, greater second-order risk (a mean-preserving spread) lowers the exploration threshold, making full first-period exploration more attractive.

3.2 Asymmetric Pure-Strategy Equilibria (Intermediate Region)

Previously, we characterized symmetric equilibria with full exploration or full exploitation. Here we study the intermediate region $\pi \in (\underline{\pi}, \bar{\pi}]$, where a pure-strategy equilibrium can feature both explorers and exploiters. Let $k \in \{1, \ldots, n-1\}$ be the equilibrium number of explorers. Such an equilibrium requires that (i) explorers prefer to keep exploring rather than deviate to exploitation, i.e., $v_k(\pi) > w_{k-1}(\pi)$, and (ii) exploiters prefer to keep exploiting rather than deviate to exploration, i.e., $w_k(\pi) \geqslant v_{k+1}(\pi)$. As a first step toward analyzing such equilibria, we show that for large values of π, the second incentive constraint (i.e., $w_k(\pi) \geqslant v_{k+1}(\pi)$) fails.

Lemma 1. *If $\pi > \tau$, then $w_k(\pi) < v_{k+1}(\pi)$ for all k.*

This lemma implies that a pure-strategy equilibrium with a nonzero number of exploiters cannot exist when $\pi > \tau$. Hence, to characterize intermediate equilibria, we restrict attention to $\pi \leqslant \tau$. The next theorem provides the belief cutoffs supporting an equilibrium with k explorers.

Theorem 2 (Asymmetric pure-strategy equilibrium)**.** *An asymmetric equilibrium with $k \in \{1, \ldots, n-1\}$ explorers exists if and only if $\pi \in (\pi_{k-1,n}, \pi_{k,n}]$, where the cutoffs are given by*

$$\pi_{k,n} \coloneqq \frac{\alpha(1-\delta)}{\beta\left((1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta))\mathsf{E}_k\left[(1-\beta)^M\right]\right)}. \tag{3.2}$$

The lower cutoff ($\pi > \pi_{k-1,n}$) follows from the explorers' incentive constraint $v_k > w_{k-1}$, while the upper cutoff ($\pi \leqslant \pi_{k,n}$) follows from the exploiters' constraint $w_k \geqslant v_{k+1}$. Deriving these conditions parallels the steps in Theorem 1, so we omit the proof..

As a result, the asymmetric equilibrium with k explorers occurs whenever $\pi_{k-1,n} < \pi \leqslant \pi_{k,n}$. Full exploitation prevails for $\pi \leqslant \pi_{0,n} \equiv \underline{\pi}$, and full exploration for $\pi > \pi_{n-1,n} \equiv \bar{\pi}_n$.[3]

Comparative statics of equilibrium k in π. Let $M_k^{(n)}$ denote the number of second-period contacts when n agents are present and k explore in period 1.[4] A simple stochastic dominance argument shows $M_k^{(n)} \geq_1 M_{k-1}^{(n)}$, hence $\pi_{k-1,n} \leqslant \pi_{k,n}$. Therefore, the equilibrium number of explorers (k) is weakly increasing in π.[5]

3.3 Symmetric Mixed-Strategy Equilibrium (Intermediate Region)

In the previous two sections, we characterized pure-strategy equilibria as a function of the initial belief π. Section 3.1 shows that at the two ends, i.e., on $[0, \underline{\pi}]$ and $(\bar{\pi}, 1]$, the unique equilibrium is full exploitation and full exploration, respectively. Section 3.2 implies that on the intermediate region $(\underline{\pi}, \bar{\pi}]$, the unique pure-strategy equilibrium is asymmetric, featuring both explorers and exploiters. This leaves open the possibility of mixed-strategy equilibria on the intermediate region.

Hence in this section, we study the *symmetric* mixed-strategy equilibrium in the intermediate region. Suppose that each agent explores the risky arm with probability μ. This will be a mixed-strategy equilibrium if the expected payoff from exploitation, namely

$$w(\pi;\mu) \coloneqq \sum_{k=0}^{n-1} \binom{n-1}{k} \mu^k (1-\mu)^{n-1-k} w_k(\pi)\,,$$

matches the expected payoff from exploration, that is

$$v(\pi;\mu) \coloneqq \sum_{k=0}^{n-1} \binom{n-1}{k} \mu^k (1-\mu)^{n-1-k} v_{k+1}(\pi)\,.$$

[3] We use the subscript n under the exploration threshold $\bar{\pi}$ when we want to be explicit about the total number of players.

[4] Depending on the context, we either use $M_k^{(n)}$ or explicitly specify the indices on the expectation operator, that is e.g., $\mathsf{E}_k^{(n)}$.

[5] The term 'weakly' is used because the equilibrium number of explorers is constant on each interval $(\pi_{k-1,n}, \pi_{k,n}]$.

Theorem 3 (Symmetric mixed-strategy equilibrium)**.** *For $\pi \in (\underline{\pi}, \bar{\pi}]$ (the intermediate region), there exists a unique symmetric mixed-strategy equilibrium. Moreover, the equilibrium probability of exploration, μ^e, is increasing in π and decreasing in n.*

The theorem has two key implications. First, it implies that the symmetric mixed-strategy equilibrium is unique. Second, it implies that the equilibrium exploration probability decreases with the number of agents, consistent with free-riding incentives in the mixed-strategy setting.

4 Large-n Limits of Equilibria in Erdős-Rényi Graphs

For the first time in the paper, we impose a restriction on the random graph structure: each pair of agents is connected with probability $p = \lambda/n$, where λ is the average number of immediate neighbors.

The random group of contacts observed in period two can be (i) immediate neighbors (the **local** case), or (ii) all agents in the connected component (the **global** case). In the global case, an agent observes signals from everyone in her component $\mathcal{C}$, with $|\mathcal{C}| = M+1$.[6] In the global case, we interpret period two as a sequence of message-passing sub-periods along paths in the graph.

We first study local connections, where $M = D$ (the degree), and then global connections, where $M = |\mathcal{C}|-1$ (component size minus one). In both cases we focus on limits as $n \to \infty$ and the effect of λ on equilibrium outcomes.

4.1 Local Connections

Recall that in the local regime $M = D$, the degree of a randomly drawn agent, follows a Binomial distribution $\mathsf{Bin}(n-1, p)$. For constant λ, this distribution converges weakly to $\mathsf{Poisson}(\lambda)$, and therefore the limiting threshold for the full exploration equilibrium in the local regime is:

$$\begin{aligned} \bar{\pi}_\infty^{\text{local}} \coloneqq \lim_{n\to\infty} \bar{\pi}_n^{\text{local}} &= \lim_{n\to\infty} \frac{\alpha(1-\delta)}{\beta\left((1-\delta)(1+\alpha)+\delta\left(\beta-\alpha(1-\beta)\right)\mathsf{E}\left[(1-\beta)^D\right]\right)} \\ &= \frac{\alpha(1-\delta)}{\beta\left((1-\delta)(1+\alpha)+\delta\left(\beta-\alpha(1-\beta)\right)e^{-\lambda\beta}\right)}, \end{aligned} \tag{4.1}$$

[6] Each player's connected component includes herself; hence in the *latter* case, M denotes the number of *other* players connected to the focal agent.

where the last equality uses the probability-generating function of the Poisson distribution. Observe that as $\lambda \to \infty$, $\bar{\pi}^{\text{local}} \uparrow \tau$.

Lemma 2. *In the local regime, the exploration threshold $\bar{\pi}_n$ is eventually increasing in n and converges to $\bar{\pi}_\infty^{\text{local}}$ defined in (4.1).*

This lemma implies that as the number of agents increases, free-riding incentives become stronger, albeit at a diminishing rate.

In the next lemma, we study the exploration cutoffs in the intermediate region that characterize the asymmetric equilibria (see Theorem 2).

Lemma 3. *In the local regime, for a fixed $k \in \mathbb{N}$ and sufficiently large n, the following ordering holds: $\pi_{k-1,n} \leqslant \pi_{k,n+1} \leqslant \pi_{k,n} \leqslant \pi_{k+1,n+1}$.*

This lemma demonstrates that for large economies with local connections adding one more individual *never* results in fewer exploring agents in the equilibrium. Put differently, the existing agents do not switch to the exploitation status when new individuals join the economy.[7]

Let k_n denote the equilibrium number of exploring agents under pure strategies. The following proposition shows that in an ER economy with local connections, the equilibrium fraction of explorers k_n/n converges as $n \to \infty$. The proof uses the incentive condition (3.2) to establish matching upper and lower bounds for k_n.

For convenience and to avoid repetition, we use the following definition.

Definition 2. Define the functions $C_1(\cdot)$ and $C_2(\cdot)$ as

$$C_1(\pi) \coloneqq -(1-\delta)\big(\pi\beta - \alpha(1-\pi\beta)\big)\,, \qquad C_2(\pi) \coloneqq \delta\pi(\beta - \alpha(1-\beta))\beta\,,$$

both positive for $\pi \leqslant \tau$.

Proposition 4 (Limiting fraction of explorers)**.** *Let $k_n(\pi)$ be the equilibrium number of exploring agents in an Erdős-Rényi economy of n individuals with local connections, then:*

$$\lim_{n\to\infty} \frac{k_n(\pi)}{n} = \kappa(\pi) \coloneqq \begin{cases} 0 & \pi \leqslant \underline{\pi} \\ \frac{1}{\lambda\beta} \log \frac{C_2(\pi)}{C_1(\pi)} & \underline{\pi} < \pi < \bar{\pi}_\infty^{\text{local}} \\ 1 & \pi \geqslant \bar{\pi}_\infty^{\text{local}} \end{cases} \tag{4.2}$$

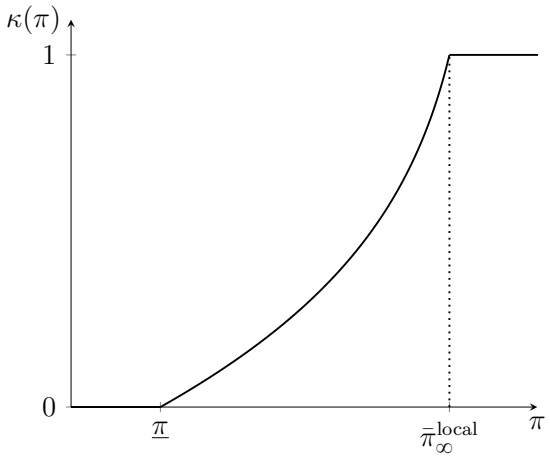


Figure 5: Limiting fraction of explorers

Figure 5 illustrates the limiting fraction of exploring agents, $\kappa(\pi)$, as a function of the initial belief π. The function exhibits two kinks at $\underline{\pi}$ and $\bar{\pi}_\infty^{\text{local}}$, corresponding to equilibrium regime changes from full exploitation to the intermediate asymmetric region and then to full exploration. Notably, the graph is convex, indicating that the equilibrium fraction of explorers grows at an increasing rate as the initial belief rises.

We next characterize the finite-n equilibrium exploration probability μ_n^e in the intermediate region $[\underline{\pi}, \bar{\pi}_n^{\text{local}}]$ where the symmetric equilibrium is mixed (Theorem 3), and then find its limit as $n \to \infty$.

Proposition 5 (Equilibrium mixing probability)**.** *For finite n and every $\pi \in [\underline{\pi}, \bar{\pi}_n^{\text{local}}]$,*

$$\mu_n^e = \frac{n}{\lambda\beta}\left[1 - \left(\frac{C_1(\pi)}{C_2(\pi)}\right)^{1/(n-1)}\right], \tag{4.3}$$

with limit

$$\lim_{n\to\infty} \mu_n^e = \frac{1}{\lambda\beta} \log \frac{C_2(\pi)}{C_1(\pi)}. \tag{4.4}$$

This proposition shows, interestingly, that the limiting exploration probability along a sequence of mixed-strategy equilibria equals the asymptotic fraction of explorers along a sequence of pure-strategy equilibria—an instance of the *exact law of large numbers* in the sense of Sun (2006).

[7] It is important to note that this conclusion primarily rests on keeping the average degree λ constant while expanding the size of the economy.

4.2 Global Connections

The analysis in the global regime (where $M = |\mathcal{C}|-1$) is more intricate. In this regime, an agent observes all members of her connected component in the second period. This setting offers greater potential for knowledge diffusion due to the increased number of connections, but also creates stronger free-riding incentives, leading to lower levels of exploration.

A coupling argument (e.g., Theorem 2.1 in Bollobás (2001)) shows that the distribution of the connected component size $|\mathcal{C}|$ of a *typical* vertex in an ER random graph is first-order stochastically increasing in λ. Since $x \mapsto (1-\beta)^x$ is decreasing, equation (3.1) implies that the exploration threshold $\bar{\pi}$ is increasing in λ, confirming the presence of free-riding in the global case.

To study the limiting behavior of the exploration threshold, we need the asymptotic distribution of $|\mathcal{C}|$, the size of a typical component. A classical result in random graph theory establishes that the giant component undergoes a phase transition: its size is $O(\log n)$ in the subcritical regime ($\lambda < 1$) and $O(n)$ in the supercritical regime ($\lambda > 1$). This phenomenon in turn impacts the distribution of the size of a *typical* (not necessarily giant) component containing a uniformly chosen vertex.

Let $B(\lambda)$ denote the *total* number of descendants in a branching process with $\mathsf{Poisson}(\lambda)$ offspring distribution. In the (sub)critical regime ($\lambda \leqslant 1$), its distribution is known as the $\mathsf{Borel}(\lambda)$ distribution:

$$\mathsf{P}\left(B(\lambda) = k\right) = \frac{\mathrm{e}^{-\lambda k}(\lambda k)^{k-1}}{k!}$$

In the supercritical regime ($\lambda > 1$), the total population becomes extinct with probability $\zeta(\lambda)$, which is the unique positive solution to the following fixed-point relation:

$$\zeta = \mathrm{e}^{-\lambda(1-\zeta)} \tag{4.5}$$

In the supercritical regime, a uniformly random vertex falls in the giant (non-extinct) component with probability $1-\zeta$, and hence the asymptotic size of its component is ∞. With probability ζ, it belongs to a finite-size extinct component whose size follows $\mathsf{Borel}(\lambda\zeta)$. We use the following notation to refer to this mixture distribution:

$$\mathsf{Borel}_{\zeta}(\lambda) \coloneqq \begin{cases} \infty & \text{with probability } 1-\zeta \\ \mathsf{Borel}(\lambda\zeta) & \text{with probability } \zeta \end{cases}$$

We can further define $\zeta(\lambda) \equiv 1$ for all $\lambda \leqslant 1$, so that the standard Borel distribution can be represented as a special case of the mixture Borel: $\mathsf{Borel}(\lambda) = \mathsf{Borel}_{\zeta(\lambda)}(\lambda)$. We denote a random variable with the standard Borel distribution by $B(\lambda)$, and one with the mixture Borel distribution with extinction probability ζ by $B_\zeta(\lambda)$.

Using these observations, we completely characterize the asymptotic distribution of the size of a typical connected component $|\mathcal{C}|$ in the following proposition:

Proposition 6. *Let $p = \lambda/n$, and denote the size of a typical connected component by $|\mathcal{C}|$. Then:*

(i) $|\mathcal{C}|$ converges in distribution to $\mathsf{Borel}_{\zeta(\lambda)}(\lambda)$.

(ii) As $n \to \infty$, the full exploration threshold converges to

$$\bar{\pi}_\infty^{\text{global}} \coloneqq \lim_{n\to\infty} \bar{\pi}_n^{\text{global}} = \frac{\alpha(1-\delta)}{\beta\left((1+\alpha)(1-\delta)+\delta\left(\beta-\alpha(1-\beta)\right)\mathsf{E}\left[(1-\beta)^{B_{\zeta(\lambda)}(\lambda)-1}\right]\right)}. \tag{4.6}$$

To further analyze $\bar{\pi}_\infty^{\text{global}}$ derived from equation (4.6), we need to determine the probability-generating function (henceforth, PGF) of the mixture Borel distribution $B_{\zeta(\lambda)}(\lambda)$. In the following lemma, we characterize that through a fixed-point argument.

Lemma 4 (Alon and Spencer (2000))**.** *Denote the probability-generating function (PGF) of the mixture Borel distribution by $\psi_\lambda(z) \coloneqq \mathsf{E}\left[z^{B_{\zeta(\lambda)}(\lambda)}\right]$ for $z \in [0,1]$, where $\zeta(\lambda)$ solves (4.5). Then $\psi(z)$ satisfies the following fixed-point relation:*

$$\psi_\lambda(z) = z\mathrm{e}^{\lambda(\psi_\lambda(z)-1)} \tag{4.7}$$

Rapid fall of exploration in the global regime (small β and $\lambda \approx 1$). As Figure 6 illustrates, there is a rapid tightening of the exploration region in the case of global connections as λ increases from values slightly below 1 to values just above. More precisely, the marginal effect of increasing λ on the exploration threshold $\bar{\pi}_\infty^{\text{global}}$ undergoes a significant shift at $\lambda = 1$. This effect is more significant when β is close to zero, which is the most relevant region in the innovation and entrepreneurship research, when the probability of success is extremely small.

The emergence of a giant connected component for $\lambda > 1$ is a classic result in random graph theory (Erdős and Rényi, 1960). In our setting, this connects the focal agent to a component of size $\Theta(n)$, expands the scope for free-riding, and raises the global exploration threshold $\bar{\pi}_\infty^{\text{global}}$. For $\lambda < 1$, components remain small—$O(\log n)$—so free-riding is limited

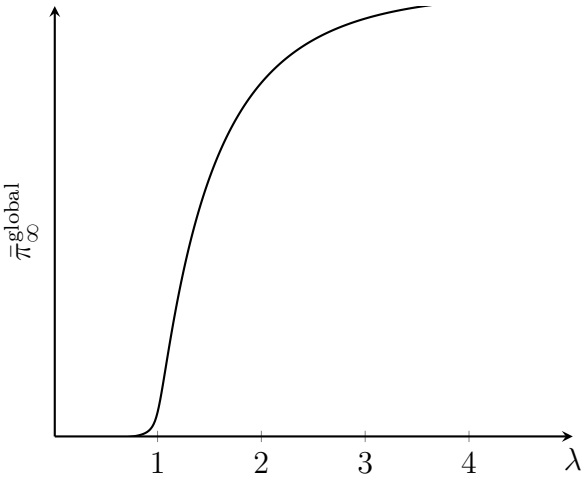


Figure 6: Rapid tightening of the exploration region $[\delta = 0.1, \alpha = 0.001, \beta = 0.002]$

and the threshold stays low. The key implication is that $\bar{\pi}_\infty^{\text{global}}$ changes sharply near $\lambda = 1$: as $\beta \to 0$, the change in the threshold is concentrated in a critical window of width $\Theta(\sqrt{\beta})$ and magnitude $\Theta(\beta^{3/2})$. The next theorem formalizes this scaling and shows that, after rescaling, the threshold curves collapse onto a single explicit "crossover function".

Theorem 4 (Scaling limits)**.** *Let $\beta \to 0$ with $\tau(\alpha, \beta) = \alpha/\beta(1+\alpha) \to \tau_0 \in (0, 1)$. Then for every fixed $x \in \mathbb{R}$:*

$$\lim_{\beta \downarrow 0} \beta^{-3/2}\Big(\bar{\pi}_\infty^{\text{global}}(1+x\sqrt{\beta}, \beta) - \bar{\pi}_\infty^{\text{global}}(1, \beta)\Big) = \frac{\tau_0(1-\tau_0)\delta}{1-\delta}\left(c(x) - \sqrt{2}\right),$$

where $c(x) \coloneqq x + \sqrt{x^2+2}$.

Figure 7 plots the crossover function $c(x) = x + \sqrt{x^2+2}$, which governs the behavior of $\bar{\pi}_\infty^{\text{global}}$ near the phase transition of ER random graphs. The proposition identifies a critical window $\lambda = 1 + x\sqrt{\beta}$ within which $\bar{\pi}_\infty^{\text{global}}$ shifts by $\Theta(\beta^{3/2})$; after rescaling by $\beta^{-3/2}$, the entire family of threshold curves collapses onto the closed-form profile $c(x) - \sqrt{2}$.

Outside this window, the PGF term driving the denominator of $\bar{\pi}_\infty^{\text{global}}$ is essentially stable. For $\lambda < 1$, the absence of a giant component means extra links barely affect the component-size distribution, so the threshold is nearly flat—consistent with $c(x) \to 0$ as $x \to -\infty$. For $\lambda > 1$, a giant component already exists and further increases in λ affect the PGF only gradually, yielding the approximately linear right tail $c(x) \sim 2x$ as $x \to +\infty$.

The crossover function is *asymmetric* around $x = 0$: it flattens quickly for $x < 0$ but continues to rise for $x > 0$. This reflects the underlying phase transition—crossing $\lambda = 1$

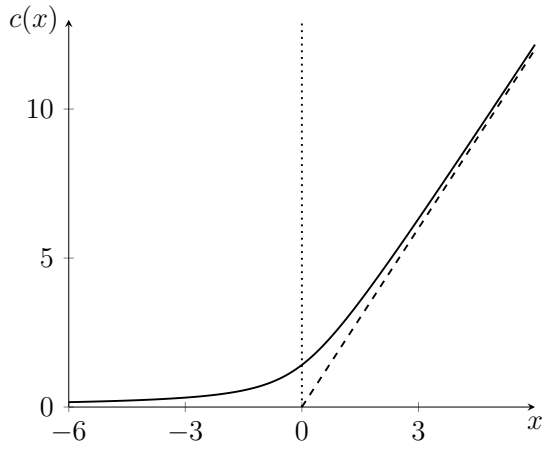


Figure 7: Crossover function $c(x) = x+\sqrt{x^2+2}$.

upward creates access to a giant component and sharply expands free-riding incentives, while moving further below $\lambda = 1$ produces no analogous jump, since component sizes remain logarithmic in n.

5 Social Surplus in Erdős-Rényi Graphs with Local Connections

We examine the social surplus in the local-connection economy of Section 4.1. We first analyze the equilibrium social surplus, under pure- and mixed strategies, in Section 5.1, characterize the socially optimal thresholds (Section 5.2), and show that over-exploitation and under-exploration persist as robust features despite the large number of players. Section 5.3 then studies the optimal mixing probability μ^* and compares it to the equilibrium mixed strategy μ^e from Theorem 3.

Suppose that out of n players, k agents choose the risky arm in the first period, and let the resulting social surplus be denoted as $u_{k,n}(\pi)$. Further, in the ER graphs with local connections, let $q_a(b) = \binom{a}{b}p^b(1-p)^{a-b}$ represent the probability of meeting b agents out of a

specific set of a individuals in the second period, then

$$\begin{aligned} u_{k,n}(\pi) &= (1-\delta)k\,(\pi\beta-\alpha(1-\pi\beta)) \\ &+\delta k\pi(\beta-\alpha(1-\beta))\beta+\delta k\sum_{m=0}^{k-1} q_{k-1}(m)\left[\pi(\beta-\alpha(1-\beta))(1-\beta)^{m+1}-\alpha(1-\pi)\right]^{+} \\ &+\delta k\sum_{m=0}^{k-1} q_{k-1}(m)\pi(\beta-\alpha(1-\beta))(1-\beta)\,(1-(1-\beta)^{m}) \\ &+\delta(n-k)\sum_{m=0}^{k} q_{k}(m)\left[\pi(\beta-\alpha(1-\beta))(1-\beta)^{m}-\alpha(1-\pi)\right]^{+} \\ &+\delta(n-k)\sum_{m=0}^{k} q_{k}(m)\pi(\beta-\alpha(1-\beta))\,(1-(1-\beta)^{m})\,. \end{aligned} \tag{5.1}$$

The first line of $u_{k,n}$ is the first-period payoff earned by the k exploring agents. The next two lines give their discounted second-period payoff, which has three components: the expected payoff when an agent herself received a conclusive signal in the first period; the expected payoff when neither she nor any of her second-period contacts did; and the expected payoff when she did not receive a high output but at least one contact did. The last two lines give the discounted second-period payoff of the remaining $n-k$ exploiting agents, decomposed into two components: their payoff when none of their contacts in the exploring group received a high output, and their payoff when at least one did.

The following lemma examines the marginal value of adding one more explorer, $\Delta u_k \coloneqq u_{k+1}-u_k$, which will prove useful when we characterize the equilibrium social surplus and the social optimum. We use the notation $Q_a(b) \coloneqq \sum_{m\leqslant b} q_a(m)$ for the cumulative distribution of q_a, with the convention $Q_0(0) = q_0(0) = 1$.

Lemma 5. *The marginal value of one more exploring agent takes the following form:*

(i) On $\frac{\pi}{1-\pi} \leqslant \frac{\alpha}{\beta-\alpha(1-\beta)}$*, or equivalently* $\pi \leqslant \tau$*, it holds that*

$$\begin{aligned} \Delta u_k(\pi) &= (1-\delta)\big(\pi\beta-\alpha(1-\pi\beta)\big) \\ &+\delta\pi(\beta-\alpha(1-\beta))\beta(1-p\beta)^{k}\left(1+(n-1)p-\frac{kp(1-p)\beta}{1-p\beta}\right). \end{aligned} \tag{5.2}$$

(ii) On $\frac{\pi}{1-\pi} \geqslant \frac{\alpha}{(\beta-\alpha(1-\beta))(1-\beta)^{k+1}}$*, it holds that* $\Delta u_k(\pi) = (1-\delta)\big(\pi\beta-\alpha(1-\pi\beta)\big)$.

(iii) On $\frac{\alpha}{(\beta-\alpha(1-\beta))(1-\beta)^r} \leqslant \frac{\pi}{1-\pi} \leqslant \frac{\alpha}{(\beta-\alpha(1-\beta))(1-\beta)^{r+1}}$ *and* $k \geqslant r \geqslant 0$*, it holds that*

$$\Delta u_k(\pi) = (1-\delta)(\pi\beta-\alpha(1-\pi\beta))+\delta\big[\pi(\beta-\alpha(1-\beta))B_k-\alpha(1-\pi)A_k\big],$$

where

$$\begin{aligned} A_k(\pi) &\coloneqq (k+1)Q_k(r-1)-kQ_{k-1}(r-1)-(n-k)Q_k(r)+(n-k-1)Q_{k+1}(r) \\ B_k(\pi) &\coloneqq -(k+1)(1-\beta)\sum_{m=r}^{k} q_k(m)(1-\beta)^m+k(1-\beta)\sum_{m=r}^{k-1} q_{k-1}(m)(1-\beta)^m \\ &\quad +(n-k)\sum_{m=r+1}^{k} q_k(m)(1-\beta)^m-(n-k-1)\sum_{m=r+1}^{k+1} q_{k+1}(m)(1-\beta)^m \end{aligned} \tag{5.3}$$

The proof follows directly by observing that the piecewise linear components in (5.1) are positive whenever $m+1 \leqslant r$ and $m \leqslant r$ in the first and second components, respectively, and is therefore omitted.

5.1 Equilibrium Social Surplus

5.1.1 Pure-Strategy

In this part we study the equilibrium social surplus when agents invoke pure strategies. For $\pi \leqslant \underline{\pi}$, no agent explores and the equilibrium social surplus is zero. Figure 8 plots $u_{k_n,n}(\pi)/n$ for finite n and $\pi \in (\underline{\pi}, \tau)$, where the asymmetric pure-strategy equilibrium prevails. Within a fixed equilibrium region (with k_n constant), an increase in λ shifts the distribution of M in the sense of first-order stochastic dominance, which by the above representation *increases* the equilibrium social surplus. The next proposition shows that at every threshold where the economy undergoes an equilibrium regime change, the social surplus falls—confirming the intuition from the two-player case.

Proposition 7. *The pure-strategy equilibrium social surplus falls discontinuously at every* λ *where the economy undergoes an equilibrium regime change.*

Figure 8 shows that the pure-strategy equilibrium social surplus increases in λ within each equilibrium region, with discontinuous drops at regime-change thresholds. The largest drop occurs at the transition from full exploration to the intermediate region. This figure mirrors panel (b) of Figure 4, with multiple drops reflecting the richer equilibrium structure for $n > 2$.

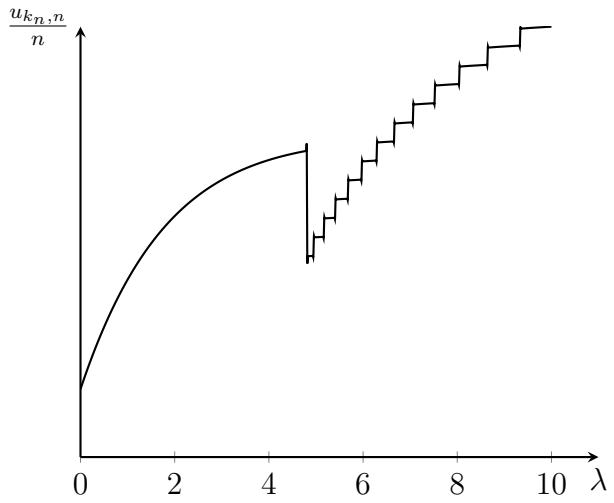


Figure 8: Finite-n average of pure-strategy equilibrium social surplus [$\pi = 0.5, \delta = 0.045, \alpha = 0.45, \beta = 0.6, n = 16$]

Large n limit. Define

$$\bar{u}_\infty(\pi) \coloneqq \lim_{n\to\infty} \frac{u_{k_n,n}(\pi)}{n}\,. \tag{5.4}$$

By Proposition 4, as $n \to \infty$, the fraction $k_n/n \to \kappa$ and we use this to show in the next proposition that

$$\bar{u}_\infty(\pi) = \begin{cases} 0\,, & \pi \leqslant \underline{\pi}\,, \quad (5.5a) \\ (1-\delta)\beta^{-1}(\pi\beta-\alpha(1-\pi\beta))+\delta\pi(\beta-\alpha(1-\beta))\beta\,, & \underline{\pi} \leqslant \pi < \bar{\pi}_\infty^{\text{local}}\,, \quad (5.5b) \\ (1-\delta)(\pi\beta-\alpha(1-\pi\beta))+\delta\pi(\beta-\alpha(1-\beta))\big(1-(1-\beta)e^{-\lambda\beta}\big) & \\ \quad +\delta\,\mathsf{E}_{M\sim\mathsf{Pois}(\lambda)}\Big[\big(\pi(\beta-\alpha(1-\beta))(1-\beta)^{M+1}-\alpha(1-\pi)\big)^+\Big], & \pi \geqslant \bar{\pi}_\infty^{\text{local}}\,. \quad (5.5c) \end{cases}$$

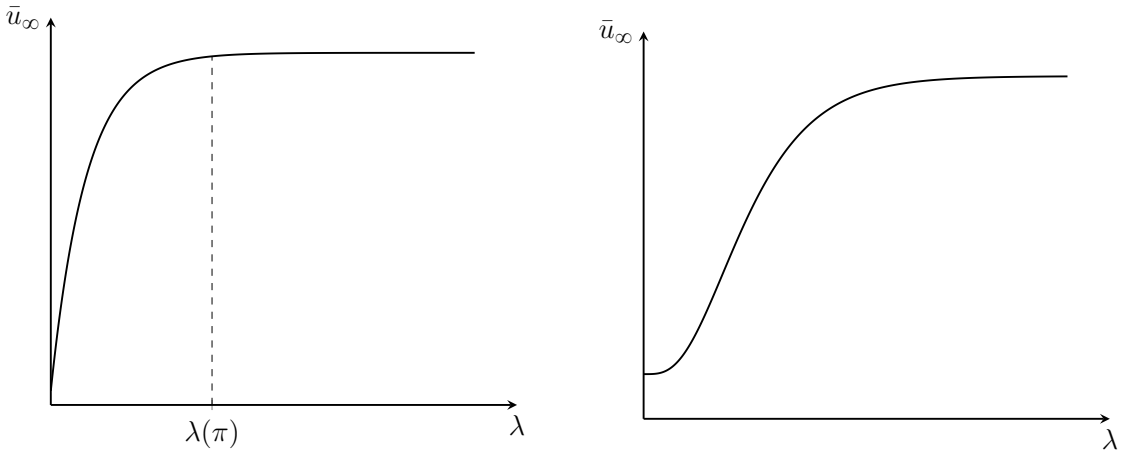
Proposition 8 (Large-n limit of the average pure-strategy equilibrium social surplus)**.** *$\bar{u}_\infty$ follows (5.5) and is weakly increasing in λ for every fixed π. Moreover, as a function of λ:*

(i) For $\pi \in (\underline{\pi}, \tau)$, there exists a threshold $\lambda(\pi)$ such that $\bar{u}_\infty(\pi, \lambda)$ is constant in λ for $\lambda \geqslant \lambda(\pi)$, and follows (5.5c) for $\lambda < \lambda(\pi)$.

(ii) For $\pi \geqslant \tau$, $\bar{u}_\infty$ follows (5.5c) for all λ.

Figure 9 illustrates the asymptotic average equilibrium social surplus against λ for two values of π. For intermediate $\pi \in (\underline{\pi}, \tau)$, the surplus becomes flat in λ once $\lambda \geqslant \lambda(\pi)$ (Panel 9a). The intuition is that an increase in λ induces more free-riding and fewer explorers, reducing

the social cost of first-period exploration but also shrinking the benefits from second-period information exchange; in the large-n limit, these two effects exactly offset. This is also the region where, for finite n, the surplus exhibits discontinuous jumps at each regime change (Figure 8); as $n \to \infty$, these jumps vanish and the per-capita surplus becomes flat in λ.



(a) $\underline{\pi} < \pi < \tau$ (b) $\pi \geqslant \tau$

Figure 9: Effect of λ on $\bar{u}_\infty$

5.1.2 Mixed-Strategy

We now study the equilibrium social surplus when agents use symmetric mixed strategies in the intermediate region, each exploring independently with probability μ_n^e as characterized in (4.3). Let $k_n^{\text{mixed}} \sim \mathsf{Bin}(n, \mu_n^e)$ denote the resulting number of explorers.[8]

The average equilibrium social surplus under symmetric mixed strategies is

$$\bar{u}_n^{\text{mixed}} := \frac{u_{k_n^{\text{mixed}},n}}{n}.$$

Our main result here is that $\bar{u}_n^{\text{mixed}}$ converges *almost surely* to $\bar{u}_\infty$, the limiting average social surplus under pure strategies (5.4).

Theorem 5 (Large-n limit of the average mixed-strategy equilibrium social surplus)**.** $\bar{u}_n^{\text{mixed}}$ *converges almost surely to* $\bar{u}_\infty$.

The proof applies a version of the continuous mapping theorem, exploiting the fact that

[8] This differs from the pure-strategy equilibrium number of explorers k_n in Proposition 4.

$\mu_n^e \to \kappa(\pi)$ on the intermediate region (Proposition 5), so that k_n^{mixed}/n converges almost surely to $\kappa(\pi)$, driving the same limiting average social surplus as in the pure-strategy case.

5.2 Optimal Thresholds

One would expect the social optimum to mirror the two-player case: exploration (respectively, exploitation) is socially optimal when the initial belief exceeds (respectively, falls below) some threshold. Establishing this in the multi-agent economy, however, requires a more involved analysis.

To determine when adding one more explorer raises social welfare, we need to sign $\Delta u_k = u_{k+1} - u_k$. Lemma 5 decomposed Δu_k into two components A and B. The following lemma is the cornerstone of the social optimum analysis, and its proof relies on the first-order stochastic dominance relation $\mathsf{Bin}(k+1, p) \succeq \mathsf{Bin}(k, p)$.

Lemma 6. *For every $k \geqslant r \geqslant 0$,*

$$B_k \geqslant \max\left\{(1-\beta)^r A_k, (1-\beta)^{r+1} A_k\right\}. \tag{5.6}$$

Lemma 6 gives tight control over Δu_k on $[\tau, 1]$. For $\pi \leqslant \tau$, we need an additional result:

Lemma 7. *For every fixed $\pi \leqslant \tau$, the marginal value $\Delta u_k(\pi)$ is decreasing in k.*

This follows immediately from part (i) of Lemma 5. It falls short of establishing diminishing returns globally, confining the claim to the region where the initial belief is small. Together with Lemmas 5 and 6, however, it suffices to characterize the regions where full exploitation and full exploration are socially optimal.

Theorem 6 (Social optimum)**.** *The socially optimal outcome is full exploitation if and only if $\pi \leqslant \underline{\pi}^*$, and full exploration if and only if $\pi \geqslant \bar{\pi}^*$. Moreover, on $[0, \underline{\pi}^*]$ the social surplus is decreasing in k ($\Delta u_k \leqslant 0$), and on $[\bar{\pi}^*, 1]$ it is increasing in k ($\Delta u_k \geqslant 0$). The cutoff points are:*

$$\underline{\pi}_n^* = \frac{\alpha(1-\delta)}{\beta\left[(1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta))(1+(n-1)p)\right]},$$

$$\bar{\pi}_n^* = \frac{\alpha(1-\delta)}{\beta\left[(1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta))\,(1-p\beta)^{n-1}\left(1+(n-1)p-\frac{(n-1)\,p(1-p)\beta}{1-p\beta}\right)\right]}.$$

Recall that $p = \lambda/n$. Taking $n \to \infty$, the limiting lower and upper cutoff points for the optimality of full exploitation and full exploration are:

$$\underline{\pi}^*_\infty \coloneqq \lim_{n\to\infty} \underline{\pi}^*_n = \frac{\alpha(1-\delta)}{\beta\left[(1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta))(1+\lambda)\right]},$$
$$\bar{\pi}^*_\infty \coloneqq \lim_{n\to\infty} \bar{\pi}^*_n = \frac{\alpha(1-\delta)}{\beta\left[(1+\alpha)(1-\delta)+\delta(\beta-\alpha(1-\beta))\, e^{-\lambda\beta}\,(1+\lambda(1-\beta))\right]}.$$

Recall that the equilibrium exploitation threshold in the multi-agent economy coincides with that of the two-player case, namely $\underline{\pi}$ from Proposition 2. Comparing the socially optimal exploitation threshold $\underline{\pi}^*_\infty$ with $\underline{\pi}$ confirms that the equilibrium exhibits over-exploitation relative to the social optimum. Comparing the socially optimal exploration threshold $\bar{\pi}^*_\infty$ with the asymptotic equilibrium exploration threshold (4.1) similarly confirms under-exploration.

Effect of λ on the optimal exploration cutoff. The optimal exploration cutoff $\bar{\pi}^*_\infty$ initially decreases in λ and then increases. To understand this non-monotonicity, we examine the marginal contribution of the n-th explorer to social surplus (i.e., Δu_{n-1}), and specifically its effect on the positive externality of community exploration for an agent whose first-period exploration failed. This is captured by the marginal change in the third line of (5.1), namely

$$\delta\pi(\beta-\alpha(1-\beta))(1-\beta)\left[n\sum_{m=0}^{n-1} q_{n-1}(m)\big(1-(1-\beta)^m\big)-(n-1)\sum_{m=0}^{n-2} q_{n-2}(m)\big(1-(1-\beta)^m\big)\right]. \tag{5.7}$$

We employ a coupling argument to further illuminate the marginal change in (5.7) and its response to λ. Suppose that in the high state, an agent who chose the risky arm failed in the first period, which occurs with probability $\pi(1-\beta)$. Let $X \sim \mathsf{Bin}(n-2, \lambda/n)$ denote the number of his second-period contacts excluding himself and the candidate n-th agent. Setting aside the base probability $\pi(1-\beta)$ and expected payoff $(\beta-\alpha(1-\beta))$, the bracketed difference in (5.7) is approximately

$$n\Big(\mathsf{E}_{X,Z}\left[1-(1-\beta)^{X+Z}\right]-\mathsf{E}_X\left[1-(1-\beta)^X\right]\Big),$$

where $Z \sim \mathsf{Bernoulli}(\lambda/n)$ represents the exploration outcome of the n-th agent. This simplifies to

$$n\ \mathsf{E}_Z\left[1-(1-\beta)^Z\right]\mathsf{E}_X\left[(1-\beta)^X\right] = n\cdot\frac{\lambda}{n}\beta\cdot(1-\lambda\beta/n)^{n-2} \to \lambda\beta e^{-\lambda\beta}.$$

This expression shows that the positive externality of the n-th agent's exploration is proportional to the expected number of successful contacts among her immediate neighbors, $np\beta = \lambda\beta$, and the probability of group failure among the remaining $n-2$ explorers, $(1-\lambda\beta/n)^{n-2}$. For any fixed $\pi \leqslant \tau$, the marginal impact of the n-th agent's exploration (Δu_{n-1}) is therefore initially increasing in λ and then decreasing, which translates into the opposite pattern for the optimal full exploration cutoff. Figure 10 plots the large-n limits of the equilibrium and optimal exploration cutoffs as a function of λ in the local economy.

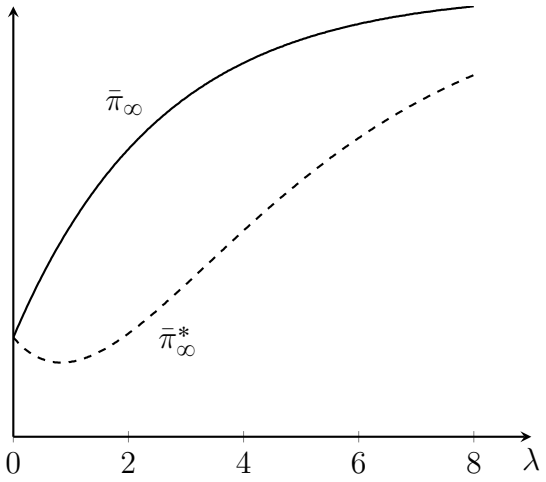


Figure 10: Effect of λ on equilibrium and optimal exploration thresholds [$\alpha = 0.5$, $\beta = 0.4$, $\delta = 0.25$]

5.3 Optimal Mixing Probability

Having characterized the optimal exploitation and exploration thresholds in the previous section, we now turn to the optimal exploration probability μ^*—the mixing probability prescribed by the planner—and compare it to the equilibrium outcome μ^e.

A benevolent planner who randomizes symmetrically across players with probability μ induces $k \sim \mathsf{Binomial}(n, \mu)$ explorers, yielding the expected social surplus:

$$S(\pi; \mu) \coloneqq \mathsf{E}_{k\sim\mathsf{Bin}(n,\mu)}\left[u_{k,n}(\pi)\right] .$$

Since the expected first-period payoff of exploration is positive for $\pi > \tau$, both the equilibrium and optimal strategy prescribe full exploration there. We therefore restrict the welfare analysis under mixed strategies to $[0, \tau]$. By Lemma 7, the mapping $k \mapsto u_{k,n}(\pi)$ is concave on this

region. The following lemma shows that this concavity carries over to $\mu \mapsto S(\pi;\mu)$ and provides a complete characterization of an interior optimum $\mu^* \in (0,1)$; the proof follows by differentiating $S(\pi;\mu)$ with respect to μ and applying a classical result on Bernstein polynomials (Lorentz, 2012), and is therefore omitted.

Lemma 8 (Lorentz (2012))**.** *The mapping $\mu \mapsto S(\pi;\mu)$ is concave for $\pi \in [0,\tau]$, so the planner's problem $\max_{\mu\in[0,1]} S(\pi;\mu)$ has a unique maximizer. Thus a necessary and sufficient condition for an interior optimum $\mu^* \in (0,1)$ is the first-order condition:*

$$\mathsf{E}_{k\sim\mathsf{Bin}(n-1,\mu^*)}\left[\Delta u_k(\pi)\right] = 0\,. \tag{5.8}$$

Theorem 7 (Equilibrium vs. optimum)**.** *Assume the local Erdős-Rényi connections, and $\pi \leqslant \tau$. Let $q \coloneqq 1-p\beta$. Then, the equilibrium μ^e and optimal μ^* satisfy, respectively,*

$$C_1(\pi) = C_2(\pi)\ \mathsf{E}_{k\sim\mathsf{Bin}(n-1,\mu^e)}\left[q^k\right]\,, \tag{5.9a}$$

$$C_1(\pi) = C_2(\pi)\ \mathsf{E}_{k\sim\mathsf{Bin}(n-1,\mu^*)}\left[q^k\left(1+(n-1)p-\frac{kp(1-p)\beta}{1-p\beta}\right)\right]\,. \tag{5.9b}$$

Proof. The identity for the equilibrium probability μ^e in (5.9a) is already shown in Proposition 5. The condition for μ^* in (5.9b) follows by substituting Δu_k from (5.2) into the first-order condition (5.8). □

The term $\left(1+(n-1)p-\frac{kp(1-p)\beta}{1-p\beta}\right)$ in (5.9b) captures the positive externality of an additional explorer. For $\pi \leqslant \tau$, the equilibrium condition (5.9a) reflects that agents explore only when the informational gain (C_2) outweighs the first-period loss (C_1), but they ignore the information they provide to others—giving rise to the wedge between μ^e and μ^*. The following corollary shows that the equilibrium mixing probability is always below the social optimum, confirming that under-exploration is robust to mixed strategies.

Corollary 1. For every $\pi \in [0,\tau)$, $\mu^* \geqslant \mu^e$.

Proof. Define the externality factor $\Gamma(k) \coloneqq 1+(n-1)p-\frac{kp(1-p)\beta}{1-p\beta}$, representing the marginal social value of an agent's information. Since $k \leqslant n-1$, we have $\Gamma(k) \geqslant 1$. Let

$$G(\pi;\mu) \coloneqq \mathsf{E}_{k\sim\mathsf{Bin}(n-1,\mu)}\left[q^k\right] \quad \text{and} \quad H(\pi;\mu) \coloneqq \mathsf{E}_{k\sim\mathsf{Bin}(n-1,\mu)}\left[q^k\Gamma(k)\right].$$

Since $\Gamma(k) \geqslant 1$, we have $G(\pi;\mu) \leqslant H(\pi;\mu)$ for all μ. Moreover, since $q < 1$, both q^k and $q^k\Gamma(k)$ are decreasing in k, so by first-order stochastic dominance, both $G(\pi;\cdot)$ and $H(\pi;\cdot)$ are

decreasing in μ. The equilibrium and optimality conditions give $C_1(\pi)/C_2(\pi) = G(\pi;\mu^e) = H(\pi;\mu^*)$, from which $\mu^* \geqslant \mu^e$ follows. □

6 Conclusion and Additional Discussion

The tension between information diffusion and production in organizations and societies represents a complex and evolving challenge. On one hand, greater connectivity has allowed for unprecedented access to knowledge and ideas fostering informed decision-making and quicker adoption of innovation. At the same time, this proliferation of information can undermine knowledge production. In a better connected organization or society, individuals are tempted to rely on knowledge that is shared through the network instead of experimenting with new ideas. Greater connectivity and knowledge diffusion can thus lead to lower knowledge production, reducing overall social welfare.

Our analysis begins with a two-player economy, where equilibrium reveals three distinct regions based on initial beliefs, each reflecting different degrees of exploitation and exploration. free-riding leads to over-exploitation and under-exploration relative to the social optimum, and equilibrium social surplus behaves non-monotonically in the connection probability. Our model is set in two periods; we expect the non-monotonicity of equilibrium social surplus to persist in longer horizons—as studied in continuous time by Board and Meyer-ter Vehn (2024)—and that as agents become more patient ($\delta \to 1$), the exploration threshold decreases, since it becomes preferable to bear the exploration cost sooner rather than later.

We then analyze multi-agent economies and explore different network structures. With local connections each agent only observes the experimentation outcomes of her immediate neighbors, whereas with global connections each agent's observable circle includes the entire set of agents who are (directly or indirectly) connected to her. In both structures, the tension between information sharing and private exploration remains a significant factor. In particular, higher connectivity can exacerbate free-riding and reduce social welfare. Additionally, we discuss the asymptotic effects of connectivity on equilibrium and highlight how the size of the connected component in the network significantly impacts exploration behavior.

Although we showed that the tension between information diffusion and production is prevalent in different settings, interesting variations remain to be studied. For example, what happens when agents have different preferences or endowments? What happens when societies and organizations have network structures with differently connected agents? We

hope to address these issues in future work.

A Proof of Results in Section 3

A.1 Proof of Theorem 1

Suppose all but one individual explore in period one. Let L be the number of successes ($y = 1$) the focal agent observes; clearly $L \leqslant M$ (her period-two contacts). Define $\pi_{\ell,m} \coloneqq \mathsf{P}(\theta = 1 \mid L = \ell, M = m)$. Then $\pi_{\ell,m} = 1$ for $\ell \geqslant 1$, and if $\ell = 0$:

$$\frac{\pi_{0,m}}{1-\pi_{0,m}} = \frac{\pi}{1-\pi}(1-\beta)^m .$$

In period two, the focal agent pulls the risky arm iff $\pi_{\ell,m} > \tau$, yielding

$$\left[\pi_{\ell,m}\beta - \alpha\big(1-\pi_{\ell,m}\beta\big)\right]^+ .$$

Equivalently,

$$\left[\pi_{\ell,m}\beta - \alpha(1-\pi_{\ell,m}\beta)\right]^+ = \mathsf{E}\left[\theta\beta - \alpha(1-\theta\beta) \mid L = \ell, M = m\right]^+ .$$

If all others explore in period one, the focal agent's exploitation payoff is

$$\begin{aligned} w_{n-1}(\pi) &= \delta \sum_{m=0}^{n-1} \sum_{\ell=0}^{m} \mathsf{P}\left(L = \ell, M = m\right) \mathsf{E}\left[\theta\beta - \alpha(1-\theta\beta) \middle| L = \ell, M = m\right]^+ \\ &= \delta \sum_{m=0}^{n-1} \sum_{\ell=0}^{m} \mathsf{E}\left[\theta\beta - \alpha(1-\theta\beta);\, L = \ell, M = m\right]^+ . \end{aligned} \tag{A.1}$$

Let $q(m) \coloneqq \mathsf{P}(M = m)$ be the probability that a randomly selected agent observes the exploration outcomes of m others. Then

$$\begin{aligned} w_{n-1}(\pi) &= \delta \sum_{m=0}^{n-1} \sum_{\ell=0}^{m} q(m) \left[\pi(\beta - \alpha(1-\beta)) \binom{m}{\ell} \beta^\ell (1-\beta)^{m-\ell} - \alpha(1-\pi) 1_{\{\ell=0\}}\right]^+ \\ &= \delta \sum_{m=0}^{n-1} \underbrace{q(m)\left[\pi(\beta - \alpha(1-\beta))(1-\beta)^m - \alpha(1-\pi)\right]^+}_{\mathsf{E}[\theta\beta - \alpha(1-\theta\beta);\, M=m, L=0]^+} + \underbrace{\delta\pi(\beta - \alpha(1-\beta)) \sum_{m=0}^{n-1} q(m)\left(1-(1-\beta)^m\right)}_{\delta\mathsf{E}[\theta\beta - \alpha(1-\theta\beta);\, L>0]} . \end{aligned}$$

Thus $w_{n-1}(\pi)$ splits into the payoff when $L = 0$ and when $L > 0$.

If the agent explores in period one, let $y_0 \in \{-\alpha, 1\}$ denote her realization of the risky

arm. Her expected payoff is

$$v_n(\pi) = (1-\delta)\left(\pi\beta - \alpha(1-\pi\beta)\right) + \delta \sum_{m=0}^{n-1} \sum_{\ell=0}^{m} \sum_{y\in\{-\alpha,1\}} \mathsf{P}\left(M=m, L=\ell, y_0=y\right) \mathsf{E}\left[\theta\beta - \alpha(1-\theta\beta) \mid M=m, L=\ell, y_0=y\right]^+ . \tag{A.2}$$

The discounted period-2 term splits by the period-1 realization y_0:

$$\begin{aligned}\text{discounted expected payoff} = \delta \sum_{m,\ell} \mathsf{E}\left[\theta\beta - \alpha(1-\theta\beta);\, M=m, L=\ell, y_0=1\right]^+ \\ + \delta \sum_{m,\ell} \mathsf{E}\left[\theta\beta - \alpha(1-\theta\beta);\, M=m, L=\ell, y_0=-\alpha\right]^+ .\end{aligned}$$

The first (second) term conditions on $y_0 = 1$ ($y_0 = -\alpha$). Further rearrangements imply that the sum above is equal to

$$\begin{aligned}&\delta\pi(\beta-\alpha(1-\beta))\beta + \delta \sum_{m,\ell} q(m) \left[\pi(\beta-\alpha(1-\beta))\binom{m}{\ell}\beta^\ell(1-\beta)^{m+1-\ell} - \alpha(1-\pi)1_{\{\ell=0\}}\right]^+ \\ &= \underbrace{\delta\pi(\beta-\alpha(1-\beta))\beta}_{\delta\mathsf{E}[\theta\beta-\alpha(1-\theta\beta);\, y_0=1]} + \delta \sum_{m=0}^{n-1} \underbrace{q(m)\left[\pi(\beta-\alpha(1-\beta))(1-\beta)^{m+1} - \alpha(1-\pi)\right]^+}_{\mathsf{E}[\theta\beta-\alpha(1-\theta\beta);\, M=m, L=0, y_0=-\alpha]^+} \\ &\quad + \underbrace{\delta\pi(\beta-\alpha(1-\beta))(1-\beta) \sum_{m=0}^{n-1} q(m)\left(1-(1-\beta)^m\right)}_{\delta\mathsf{E}[\theta\beta-\alpha(1-\theta\beta);\, L>0, y_0=-\alpha]} .\end{aligned}$$

Formally, exploration occurs iff $v_n(\pi) > w_{n-1}(\pi)$, i.e., when the discounted gains from conclusive signals ($L > 0$ or $y_0 = 1$) outweigh the opportunity cost when signals are inconclusive ($L = 0$ and $y_0 = -\alpha$):

$$\begin{aligned}&\overbrace{(1-\delta)\left(\pi\beta-\alpha(1-\pi\beta)\right)}^{\text{present payoff}} \\ &+\delta\Big(\mathsf{E}\left[\theta\beta-\alpha(1-\theta\beta); y_0=1\right] + \mathsf{E}\left[\theta\beta-\alpha(1-\theta\beta); L>0, y_0=-\alpha\right] - \mathsf{E}\left[\theta\beta-\alpha(1-\theta\beta); L>0\right]\Big) \\ &> \delta \sum_{m=0}^{n-1} \Big(\mathsf{E}\left[\theta\beta-\alpha(1-\theta\beta); M=m, L=0\right]^+ - \mathsf{E}\left[\theta\beta-\alpha(1-\theta\beta); M=m, L=0, y_0=-\alpha\right]^+\Big) \\ &= \text{discounted opportunity cost of exploration absent conclusive signals.}\end{aligned} \tag{A.3}$$

If $\pi > \tau$, the agent explores in period one: the expected first period's payoff is positive and exploration yields informational benefits, so $v_n(\pi) > w_{n-1}(\pi)$. Hence the cutoff satisfies

$\bar{\pi} \leqslant \tau$, and we restrict attention to $\pi \leqslant \tau$. On $\pi \leqslant \tau$, all $[\cdot]^+$ terms equal zero because, for any $k \geqslant 0$,

$$\pi\left(\beta-\alpha(1-\beta)\right)(1-\beta)^k \leqslant \pi\left(\beta-\alpha(1-\beta)\right) \leqslant \alpha(1-\pi)\,,$$

where the last inequality uses $\pi \leqslant \tau$. Thus $v_n(\pi) > w_{n-1}(\pi)$ reduces to

$$(1-\delta)\left(\pi\beta-\alpha(1-\pi\beta)\right)+\delta\pi\beta(\beta-\alpha(1-\beta))\mathsf{E}\left[(1-\beta)^M\right] > 0\,. \tag{A.4}$$

This is equivalent to $\pi > \bar{\pi}$, proving (3.1). □

A.2 Proof of Lemma 1

Let $q_k(m) \coloneqq \mathsf{P}_k(M = m)$ be the probability of observing m exploration outcomes among the k first-period explorers. Following the recipe of (A.1) and (A.2), the payoff functions are

$$\begin{aligned} w_k(\pi) = \delta\mathsf{E}_k\left[\left(\pi(\beta-\alpha(1-\beta))(1-\beta)^M-\alpha(1-\pi)\right)^+\right]+\delta\pi(\beta-\alpha(1-\beta))\mathsf{E}_k\left[1-(1-\beta)^M\right]\,,\\ v_k(\pi) = (1-\delta)\left(\pi\beta-\alpha(1-\pi\beta)\right)+\delta\pi(\beta-\alpha(1-\beta))\beta\\ +\delta\mathsf{E}_{k-1}\left[\left(\pi(\beta-\alpha(1-\beta))(1-\beta)^{M+1}-\alpha(1-\pi)\right)^+\right]\\ +\delta\pi(\beta-\alpha(1-\beta))(1-\beta)\mathsf{E}_{k-1}\left[1-(1-\beta)^M\right]\,. \end{aligned} \tag{A.5}$$

Throughout, the distribution of M is the one indexed by the outer expectation: E_k means $\mathsf{P}(M = m) = q_k(m)$.

Consider the difference

$$\begin{aligned} v_{k+1}(\pi)-w_k(\pi) = (1-\delta)\left(\pi\beta-\alpha(1-\pi\beta)\right)+\delta\pi\left(\beta-\alpha(1-\beta)\right)\beta\mathsf{E}_k\left[(1-\beta)^M\right]\\ +\delta\mathsf{E}_k\left[\left(\pi\left(\beta-\alpha(1-\beta)\right)(1-\beta)^{M+1}-\alpha(1-\pi)\right)^+\right]-\delta\mathsf{E}_k\left[\left(\pi\left(\beta-\alpha(1-\beta)\right)(1-\beta)^M-\alpha(1-\pi)\right)^+\right]. \end{aligned}$$

Since $\pi > \tau$, both $[\cdot]^+$ terms are zero, so

$$v_{k+1}(\pi)-w_k(\pi) = (1-\delta)\left(\pi\beta-\alpha(1-\pi\beta)\right)+\delta\pi\left(\beta-\alpha(1-\beta)\right)\beta\mathsf{E}_k\left[(1-\beta)^M\right] > 0\,.$$

□

A.3 Proof of Theorem 3

To prove the theorem, we first demonstrate that on the intermediate region there exists a unique μ satisfying $v(\pi;\mu) = w(\pi;\mu)$. For existence, observe that on this region $v(\pi;0) = v_1(\pi) > w_0(\pi) = w(\pi;0)$ and $v(\pi;1) = v_n(\pi) \leqslant w_{n-1}(\pi) = w(\pi;1)$. Therefore, there exists at least one $\mu^e \in (0,1]$ satisfying $v(\pi;\mu^e) = w(\pi;\mu^e)$.

For uniqueness, observe that the $[\cdot]^+$ terms in equation (A.5) vanish on the intermediate region, yielding:

$$\begin{aligned} w(\pi;\mu) &= \delta\pi(\beta-\alpha(1-\beta))\sum_{k=0}^{n-1}\binom{n-1}{k}\mu^k(1-\mu)^{n-1-k}\mathsf{E}_k\left[1-(1-\beta)^M\right], \\ v(\pi;\mu) &= (1-\delta)\left(\pi\beta-\alpha(1-\pi\beta)\right)+\delta\pi(\beta-\alpha(1-\beta))\beta \\ &+\delta\pi(\beta-\alpha(1-\beta))(1-\beta)\sum_{k=0}^{n-1}\binom{n-1}{k}\mu^k(1-\mu)^{n-1-k}\mathsf{E}_k\left[1-(1-\beta)^M\right]. \end{aligned}$$

Hence $v(\pi;\mu) = w(\pi;\mu)$ is equivalent to

$$\begin{aligned} \frac{(1-\delta)\left(\alpha(1-\pi\beta)-\pi\beta\right)}{\delta\pi(\beta-\alpha(1-\beta))\beta} &= \sum_{k=0}^{n-1}\binom{n-1}{k}\mu^k(1-\mu)^{n-1-k}\,\mathsf{E}_k\left[(1-\beta)^M\right] \\ &= \mathsf{E}_{k\sim\mathsf{Bin}(n-1,\mu)}\left[\mathsf{E}_k\left[(1-\beta)^M\right]\right]. \end{aligned} \tag{A.6}$$

Since the binomial distribution $\mathsf{Bin}(n-1,\mu)$ is increasing in the sense of FOSD with respect to μ, and the mapping $k \mapsto \mathsf{E}_k\left[(1-\beta)^M\right]$ is decreasing in k (as can be shown by a simple coupling argument), the RHS of (A.6) is decreasing in μ. This establishes the uniqueness of μ^e.

In addition, the LHS of (A.6) is decreasing in π. Given that its RHS is decreasing in μ, the equilibrium mixing probability μ^e must be increasing in π.

Lastly, observe that for a fixed μ, one has $\mathsf{Bin}(n,\mu) \geq_1 \mathsf{Bin}(n-1,\mu)$. Because the mapping $k \mapsto \mathsf{E}_k\left[(1-\beta)^M\right]$ is decreasing in k, the RHS of (A.6) is decreasing in both μ and n. Since the LHS of (A.6) is unaffected by n, μ^e must be decreasing in n. □

B Proof of Results in Section 4

B.1 Proof of Lemma 2

To show that $\bar{\pi}_{n+1} > \bar{\pi}_n$, we use equation (3.1) along with the notation in footnote 4 to establish that

$$\mathsf{E}_{n-1}^{(n)}\left[(1-\beta)^M\right] > \mathsf{E}_{n}^{(n+1)}\left[(1-\beta)^M\right].$$

This holds because $\mathsf{E}_{n-1}^{(n)}\left[(1-\beta)^M\right] = \left(1-\frac{\lambda\beta}{n}\right)^{n-1}$, which is eventually decreasing in n (since $x \mapsto (x-1)\log(1-\lambda\beta/x)$ has negative derivative for large x). □

B.2 Proof of Lemma 3

Since $p_n = \frac{\lambda}{n} \geqslant \frac{\lambda}{n+1} = p_{n+1}$, a coupling argument shows that on the same probability space $M_k^{(n)} \geqslant M_k^{(n+1)}$ almost surely, and therefore $\mathsf{E}\left[(1-\beta)^{M_k^{(n)}}\right] \leqslant \mathsf{E}\left[(1-\beta)^{M_k^{(n+1)}}\right]$. This in turn implies $\pi_{k,n+1} \leqslant \pi_{k,n}$.

Next, observe that with local connections,

$$\begin{aligned}\mathsf{E}_{k+1}^{(n+1)}\left[(1-\beta)^M\right] = \left(1-\frac{\lambda\beta}{n+1}\right)^{k+1} &\leqslant \left(1-\frac{\lambda\beta}{n+1}\right)\left(1+\frac{\lambda\beta}{n+1}\right)^{-k} \\ &\leqslant \left(1-\frac{\lambda\beta}{n+1}\right)\left(1+\frac{k\lambda\beta}{n+1}\right)^{-1}.\end{aligned}$$

Moreover,

$$\mathsf{E}_{k}^{(n)}\left[(1-\beta)^M\right] = \left(1-\frac{\lambda\beta}{n}\right)^{k} \geqslant 1-\frac{k\lambda\beta}{n}.$$

For large n, one can readily verify that

$$\left(1-\frac{\lambda\beta}{n+1}\right) \leqslant \left(1-\frac{k\lambda\beta}{n}\right)\left(1+\frac{k\lambda\beta}{n+1}\right).$$

Therefore,

$$\mathsf{E}_{k+1}^{(n+1)}\left[(1-\beta)^M\right] \leqslant \mathsf{E}_{k}^{(n)}\left[(1-\beta)^M\right],$$

which implies $\pi_{k+1,n+1} \geqslant \pi_{k,n}$ for sufficiently large n. A similar argument shows $\pi_{k,n+1} \geqslant \pi_{k-1,n}$, concluding the proof. □

B.3 Proof of Proposition 4

For every $\pi \leqslant \underline{\pi}$, the full exploitation equilibrium prevails, thus $k_n(\pi) = 0$. Also, for every $\pi \geqslant \bar{\pi}_\infty^{\text{local}}$, due to Lemma 2, it follows that $\pi > \bar{\pi}_n$ for large enough n, hence $k_n(\pi) = n$. Therefore, it remains to examine the limiting behavior of $k_n(\pi)/n$ on the intermediate region $(\underline{\pi}, \bar{\pi}_\infty^{\text{local}})$, where asymmetric equilibria prevail. By Theorem 2, there will be k_n explorers in the equilibrium if and only if $\pi_{k_n-1,n} < \pi \leqslant \pi_{k_n,n}$. According to equation (3.2), this condition translates to

$$\begin{aligned} \mathsf{E}_{k_n}^{(n)}\left[(1-\beta)^M\right] \leqslant \frac{(1-\delta)\left(\alpha(1-\pi\beta)-\pi\beta\right)}{\delta\pi\beta(\beta-\alpha(1-\beta))} < \mathsf{E}_{k_n-1}^{(n)}\left[(1-\beta)^M\right] \\ \Leftrightarrow (k_n-1) < \frac{\log\left(\frac{(1-\delta)(\alpha(1-\pi\beta)-\pi\beta)}{\delta\pi\beta(\beta-\alpha(1-\beta))}\right)}{\log\left(1-\lambda\beta/n\right)} \leqslant k_n\,. \end{aligned}$$

Therefore,

$$\lim_{n\to\infty}\frac{k_n}{n} = \frac{\log\left(\frac{(1-\delta)(\alpha(1-\pi\beta)-\pi\beta)}{\delta\pi\beta(\beta-\alpha(1-\beta))}\right)}{\lim_{n\to\infty} n\log\left(1-\lambda\beta/n\right)} = \frac{1}{\lambda\beta}\log\frac{\delta\pi\beta(\beta-\alpha(1-\beta))}{(1-\delta)\left(\alpha(1-\pi\beta)-\pi\beta\right)} = \frac{1}{\lambda\beta}\log\frac{C_2(\pi)}{C_1(\pi)}\,.$$

□

B.4 Proof of Proposition 5

From (A.6),

$$C_1(\pi) = C_2(\pi)\ \mathsf{E}_{k\sim\mathsf{Bin}(n-1,\mu)}\left[\mathsf{E}_k\left[(1-\beta)^M\right]\right].$$

In the local ER graph with k explorers, the number contacts $M \sim \mathsf{Bin}(k,p)$ where $p = \lambda/n$, so the inner expectation equals $(1-\beta p)^k$. Therefore,

$$C_1(\pi) = C_2(\pi)\ \mathsf{E}_{k\sim\mathsf{Bin}(n-1,\mu)}\left[(1-\beta p)^k\right] = C_2(\pi)(1-\mu\beta p)^{n-1}.$$

Taking logarithms of both sides yields (4.3).

To find the limiting μ_n^e, set $R(\pi) \coloneqq C_1(\pi)/C_2(\pi) \in \mathbb{R}_+$. As $n \to \infty$,

$$R(\pi)^{1/(n-1)} = \mathrm{e}^{\frac{1}{n-1}\log R(\pi)} \approx 1 + \frac{\log D(\pi)}{n-1}\,,$$

and therefore

$$\lim_{n\to\infty} \mu_n^e = \lim_{n\to\infty} \frac{n}{\lambda\beta}\left(1-1-\frac{1}{n-1}\log\frac{C_1(\pi)}{C_2(\pi)}\right) = \frac{1}{\lambda\beta}\log\frac{C_2(\pi)}{C_1(\pi)}\,,$$

which justifies (4.4). □

B.5 Proof of Proposition 6

We only present the proof in the subcritical regime ($\lambda \leqslant 1$ and thus $\zeta(\lambda) = 1$), showing that the asymptotic distribution of the size of a typical connected component is $B_1(\lambda) \stackrel{d}{=} \mathsf{Borel}(\lambda)$. The convergence of $|\mathcal{C}|$ to the mixture Borel distribution in the supercritical regime follows by simple conditioning on the extinction event. Henceforth, we assume $\lambda \leqslant 1$.

First, we show how the size of the connected component $|\mathcal{C}|$ in an ER random graph with parameters $(n, p = \lambda/n)$ can be approximated by the total number of descendants in a branching process with $\mathsf{Bin}(n, p)$ offspring distribution, which we denote by B. We use $\mathsf{P}_{n,p}$ to refer to the distribution of B. Theorems 4.2 and 4.3 of Van Der Hofstad (2016) jointly state that:

$$\mathsf{P}_{n-k,p}\left(B \geqslant k\right) \leqslant \mathsf{P}\left(|\mathcal{C}| \geqslant k\right) \leqslant \mathsf{P}_{n,p}\left(B \geqslant k\right)\,.$$

Next, we examine how the total number of progenies in a binomial branching process with parameters (n, p) can be approximated by a branching process with $\mathsf{Poisson}(np)$ offspring distribution, which we denote by T. We use P_λ to denote the distribution of the branching process with $\mathsf{Poisson}(\lambda)$ offspring distribution. Let $\lambda = np$ and fix $k \in \mathbb{N}$. Then, Theorem 3.20 in Van Der Hofstad (2016) implies that

$$\left|\mathsf{P}_{n,p}(B \geqslant k) - \mathsf{P}_\lambda(T \geqslant k)\right| \leqslant \frac{\lambda^2 k}{n}\,.$$

Combining the last two relations yields

$$\mathsf{P}_{\lambda(1-kn^{-1})}(T \geqslant k) - \frac{\lambda^2(n-k)k}{n^2} \leqslant \mathsf{P}\left(|\mathcal{C}| \geqslant k\right) \leqslant \mathsf{P}_\lambda(T \geqslant k) + \frac{\lambda^2 k}{n}\,.$$

For a fixed $k \in \mathbb{N}$, let $\lambda_n \coloneqq \lambda(1-kn^{-1})$. Then, using the method of characteristic functions, one can show P_{λ_n} weakly converges to P_λ as $n \to \infty$ (Theorem 6.3 in Kallenberg (2021)). This in turn implies $\mathsf{P}_{\lambda_n}(T < k) \to \mathsf{P}_\lambda(T < k)$, and hence $\mathsf{P}_{\lambda_n}(T \geqslant k) \to \mathsf{P}_\lambda(T \geqslant k)$. Combining

this with the above inequality, we conclude that for every $k \in \mathbb{N}$,

$$\lim_{n\to\infty} \mathsf{P}(|\mathcal{C}| \geqslant k) = \mathsf{P}_\lambda(T \geqslant k)\,. \tag{B.1}$$

Let $\bar{\mathbb{N}} = \mathbb{N} \cup \{\infty\}$; then $|\mathcal{C}|$ and T are $\bar{\mathbb{N}}$-valued random variables, where $\bar{\mathbb{N}}$ is a discrete metric space (so every function on it is continuous). The limiting result in (B.1) therefore implies weak convergence of $|\mathcal{C}|$ to T. The distribution of the total progeny in a Poisson branching process is known to follow the Borel distribution $\mathsf{Borel}_1(\lambda)$ (see Theorem 3.16 of Van Der Hofstad (2016)), concluding the of part (i).

A similar argument combined with conditioning on the extinction event in the supercritical regime implies that $|\mathcal{C}|$ converges in distribution to $\mathsf{Borel}_{\zeta(\lambda)}(\lambda)$ for $\lambda > 1$. Given our definition of $\zeta(\lambda) \equiv 1$ for $\lambda \leqslant 1$, we obtain convergence across all $\lambda \geqslant 0$, proving part (i).

Part (ii) follows immediately since every function on $\bar{\mathbb{N}}$ is continuous. In particular, $x \mapsto (1-\beta)^{x-1}$ is bounded and continuous, so by the weak convergence established above,

$$\lim_{n\to\infty} \mathsf{E}\left[(1-\beta)^{|\mathcal{C}|-1}\right] = \mathsf{E}\left[(1-\beta)^{B_{\zeta(\lambda)}(\lambda)-1}\right]\,,$$

justifying equation (4.6). □

B.6 Proof of Lemma 4

We consider the subcritical and supercritical regimes separately.

Subcritical regime ($\lambda \leqslant 1$). In this case $\zeta(\lambda) = 1$, so $B_{\zeta(\lambda)}(\lambda) = B(\lambda)$ and $\psi_\lambda(z) = \mathsf{E}\left[z^{B(\lambda)}\right]$. Since the total progeny of a branching process satisfies the distributional recursion $B(\lambda) = 1 + \sum_{i=1}^{N} B_i(\lambda)$, where $N \sim \mathsf{Poisson}(\lambda)$ and the $B_i(\lambda)$ are i.i.d. copies of $B(\lambda)$ independent of N, we have

$$\psi_\lambda(z) = \mathsf{E}\left[z^{1+\sum_{i=1}^{N} B_i(\lambda)}\right] = z\,\mathsf{E}\left[\prod_{i=1}^{N} z^{B_i(\lambda)}\right] = z\,\mathsf{E}\left[\psi_\lambda(z)^N\right] = z\,\mathrm{e}^{\lambda(\psi_\lambda(z)-1)},$$

where the last step uses the probability-generating function of the Poisson distribution.

Supercritical regime ($\lambda > 1$). By definition of the mixture Borel distribution, with probability $1-\zeta(\lambda)$ the component size is infinite (contributing $z^\infty = 0$ for $z \in [0,1)$), and

with probability $\zeta(\lambda)$ the size follows $\mathsf{Borel}(\lambda\zeta(\lambda))$. Therefore,

$$\psi_\lambda(z) = \zeta(\lambda)\cdot\mathsf{E}\left[z^{B(\lambda\zeta(\lambda))}\right].$$

We claim that $\lambda\zeta(\lambda) < 1$, so that $B(\lambda\zeta(\lambda))$ has the standard (subcritical) $\mathsf{Borel}(\lambda\zeta(\lambda))$ distribution. To see this, consider the convex function $f(x) = \mathrm{e}^{-\lambda(1-x)}$ on $[0,1]$. The fixed-point relation (4.5) states that $\zeta(\lambda) = f(\zeta(\lambda))$. In the supercritical regime, f has two fixed points in $[0,1]$: the smaller one is $\zeta(\lambda)$ and the larger one is 1. Since f is convex and $\zeta(\lambda) < 1$, the graph of f must cross the diagonal from above at $x = \zeta(\lambda)$, which means $f'(\zeta(\lambda)) < 1$. Computing $f'(\zeta(\lambda)) = \lambda\mathrm{e}^{-\lambda(1-\zeta(\lambda))} = \lambda\zeta(\lambda)$, we conclude that $\lambda\zeta(\lambda) < 1$.

Now, by the subcritical case, $g(z) \coloneqq \mathsf{E}\left[z^{B(\lambda\zeta(\lambda))}\right]$ satisfies

$$g(z) = z\,\mathrm{e}^{\lambda\zeta(\lambda)(g(z)-1)}.$$

Substituting $g(z) = \psi_\lambda(z)/\zeta(\lambda)$ gives

$$\frac{\psi_\lambda(z)}{\zeta(\lambda)} = z\,\mathrm{e}^{\lambda\zeta(\lambda)\left(\frac{\psi_\lambda(z)}{\zeta(\lambda)}-1\right)} = z\,\mathrm{e}^{\lambda(\psi_\lambda(z)-\zeta(\lambda))}.$$

Multiplying both sides by $\zeta(\lambda)$ and using $\zeta(\lambda) = \mathrm{e}^{-\lambda(1-\zeta(\lambda))}$ from (4.5), we obtain

$$\psi_\lambda(z) = z\,\zeta(\lambda)\,\mathrm{e}^{\lambda(\psi_\lambda(z)-\zeta(\lambda))} = z\,\mathrm{e}^{\lambda(\psi_\lambda(z)-1)}.$$

Finally, at $z = 1$ the identity $\psi_\lambda(1) = \zeta(\lambda)$ is consistent with (4.7), since $\zeta(\lambda) = 1\cdot\mathrm{e}^{\lambda(\zeta(\lambda)-1)}$ is precisely the fixed-point relation (4.5). □

B.7 Proof of Theorem 4

To prove the theorem, we need the following perturbative result:

Lemma 9. *Let $z = 1-\beta$, and recall that the PGF of the mixture Borel distribution satisfies (4.7). Then as $\beta \to 0$, for every $x \in \mathbb{R}$:*

$$\psi_{1+x\sqrt{\beta}}(1-\beta) = 1 - c(x)\sqrt{\beta} + o(\sqrt{\beta}), \tag{B.2}$$

where $c(x) \coloneqq x+\sqrt{x^2+2}$ is the crossover function.

Proof. Fix $x \in \mathbb{R}$ and set $\lambda_\beta \coloneqq 1+x\sqrt{\beta}$, $z \coloneqq 1-\beta$. Let $\psi_\beta \coloneqq \psi_{\lambda_\beta}(z) \in (0,1]$, and

$\varepsilon_\beta \coloneqq 1-\psi_\beta \in [0,1)$. The fixed point $\psi = ze^{\lambda(\psi-1)}$ becomes

$$1-\varepsilon_\beta = (1-\beta)\exp(-\lambda_\beta \varepsilon_\beta)$$

Taking logs and using $\log(1-y) = -(y+y^2/2)+O(y^3)$ yields

$$(1-\lambda_\beta)\varepsilon_\beta + \frac{\varepsilon_\beta^2}{2} = \beta + O(\beta^2+\varepsilon_\beta^3).$$

Since $\lambda_\beta = 1+O(\sqrt{\beta})$, the left-hand side is larger than $\varepsilon_\beta^2/4$ for β small, hence $\varepsilon_\beta = O(\sqrt{\beta})$. Write $\varepsilon_\beta = c_\beta\sqrt{\beta}$ and substitute $1-\lambda_\beta = -x\sqrt{\beta}$:

$$-xc_\beta + \frac{c_\beta^2}{2} = 1+O(\sqrt{\beta}).$$

Thus $c_\beta \to c$, where c solves $c^2-2xc-2 = 0$. Since $\varepsilon_\beta \geqslant 0$, we take the positive root $c(x) = x+\sqrt{x^2+2}$, which justifies the definition of the crossover function. Therefore $\varepsilon_\beta = c(x)\sqrt{\beta}+o(\sqrt{\beta})$, i.e.

$$\psi_{1+x\sqrt{\beta}}(1-\beta) = 1-c(x)\sqrt{\beta}+o(\sqrt{\beta}),$$

as claimed. □

Proof of Theorem 4. Since $\tau(\alpha,\beta) \to \tau_0 \in (0,1)$ as $\beta \to 0$, the parameter α is implicitly a function of β along this limit. We may then write (4.6) as

$$\bar{\pi}_\infty^{\text{global}}(\lambda,\beta) = \frac{A_\beta}{\beta\big(a_\beta + b_\beta \varphi_\beta(\lambda)\big)},$$

where

$$A_\beta \coloneqq \alpha(1-\delta), \quad a_\beta \coloneqq (1+\alpha)(1-\delta), \quad b_\beta \coloneqq \delta(\beta-\alpha(1-\beta)),$$
$$\varphi_\lambda(z) \coloneqq \mathsf{E}\left[z^{B_{\zeta(\lambda)}(\lambda)-1}\right] = \psi_\lambda(z)/z.$$

Hence

$$\bar{\pi}_\infty^{\text{global}}(\lambda,\beta) - \bar{\pi}_\infty^{\text{global}}(1,\beta) = -\frac{A_\beta}{\beta}\cdot\frac{b_\beta\big(\varphi_\lambda(1-\beta)-\varphi_1(1-\beta)\big)}{\big(a_\beta+b_\beta\varphi_\lambda(1-\beta)\big)\big(a_\beta+b_\beta\varphi_1(1-\beta)\big)}. \tag{B.3}$$

From $\tau(\alpha,\beta) = \alpha/(\beta(1+\alpha)) \to \tau_0 \in (0,1)$, we have $\alpha = \tau_0\beta+o(1)$. Therefore, $a_\beta = (1+\alpha)(1-\delta) = (1-\delta)+o(1)$, $b_\beta = \delta(\beta-\alpha(1-\beta)) = \delta(1-\tau_0)\beta+o(\beta)$, and $\frac{A_\beta}{\beta} = \frac{\alpha(1-\delta)}{\beta} = (1-\delta)\tau_0+o(1)$. Since $\varphi_\lambda(1-\beta) = O(1)$, it follows that $a_\beta+b_\beta\varphi_\lambda(1-\beta) = (1-\delta)+o(1)$, so the product of the two denominator factors in (B.3) is $(1-\delta)^2+o(1)$. Plugging these into (B.3) yields the

reduction:

$$\begin{aligned}\bar{\pi}_\infty^{\text{global}}(\lambda,\beta)-\bar{\pi}_\infty^{\text{global}}(1,\beta) = -\frac{\tau_0(1-\tau_0)\delta}{1-\delta}\,\beta\big(\varphi_\lambda(1-\beta)-\varphi_1(1-\beta)\big)\\ +o\left(\beta\big|\big(\varphi_\lambda(1-\beta)-\varphi_1(1-\beta)\big)\big|\right)\,.\end{aligned} \tag{B.4}$$

Since $\varphi_\lambda(1-\beta)=\psi_\lambda(1-\beta)/(1-\beta)$, and $\beta\to 0$, Lemma 9 gives $\varphi_{1+x\sqrt{\beta}}(1-\beta)=1-c(x)\sqrt{\beta}+o(\sqrt{\beta})$, and therefore

$$\varphi_{1+x\sqrt{\beta}}(1-\beta)-\varphi_1(1-\beta) = -\big(c(x)-c(0)\big)\sqrt{\beta}+o(\sqrt{\beta}) = -\big(c(x)-\sqrt{2}\big)\sqrt{\beta}+o(\sqrt{\beta})$$

Inserting this into (B.4) yields

$$\bar{\pi}_\infty^{\text{global}}(\lambda,\beta)-\bar{\pi}_\infty^{\text{global}}(1,\beta) = \frac{\tau_0(1-\tau_0)\delta}{1-\delta}\,\beta^{3/2}\big(c(x)-\sqrt{2}\big)+o(\beta^{3/2})\,.$$

Dividing by $\beta^{3/2}$ and letting $\beta\downarrow 0$ gives the claimed limit. □

C Proof of Results in Section 5

C.1 Proof of Proposition 7

Suppose that initially at $\lambda=\lambda_0$, the common belief lies in $(\pi_{k,n},\pi_{k+1,n}]$, so that $k+1$ agents explore in equilibrium. By (3.2), the belief cutoffs $\pi_{k,n}$ are increasing in λ, so there exists $\lambda_{k,n}(\pi)>\lambda_0$ at which $\pi=\pi_{k,n}$ and the equilibrium transitions to k explorers. Part (i) of Lemma 5 gives the change in social surplus at $\lambda=\lambda_{k,n}(\pi)$ as $u_{k,n}(\pi)-u_{k+1,n}(\pi)=-\Delta u_k(\pi)$. Substituting $\pi=\pi_{k,n}$ into (5.2) yields, at $p=\lambda_{k,n}(\pi)/n$,

$$u_{k,n}(\pi_{k,n})-u_{k+1,n}(\pi_{k,n}) = -\delta\pi_{k,n}\beta p(\beta-\alpha(1-\beta))(n-1-k)(1-p\beta)^k\,,$$

which is always negative. Hence the equilibrium social surplus just above $\lambda_{k,n}(\pi)$ is strictly lower than just below it, confirming the discontinuous decline.

C.2 Proof of Proposition 8

For $\pi\leqslant\underline{\pi}$, no agent explores and the social surplus is zero, as in (5.5a). In the intermediate region $\pi\in(\underline{\pi},\bar{\pi}_\infty^{\text{local}})$, a fraction $k_n/n\to\kappa(\pi)$ of agents explore, as characterized in Proposi-

tion 4. Since all terms with the $[\cdot]^+$ operator vanish in (5.1), the average equilibrium social surplus on this region is

$$\begin{aligned}\frac{u_{k_n,n}(\pi)}{n} &= (1-\delta)\,\frac{k_n}{n}\,(\pi\beta-\alpha(1-\pi\beta))+\delta\pi(\beta-\alpha(1-\beta))\beta \\ -\delta\,\frac{k_n}{n}\,\pi(\beta-\alpha(1-\beta))(1-\beta)\mathsf{E}^{(n)}_{k_n-1}\big[(1-\beta)^M\big]&-\delta\,\frac{n-k_n}{n}\,\pi(\beta-\alpha(1-\beta))\,\mathsf{E}^{(n)}_{k_n}\big[(1-\beta)^M\big]\,.\end{aligned} \tag{C.1}$$

Since $M \sim \mathsf{Bin}(k_n, \lambda/n)$ converges in distribution to $\mathsf{Poisson}(\lambda\kappa(\pi))$ as $n \to \infty$,

$$\lim_{n\to\infty} \mathsf{E}^{(n)}_{k_n-1}\left[(1-\beta)^M\right] = \lim_{n\to\infty} \mathsf{E}^{(n)}_{k_n}\left[(1-\beta)^M\right] = \mathrm{e}^{-\lambda\beta\kappa(\pi)}\,.$$

Taking limits in (C.1) and substituting $\kappa(\pi)$ from (4.2) yield (5.5b).

By (4.1), $\bar{\pi}^{\text{local}}_\infty$ is increasing in λ with $\bar{\pi}^{\text{local}}_\infty \uparrow \tau$ as $\lambda \to \infty$. Hence for every $\pi \in (\underline{\pi}, \tau)$, there exists a threshold $\lambda(\pi)$ such that $\bar{\pi}^{\text{local}}_\infty \geqslant \pi$ for all $\lambda \geqslant \lambda(\pi)$, so $\bar{u}_\infty$ follows (5.5b) and is constant in λ, proving the first claim in Part (i).

Finally, for $\pi \geqslant \bar{\pi}^{\text{local}}_\infty$, all agents explore ($\kappa(\pi) = 1$) and M converges in distribution to $\mathsf{Poisson}(\lambda)$. Using this limiting result in (5.1) yields (5.5c), completing the proof of Part (i) and establishing Part (ii). □

C.3 Proof of Theorem 5

We divide the proof into a few steps:

Step 1 (social surplus representation). Recall that the resulting number of explorers when each agent explores independently with probability μ^e_n is denoted by $k^{\text{mixed}}_n \sim \mathsf{Bin}(n, \mu^e_n)$. Similar to (C.1) the finite-n average social surplus resulted from this group is

$$\begin{aligned}\frac{u_{k^{\text{mixed}}_n,n}(\pi)}{n} &= (1-\delta)\,\frac{k^{\text{mixed}}_n}{n}\,(\pi\beta-\alpha(1-\pi\beta))+\delta\pi(\beta-\alpha(1-\beta))\beta \\ &\quad -\delta\,\frac{k^{\text{mixed}}_n}{n}\,\pi(\beta-\alpha(1-\beta))(1-\beta)\mathsf{E}^{(n)}_{k^{\text{mixed}}_n-1}\big[(1-\beta)^M\big] \\ &\quad -\delta\,\frac{n-k^{\text{mixed}}_n}{n}\,\pi(\beta-\alpha(1-\beta))\,\mathsf{E}^{(n)}_{k^{\text{mixed}}_n}\big[(1-\beta)^M\big]\,.\end{aligned}$$

Since $M \sim \mathsf{Bin}(k_n^{\text{mixed}}, \lambda/n)$, applying the PGF of the Binomial distribution and setting $\xi_n \coloneqq k_n^{\text{mixed}}/n$ reduce the above expression to

$$\frac{u_{k_n^{\text{mixed}},n}(\pi)}{n} = (1-\delta)\,\xi_n\,(\pi\beta-\alpha(1-\pi\beta))+\delta\pi(\beta-\alpha(1-\beta))\beta$$
$$-\delta\,\xi_n\,\pi(\beta-\alpha(1-\beta))(1-\beta)(1-\lambda\beta/n)^{n\xi_n-1}-\delta\,(1-\xi_n)\,\pi(\beta-\alpha(1-\beta))\,(1-\lambda\beta/n)^{n\xi_n}.$$

Step 2 (uniform convergence). We show that $\gamma_n(x) \coloneqq (1-\lambda\beta/n)^{nx}$ converges to $\gamma(x) \coloneqq \mathrm{e}^{-\lambda\beta x}$ uniformly over $x \in [0,1]$. Taking logarithms and applying the Taylor expansion $\log(1-\varepsilon) = -\varepsilon-\varepsilon^2/2+O(\varepsilon^3)$ gives

$$\log\gamma_n(x)-\log\gamma(x) = nx\log(1-\lambda\beta/n)+\lambda\beta x = -\frac{(\lambda\beta)^2 x}{2n}+x\,O(1/n^2)\,,$$

hence

$$\sup_{x\in[0,1]}|\log\gamma_n(x)-\log\gamma(x)| \leqslant \frac{(\lambda\beta)^2}{2n}+O(1/n^2)\,.$$

For fixed λ and β, the functions γ_n and γ are uniformly bounded over $n \in \mathbb{N}$ and $x \in [0,1]$. Since the exponential map $t \mapsto \mathrm{e}^t$ is Lipschitz continuous on bounded sets, there exists a constant $L > 0$, independent of n and x, such that

$$\sup_{x\in[0,1]}|\gamma_n(x)-\gamma(x)| \leqslant L\sup_{x\in[0,1]}|\log\gamma_n(x)-\log\gamma(x)| \leqslant L\left(\frac{(\lambda\beta)^2}{2n}+O(1/n^2)\right).$$

Therefore $\lim_{n\to\infty}\sup_{x\in[0,1]}|\gamma_n(x)-\gamma(x)| = 0$, establishing uniform convergence.

Step 3 (*generalized* continuous mapping). We invoke Theorem 5.27 of Kallenberg (2021), which states that: For any metric spaces S, T and set $C \in S$, consider some measurable functions $f, f_1, f_2, \ldots : S \to T$ satisfying $s_n \to s \in C \Rightarrow f_n(s_n) \to f(s)$. Then for any random elements $\xi_1, \xi_2, \ldots \in S$,

$$\xi_n \xrightarrow{d} \xi \in C \text{ a.s.} \Rightarrow f_n(\xi_n) \xrightarrow{d} f(\xi)\,.$$

In our setting, ξ_n converges almost surely, and hence in distribution, to $\kappa(\pi)$. Let

$C \coloneqq [0,1]$ and define

$$f_n(s) \coloneqq (1-\delta)\, s\, (\pi\beta - \alpha(1-\pi\beta)) + \delta\pi(\beta - \alpha(1-\beta))\beta$$
$$-\delta\, s\, \pi(\beta-\alpha(1-\beta))(1-\beta)(1-\lambda\beta/n)^{ns-1} - \delta\,(1-s)\,\pi(\beta-\alpha(1-\beta))\,(1-\lambda\beta/n)^{ns}\,,$$
$$f(s) \coloneqq (1-\delta)\, s\, (\pi\beta - \alpha(1-\pi\beta)) + \delta\pi(\beta - \alpha(1-\beta))\beta$$
$$-\delta\, s\, \pi(\beta-\alpha(1-\beta))(1-\beta)\mathrm{e}^{-\lambda\beta s} - \delta\,(1-s)\,\pi(\beta-\alpha(1-\beta))\,\mathrm{e}^{-\lambda\beta s}\,.$$

For every convergent sequence $s_n \to s \in C$,

$$|f_n(s_n) - f(s)| \leqslant |f_n(s_n) - f(s_n)| + |f(s_n) - f(s)|\ .$$

By the previous step, f_n converges uniformly to f on C, so both terms on the right vanish as $n \to \infty$, giving $\lim_{n\to\infty} f_n(s_n) = f(s)$. By the above theorem, $f_n(\xi_n) \xrightarrow{d} f(\kappa(\pi))$. Since the limit $f(\kappa(\pi))$ is a constant, convergence in distribution implies almost sure convergence: $f_n(\xi_n) \xrightarrow{\text{a.s.}} f(\kappa(\pi))$. Finally, substituting $\kappa(\pi)$ from (4.2) into $f(\cdot)$ gives $f(\kappa(\pi)) = \bar{u}_\infty$. □

C.4 Proof of Lemma 6

We separately show B_k is larger than both of the arguments of the max operator. First, observe that $B_k \geqslant (1-\beta)^r A_k$ if and only if

$$\begin{aligned}
0 \leqslant\ & k\Big[Q_{k-1}(r-1)(1-\beta)^r + \sum_{m=r}^{k-1} q_{k-1}(m)(1-\beta)^{m+1}\Big] \\
& -(k+1)\Big[Q_k(r-1)(1-\beta)^r + \sum_{m=r}^{k} q_k(m)(1-\beta)^{m+1}\Big] \\
& +(n-k)\Big[Q_k(r)(1-\beta)^r + \sum_{m=r+1}^{k} q_k(m)(1-\beta)^{m}\Big] \\
& -(n-k-1)\Big[Q_{k+1}(r)(1-\beta)^r + \sum_{m=r+1}^{k+1} q_{k+1}(m)(1-\beta)^{m}\Big]\,.
\end{aligned} \tag{C.2}$$

Below, E_k denotes expectation with respect to $\mathsf{Bin}(k,p)$, where M follows the distribution indicated by the subscript.[9]

$$\begin{aligned}\text{RHS of (C.2)} = k\mathsf{E}_{k-1}\left[(1-\beta)^{(M+1)\vee r}\right]-(k+1)\mathsf{E}_k\left[(1-\beta)^{(M+1)\vee r}\right]\\ +(n-k)\mathsf{E}_k\left[(1-\beta)^{M\vee r}\right]-(n-k-1)\mathsf{E}_{k+1}\left[(1-\beta)^{M\vee r}\right].\end{aligned}$$

Since each function inside the expectation operators is decreasing in M, first-order stochastic dominance—$\mathsf{Bin}(k,p) \succeq \mathsf{Bin}(k-1,p)$ for the first line and $\mathsf{Bin}(k+1,p) \succeq \mathsf{Bin}(k,p)$ for the second—yields the lower bound:

$$\begin{aligned}\textit{RHS of (C.2)} &\geqslant \mathsf{E}_{k+1}\left[(1-\beta)^{M\vee r}\right]-\mathsf{E}_k\left[(1-\beta)^{(M+1)\vee r}\right]\\ &= p\mathsf{E}_k\left[(1-\beta)^{(M+1)\vee r}\right]+(1-p)\mathsf{E}_k\left[(1-\beta)^{M\vee r}\right]-\mathsf{E}_k\left[(1-\beta)^{(M+1)\vee r}\right]\\ &= (1-p)\mathsf{E}_k\left[(1-\beta)^{M\vee r}-(1-\beta)^{(M+1)\vee r}\right]\geqslant 0\,,\end{aligned}$$

where the second line follows by conditioning $\mathsf{Bin}(k+1,p)$ on its last Bernoulli trial. This establishes the first part.

For the second inequality, $B_k \geqslant (1-\beta)^{r+1}A_k$, an equivalent condition is:

$$\begin{aligned}0 \leqslant k\Big[Q_{k-1}(r-1)(1-\beta)^{r+1}+\sum_{m=r}^{k-1} q_{k-1}(m)(1-\beta)^{m+1}\Big]\\ -(k+1)\Big[Q_k(r-1)(1-\beta)^{r+1}+\sum_{m=r}^{k} q_k(m)(1-\beta)^{m+1}\Big]\\ +(n-k)\Big[Q_k(r)(1-\beta)^{r+1}+\sum_{m=r+1}^{k} q_k(m)(1-\beta)^{m}\Big]\\ -(n-k-1)\Big[Q_{k+1}(r)(1-\beta)^{r+1}+\sum_{m=r+1}^{k+1} q_{k+1}(m)(1-\beta)^{m}\Big].\end{aligned} \tag{C.3}$$

Consolidating the right-hand side yields:

$$\begin{aligned}\textit{RHS of (C.3)} = (1-\beta)\Big(k\mathsf{E}_{k-1}\left[(1-\beta)^{M\vee r}\right]-(k+1)\mathsf{E}_k\left[(1-\beta)^{M\vee r}\right]\\ +(n-k)\mathsf{E}_k\left[(1-\beta)^{(M-1)\vee r}\right]-(n-k-1)\mathsf{E}_{k+1}\left[(1-\beta)^{(M-1)\vee r}\right]\Big).\end{aligned}$$

[9]The distribution of M thus varies across terms.

Using the first-order stochastic dominance once again yields the following lower bound:

$$\begin{aligned} \textit{RHS of (C.3)} &\geqslant (1-\beta)\Big(\mathsf{E}_k\left[(1-\beta)^{(M-1)\vee r}\right]-\mathsf{E}_k\left[(1-\beta)^{M\vee r}\right]\Big) \\ &= (1-\beta)\mathsf{E}_k\left[(1-\beta)^{(M-1)\vee r}-(1-\beta)^{M\vee r}\right] \geqslant 0\,. \end{aligned}$$

This establishes the second part and concludes the proof. □

C.5 Proof of Theorem 6

We first justify the lower cutoff for full exploitation. By part (i) of Lemma 5, $\Delta u_0 \leqslant 0$ on $\pi \leqslant \underline{\pi}^*$ and $\Delta u_0 > 0$ on $\underline{\pi}^* < \pi \leqslant \tau$. Since Δu_k is decreasing in k on $[0,\tau]$ by Lemma 7, we have $\Delta u_k \leqslant \Delta u_0 \leqslant 0$ on $\pi \leqslant \underline{\pi}^*$, establishing the optimality of full exploitation in this region. When $\frac{\pi}{1-\pi} \in \left[\frac{\alpha}{\beta-\alpha(1-\beta)}, \frac{\alpha}{(\beta-\alpha(1-\beta))(1-\beta)}\right]$, part (iii) of Lemma 5 (with $r=0$) identifies $\Delta u_0(\pi)$. If $A_0 \geqslant 0$, then $B_0 \geqslant 0$ by Lemma 6, and hence

$$\begin{aligned} \Delta u_0(\pi) &= (1-\delta)(\pi\beta-\alpha(1-\pi\beta))+\delta\Big(\pi(\beta-\alpha(1-\beta))\,B_0-\alpha(1-\pi)A_0\Big) \\ &\geqslant \delta\pi(\beta-\alpha(1-\pi))\left(B_0-\frac{\alpha(1-\pi)}{\pi(\beta-\alpha(1-\beta))}\,A_0\right) \geqslant \delta\pi(\beta-\alpha(1-\pi))(B_0-A_0) \geqslant 0\,. \end{aligned}$$

where the second to-last inequality holds because, $\pi \geqslant \tau$.

Alternatively, if $A_0 < 0$, then by Lemma 6,

$$\begin{aligned} \Delta u_0(\pi) &\geqslant \delta\big(\pi(\beta-\alpha(1-\beta))B_0-\alpha(1-\pi)A_0\big) \\ &= \delta\pi(\beta-\alpha(1-\pi))\left(B_0-\frac{\alpha(1-\pi)}{\pi(\beta-\alpha(1-\beta))}\,A_0\right) \\ &\geqslant \delta\pi(\beta-\alpha(1-\pi))\,(B_0-(1-\beta)A_0) \geqslant 0\,, \end{aligned}$$

where the last inequality holds because $\frac{\pi}{1-\pi} \leqslant \frac{\alpha}{(\beta-\alpha(1-\beta))(1-\beta)}$.

Finally, when $\frac{\pi}{1-\pi} > \frac{\alpha}{(\beta-\alpha(1-\beta))(1-\beta)}$, part (ii) of Lemma 5 implies $\Delta u_0(\pi) > 0$. We conclude that $u_1(\pi) > u_0(\pi)$ for all $\pi > \underline{\pi}^*$, so full exploitation is optimal if and only if $\pi \leqslant \underline{\pi}^*$.

Next, we establish the optimality of full exploration above $\bar{\pi}^*$. On the region $\pi \leqslant \tau$, part (i) of Lemma 5 shows that $\Delta u_{n-1}(\pi) < 0$ for $\pi < \bar{\pi}^*$ and $\Delta u_{n-1}(\pi) \geqslant 0$ for $\pi \in [\bar{\pi}^*, \tau]$. By Lemma 7, $\Delta u_k(\pi) \geqslant \Delta u_{n-1}(\pi) \geqslant 0$ for all $\pi \in [\bar{\pi}^*, \tau]$. For $\pi \geqslant \tau$, part (ii) of Lemma 5 directly gives $\Delta u_k(\pi) \geqslant 0$. Hence, full exploration is socially optimal if and only if $\pi \geqslant \bar{\pi}^*$.

It remains to show $\Delta u_k(\pi) \geqslant 0$ for every $\pi > \tau$. For each such π, there exists r such that

$\frac{\alpha}{(\beta-\alpha(1-\beta))(1-\beta)^r} \leqslant \frac{\pi}{1-\pi} \leqslant \frac{\alpha}{(\beta-\alpha(1-\beta))(1-\beta)^{r+1}}$. If $k < r$, part (ii) gives $\Delta u_k > 0$ directly. If $k \geqslant r$ and $A_k \geqslant 0$, Lemma 6 implies

$$\begin{aligned}\Delta u_k(\pi) &= (1-\delta)(\pi\beta-\alpha(1-\pi\beta))+\delta\big(\pi(\beta-\alpha(1-\beta))B_k-\alpha(1-\pi)A_k\big)\\ &\geqslant \delta\pi(\beta-\alpha(1-\beta))\left(B_k-\frac{\alpha(1-\pi)}{\pi(\beta-\alpha(1-\beta))}\,A_k\right)\\ &\geqslant \delta\pi(\beta-\alpha(1-\beta))\left(B_k-(1-\beta)^r A_k\right)\,.\end{aligned}$$

Similarly, if $A_k < 0$, then

$$\begin{aligned}\Delta u_k(\pi) &\geqslant \delta\big(\pi(\beta-\alpha(1-\beta))B_k-\alpha(1-\pi)A_k\big)\\ &= \delta\pi(\beta-\alpha(1-\beta))\left(B_k-\frac{\alpha(1-\pi)}{\pi(\beta-\alpha(1-\beta))}\,A_k\right)\\ &\geqslant \delta\pi(\beta-\alpha(1-\beta))\left(B_k-(1-\beta)^{r+1}A_k\right) \geqslant 0\,.\end{aligned}$$

This confirms that u_k is increasing in k on $[\bar{\pi}^*, 1]$, completing the proof. □

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