Assignment 4

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1 One-link pendulum

First, we write out the equations of motion:

1. Newton equation:

$$\begin{aligned} \sum F &= M_1 \ddot{P}_1 \\ \Rightarrow F_c + M_1 g &= M_1 \ddot{P}_1 \\ \Rightarrow M_1 \ddot{P}_1 - F_c &= M_1 g \end{aligned}$$

2. Euler equation:

$$\begin{aligned} \sum \tau &= I_{1_w} \dot{\omega}_1 + \omega_1 \times I_{1_w} \omega_1 \\ \Rightarrow r_w \times F_c &= I_{1_w} \dot{\omega}_1 + \omega_1 \times I_{1_w} \omega_1 \\ \Rightarrow I_{1_w} \dot{\omega}_1 - r_w \times F_c &= \omega_1 \times I_{1_w} \omega_1 \\ I_{1_w} \dot{\omega}_1 - \widetilde{r_w} F_c &= \omega_1 \times I_{1_w} \omega_1 \end{aligned}$$

3. Fixed point constraint:

$$\begin{aligned} & P_{1_c} = A \\ \Rightarrow \quad & \dot{P}_{1_c} = 0 \\ \Rightarrow \quad & \ddot{P}_{1_c} = 0 \\ \Rightarrow \quad & \ddot{P}_{1_c} = \ddot{P}_1 + \dot{\omega}_1 \times r_w + \omega_1 \times (\omega_1 \times r_w) = 0 \\ \Rightarrow \quad & -\ddot{P}_1 + \widetilde{r_w} \dot{\omega}_1 = \omega_1 \times (\omega_1 \times r_w) \end{aligned}$$

Now, we have to arrange them in a matrix in the form of $Ax = b$:

$$\left[\begin{array}{c|c|c} \mathrm{M} & & \begin{matrix}-1&&\\&-1&\\&&-1\end{matrix} \\ \hline & \mathrm{I} & -\widetilde{r_w} \\ \hline \begin{matrix}-1&&\\&-1&\\&&-1\end{matrix} & \widetilde{r_w} & \end{array}\right] \begin{bmatrix}\ddot{P}_1\\ \dot{\omega}_1 \\ F_c\end{bmatrix} = \begin{bmatrix} M_1 g \\ \omega_1 \times I_{1_w}\omega_1 \\ \omega_1\times(\omega_1\times r_w)\end{bmatrix}$$

Then, the only missing link is to calculate the moment of inertia. By symmetry, I_{1_L}, i.e. innertia in the local frame, must be a diagonal matrix. Also, a quick search gives the formula for a solid cuboid as:

$$I_{1_l} = \frac{m}{12}\begin{bmatrix} w^2+d^2 & 0 & 0 \\ 0 & d^2+h^2 & 0 \\ 0 & 0 & w^2+h^2 \end{bmatrix}$$

Therefore, we can get the innertia in the world frame just by applying the rotation matrix:

$$I_{1_w} = R\, I_{1_l}\, R^{-1}$$

Also, it's important to note that the r_w here refers to the distance from the center of mass of the pendulum to the fixed point in the world frame which can be calculated again using the rotation matrix:

$$r_w = R\, r_l$$