Numerical optimization for large scale problems
Unconstrained optimization

Assignment 1: Newton and finite difference Newton methods

Consider the following problem:

$$\min_{x \in \mathbb{R}^n} f(x)$$

where

$$f(x) = \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n-1} x_i x_{i+1}$$

Use your own implementation of the Newton method with line-search to solve the problem with $n = 10^4$ and $n = 10^6$, both using exact derivatives and using finite differences to approximate the gradient and the Hessian matrix. Compare the behavior of the two implementations, using the following values for the increment h:

$$h = 10^{-k}\hat{x}, \qquad k = 2, 4, 6, 8, 10, 12, 14$$

where $\hat{x}$ is the point at which the derivatives have to be approximated. The comparison should be made, for example, in terms of number of iterations and computing time.

Write a report summarizing the results with tables and/or figures, commenting the results obtained.