




Le génie pour l'industrie

MGA852

AIR NAVIGATION, GNSS AND EMBEDDED INERTIAL SYSTEMS

Technical Description of Laboratory Activities and Final Project

Professor

René Jr. Landry 

Written by:

Farzan Farhangian 

July 2022

TABLE OF CONTENT

SUMMARY OF LABORATORIES AND PROJECT	3
1 LABORATORY I: HEADING AND ORIENTATION ESTIMATION	4
1.1 Objectives	4
1.2 Simulation and Implementation Theory	4
1.3 Evaluation Criteria	4
1.4 References	5
2 LABORATORY II: PEDESTRIAN DEAD-RECKONING NAVIGATION	6
2.1 Objectives	6
2.2 Simulation and Implementation Theory	6
2.3 Evaluation Criteria	6
2.4 References	7
3 FINAL PROJECT: INDOOR NAVIGATION SOLUTION	8
3.1 Objectives	8
3.2 Experimental setup and data recording	8
3.3 Implementation, simulation and programming	9
3.4 Evaluation Criteria	9
APPENDIX I	10
APPENDIX II	15
APPENDIX III	17

SUMMARY OF LABORATORIES AND PROJECT

Summary of laboratory activities and the final project:

- I. **Laboratory I:** In this activity, we are going to implement basics of orientation estimation. Using a proper model, we can obtain the 3-axis orientation from gyroscope, accelerometer and magnetometer. Two different methods will be used in order to design and implement the orientation estimation. These methods are namely, Madgwick and basic gyro rate orientation. The results will be compared together in order to find the best method and the highest accuracy.
- II. **Laboratory II:** The Pedestrian Dead Reckoning (PDR) system can be expanded to use more informative measurement in the main Extended Kalman Filter (EKF) system. In this method, the length of each step of the pedestrian will be obtained using different length and step estimators. On the other side, the change in heading will be calculated using the magnetometer and accelerometer data. Using these two measurements (step length and heading change), and a central EKF, each position will be updated from its previous value.
- III. **Project:** The basic concept of the final project is inherited from the previous activities. The goal is to compare all the methods and find a best solution in terms of positioning accuracy with a minimum end to end and RMS error. A dual foot-mounted INS will be used by using the ZUPT, step lengths and heading deviation in a central EKF. The impact of different measurement model will be investigated, and the best measurement combination will be presented as a final solution.

LABORATORY I: HEADING AND ORIENTATION ESTIMATION

1.1. Objectives

As a basic concept of orientation and heading estimation will be implemented in MATLAB. The goal is to use the raw data of 9 degree of freedom IMU includes accelerometer, gyroscope, and magnetometer to calculate the sensor's attitude. Madgwick, regular orientation estimation method based on the gyro rate will be implemented and compared. Reaching the best heading accuracy is one of the main important objectives of this lab.

1.2. Simulation and Implementation Theory

The following section presents the essentials so that you can carry out the simulation of the various orientation estimation. Shortly, your model should follow the critical parts below:

- a. A code to obtain the orientation from gyro rate (quaternion)
- b. Converting the quaternion to Euler angles
- c. A code to estimate the orientation using the Madgwick method (quaternion)
- d. Using 9 Degree of freedom data (all accel, gyro and magnet)
- e. Data representation

1.3. Evaluation Criteria

Laboratory I will be evaluated in two parts. First, the evaluation of the model you have developed and second, the evaluation of your laboratory report and discussions. Your proposed model is not intended to assess your programming skills but to assess the following points:

- Your ability to technically apply the material presented in class.
- The accuracy of implementing the mathematical equations used in your model.
- The validity and relevance of the results presented to the user.
- Innovation and novelty in improving the current methods.
- Theoretical knowledge and understanding and using them in implementing the model.

The evaluation of your laboratory report will mainly aim to assess the following points:

- The approach you used in the laboratory.
- Your mastery of the material presented in class.
- The validity and relevance of the results presented in your lab report.
- Your ability to analyze results.
- Visualization, conclusion and discussion.

The suggested structure for the final report of laboratories is presented in Appendix 4.

1.4. References

- [1] J. Nilsson, Titterton D. H J. L Weston and Institution of Electrical Engineers. 1997. *Strapdown Inertial Navigation Technology*. London: Peter Peregrinus.

LABORATORY II: PEDESTRIAN DEAD-RECKONING NAVIGATION

2.1. Objectives

Implementing a pedestrian dead-reckoning system is the second algorithm for IMU-based indoor navigation systems which position of each timestamp relies on that value in previous timestamp added by the distance walked in special heading deviation. The bottle neck of this activity is to estimate an accurate heading varying between special clusters as well as distance of each step. It could be obtained using analyzing the data of gyroscope and accelerometer in particular window times. Later, the Kalman Filter can be designed to update and predict the final position using a proper measurement model which relates the measured distance to the location and attitude of the pedestrian.

2.2. Simulation and Implementation Theory

The following section presents the essentials so that you can carry out the simulation of the various pedestrian inertial navigation systems. Shortly, your model should follow the critical parts below:

- a. Dead-reckoning 2-dimensional kinematic model
- b. Calculating a measurement model for distance and heading variation observations
- c. Position update (without KF)
- d. Measurement model calculation (with Filter)

Please refer to Appendix 2 for more information and details about the implementation theory associated to this laboratory activity.

2.3. Evaluation Criteria

Laboratory II will be evaluated in two parts. First, the evaluation of the model you have developed and second, the evaluation of your laboratory report and discussions. Your proposed model is not intended to assess your programming skills but to assess the following points:

- Your ability to technically apply the material presented in class.
- The accuracy of implementing the mathematical equations used in your model.
- The validity and relevance of the results presented to the user.
- Innovation and novelty in improving the current methods (new length estimations)
- Theoretical knowledge and understanding and using them in implementing the model.

- Design a correct EKF design and measurement model of the PDR system

The evaluation of your laboratory report will mainly aim to assess the following points:

- The approach you used in the laboratory.
- Your mastery of the material presented in class.
- The validity and relevance of the results presented in your lab report.
- Your ability to analyze results.
- Visualization, conclusion and discussion.

2.4. References

- [1] Hou, X.; Bergmann, J. HeadSLAM: Pedestrian SLAM with Head-Mounted Sensors. *Sensors* **2022**, 22, 1593. <https://doi.org/10.3390/s22041593> Yin, H., H. Guo, X.
- [2] <https://april.eecs.umich.edu/media/pdfs/kwanmuang2015phd.pdf>

FINAL PROJECT: INDOOR NAVIGATION SOLUTION

3.1. Objectives

For the final project we are going to use all the algorithms, observations and kinematic knowledge in hand to achieve the best and the most robust and innovative pedestrian indoor navigation solution in GPS-denied urban indoors. The goal is to contribute in all activities from data gathering, modeling to implementing, simulating, coding and writing your final report. A dual or single foot mounted INS can be used as a main kinematic model. However, we can use any measurement, additional AHRS model, drift reduction, step length estimation, ZUPT, etc (in combination). To do that a new measurement model need to be defined based on your selected observations. New AHRS algorithms can be used to achieve more precise heading estimation.

3.2. Experimental setup and data recording

A [MPU9250](#) low cost IMU will be provided for each student (or group of students as a team). It includes 3-axis accelerometer, magnetometer, and gyroscope in order to give you this ability to be able to use any 6 DoF or 9 DoF navigation and orientation estimation algorithm. Figure below shows the proposed sensor. It also has a Graphical User Interface (GUI) to visualize, record different data as well as an embedded orientation estimation algorithm to give us a true orientation reference.



The sensor need to be mounted on a foot (or dual on both feet). The raw IMU data as well as the AHRS true reference need to be recorded for at least 5 minutes of real experiment. The walking should be a regular walk, in rectangular corridors or other path shapes (up to you).

3.3. Implementation, simulation and programming

Students can use both MATLAB programming or SIMULINK for implementing their algorithms. The codes need to follow these criteria:

- Simple, well-written and with description and comment for each function and part of code to be understandable and legible
- Functions need to be in separate folder and just one main function run the program. Try to reduce the complexity of the main function by defining the different modules in your program.
- Try to define variables with proper name.
- Correct visualizing with topic for each figure, legends, x and y axis names and labels as well as a white background for all the figures.
- During the experiment you need to record your correct and true position using the true ETS map.

3.4. Evaluation Criteria

Here are the most important technical keys to achieve the best grade for the final project:

- Your ability to present your understanding about (E)KF, system model and navigation.
- Good understanding of orientation estimation with applying several AHRS methods and investigating the impact and accuracy of each of them in the final results.
- Using different observation and measurements and define the best measurement model based on your system.
- Implementing different algorithms and compare their final results like different drift reduction methods, ZUPT, dual foot mounted INS, etc.
- Accuracy of your final heading and positioning results.
- Understanding the frames and coordinates of navigation.
- Different representation of orientation and understanding their transformation.
- Calculating correct errors (Root mean Square).

The evaluation of your laboratory report will mainly aim to assess the following points:

- The validity and relevance of the results presented in your lab report.
- Your ability to analyze results.
- Visualization, conclusion and discussion.
- Please refer to the Appendix 3.

APPENDIX I

Notations:

δx_k is the error state at time k.

n is the navigation frame and s the sensor frame.

b_a and b_ω are the acceleration and gyroscope biases, respectively.

a_c is the compensated acceleration.

p , v and a are respectively the position, the velocity, and the acceleration.

$R_{s\ k}^n$ is the rotation matrix which transforms sensor frame to navigation frame at time k.

g is the gravity value.

$x_{k+1|k}$ is the value of x before the EKF correction at time $k+1$ and $x_{k+1|k+1}$ is the value of x after the EKF correction at time $k+1$.

Φ_k is the state-transition matrix at time k .

Error state model (15 dimensions):

Let

$$\delta x_k = \begin{bmatrix} \delta \Phi_k \\ \delta p_k \\ \delta v_k \\ b_{\omega_k} \\ b_{a_k} \end{bmatrix}$$

be the error state at time k.

➤ Bias correction for acceleration and angular velocity:

$$\omega_{c_{k+1}}^s = \omega_{k+1}^s - \delta b_{\omega_k}^s$$

$$a_{c_{k+1}}^s = a_{k+1}^s - \delta b_{a_k}^s$$

➤ Rotation matrix integration:

$\delta\Omega_k$ denotes the skew-symmetric matrix for angular velocity.

$$\delta\Omega_k = \begin{bmatrix} 0 & -\omega_{czk}^s & \omega_{cyk}^s \\ \omega_{czk}^s & 0 & -\omega_{cxk}^s \\ -\omega_{cyk}^s & \omega_{cxk}^s & 0 \end{bmatrix}$$

$$R_{s\ k+1|k}^n = R_{s\ k|k}^n \frac{2I_{3 \times 3} + \delta\Omega_{k+1}\Delta t}{2I_{3 \times 3} - \delta\Omega_{k+1}\Delta t}$$

➤ Acceleration in navigation frame:

$$a_{c\ k+1}^s = a_{k+1}^s - \delta b_{a\ k+1}^s$$

$$a_{k+1}^n = R_{s\ k+1|k}^n a_{c\ k+1}^s - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

➤ Integration method:

$$v_{k+1|k} = v_{k|k} + (a_k^n + a_{k+1}^n) \frac{\Delta t}{2}$$

$$p_{k+1|k} = p_{k|k} + (v_k^n + v_{k+1}^n) \frac{\Delta t}{2}$$

➤ Correction after the ZUPT:

$$v_{k+1|k+1} = v_{k+1|k} - \delta v_{k+1}$$

$$p_{k+1|k+1} = p_{k+1|k} - \delta p_{k+1}$$

$\delta\Theta_k$ denotes the skew-symmetric matrix for small angles.

$$\delta\Theta_k = \begin{bmatrix} 0 & -\phi_{zk}^s & \phi_{yk}^s \\ \phi_{zk}^s & 0 & -\phi_{xk}^s \\ -\phi_{yk}^s & \phi_{xk}^s & 0 \end{bmatrix}$$

$$R_{s\ k+1|k+1}^n = R_{s\ k+1|k}^n \frac{2I_{3 \times 3} + \delta\Theta_{k+1}\Delta t}{2I_{3 \times 3} - \delta\Theta_{k+1}\Delta t}$$

➤ Extended Kalman Filter (EKF) equations:

$$\Phi_k = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & \Delta t R_{s_{k+1|k}}^n & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} & \Delta t I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ -\Delta t A_{k+1} & 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} & \Delta t R_{s_{k+1|k}}^n \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}$$

With A_k defined as

$$A_k = \begin{bmatrix} 0 & -a_{z_k}^s & a_{y_k}^s \\ a_{z_k}^s & 0 & -a_{x_k}^s \\ -a_{y_k}^s & a_{x_k}^s & 0 \end{bmatrix}$$

P stands for the error covariance matrix, and Q for the system noise covariance matrix. Here, the measurement vector and the measurement matrix are defined as below.

$$y = - \begin{bmatrix} \omega_x^s \\ \omega_y^s \\ \omega_z^s \\ v_x \\ v_y \\ v_z \end{bmatrix}$$

$$H = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & -I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & -I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}$$

The Kalman gain K is calculated with the formula below. R is measurement covariance matrix.

$$K = P_{k+1|k} H^T (H P_{k+1|k} H^T + R)^{-1}$$

➤ State update:

$$\delta x_{k+1} = Ky$$

$$P_{k+1|k+1} = (I - KH)P_{k+1|k}(I - KH)^T + KRK^T$$

Stance Phase Detection:

Using Angular Rate (presented as ARED):

This method is only using gyroscope data. Raw data is divided on segments of n samples (to obtain a 0.1-0.5 s recording). Then the algorithm computes the magnitude of each sample following the next formula:

$$\omega_{\{s,i\}} = \sqrt{\{\omega_{\{x,i\}}^2 + \omega_{\{y,i\}}^2 + \omega_{\{z,i\}}^2\}}$$

If some magnitude sample are under a certain threshold during the time segment denoted T, the moment where the magnitude is minimal on the time segment T is chosen and denoted Ts. ZUPT is performed at that instance. If no magnitude sample is under the threshold, no stance phase is detected during this period T. This procedure is repeated for each segment.

Using Angular Rate and acceleration (presented as SHOE):

Three conditions have to be simultaneously satisfied to detect a stance phase. Data is filtered using a median filter with a window of 11 samples to prevent faulty stance phase detection. The first condition is on acceleration.

$$a_{\{i\}} = \sqrt{\{a_{\{x,i\}}^2 + a_{\{y,i\}}^2 + a_{\{z,i\}}^2\}}$$

if $th_1 < a_{\{i\}} < th_2$, the first condition is satisfied. (th_1 and th_2 are empirical thresholds like 9 and 11 ms^{-2}). The second condition is on the local acceleration variance calculated with:

$$\sigma_{\{a_i\}}^2 = \frac{1}{2s+1} \sum_{l=i-s}^{i+s} a_l$$

The second condition is satisfied if the variance $\sigma_{\{a_i\}}^2$ is lower than a certain threshold ($15 \text{ m}^2 \text{s}^{-4}$ for a window of $s=3$ samples). The last condition is the same as for ARED algorithm.

$$\omega_{\{s,i\}} = \sqrt{\{\omega_{\{x,i\}}^2 + \omega_{\{y,i\}}^2 + \omega_{\{z,i\}}^2\}}$$

Heuristic Drift Reduction:

This method reduces the drift considering the fact that walking paths for many applications are straight. It introduces the yaw angle variation as a measurement into the EKF. Let ψ_k be the yaw angle at time k .

$$\psi_k = \arctan\left(\frac{R_{s\ k|k}^n(2,1)}{R_{s\ k|k}^n(1,1)}\right)$$

You can use *arctan2* function in MATLAB as well. Let $T_1, T_2 \dots$ be the time instances when a step is detected.

$$\Delta\psi_{T_n+i} = \psi_{T_n+i} + \frac{\psi_{T_{n-1}+i} + \psi_{T_{n-2}+i}}{2}$$

If the variation is under a certain threshold (0.06 rad.s^{-1}), the measurement y_3 is $\Delta\psi_{T_n+i}$. Otherwise, it is 0. The measurement matrix is becoming:

$$H = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & -I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & -I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ (0,0,1) & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} \end{bmatrix}$$

- [1] J. Nilsson, I. Skog, P. Händel and K. V. S. Hari, "Foot-mounted INS for everybody - an open-source embedded implementation," *Proceedings of the 2012 IEEE/ION Position, Location and Navigation Symposium*, 2012, pp. 140-145, doi: 10.1109/PLANS.2012.6236875.
- [2] Yin, H., H. Guo, X. Deng, M. Yu, and J. Xiong. "Pedestrian dead reckoning indoor positioning with step detection based on foot-mounted IMU." In *Proceedings of the 2014 International Technical Meeting of The Institute of Navigation*, pp. 186-192. 2014.

APPENDIX II

Pedestrian Dead Reckoning system is based in estimating the length of each step as well as the attitude estimation. To estimate the Step Length (SL), we can use the magnitude of the accelerometer sensor with the Weinberg algorithm. The attitude and heading also can be estimated using the data of gyroscope with combination with magnetometer or accelerometer, based on the utilized AHRS method.

Weinberg SL estimation:

- Magnitude of the acceleration:

$$a_i = \sqrt{\{a_{\{x,i\}}^2 + a_{\{y,i\}}^2 + a_{\{z,i\}}^2\}}$$

- Low Pass Filter (LPF):

$$\tilde{a}_i = LP\{a_i\}$$

- Weinberg hip bounce model:

By assuming that the SL is proportional to the hip bounce, which can be estimated from the largest acceleration.

$$SL_{weinberg} = K \cdot \left\{ \max_{j=[i(k) \pm w]}^{\tilde{a}_j} - \min_{j=[i(k) \pm w]}^{\tilde{a}_j} \right\}^{1/4}$$

Attitude estimation:

- Orientation from gyroscope:

$$\omega^b = (\omega_x^b, \omega_y^b, \omega_z^b)$$

$$\dot{C}_b^n = C_b^n \Omega_{nb}^b \quad \text{when} \quad \Omega_{nb}^b = \begin{bmatrix} 0 & -\omega_z^b & \omega_y^b \\ \omega_z^b & 0 & -\omega_x^b \\ -\omega_y^b & \omega_x^b & 0 \end{bmatrix}$$

➤ Orientation from accelerometer and magnetometer:

$$m^b = (m_x^b, m_y^b, m_z^b)$$

$$a^b = (a_x^b, a_y^b, a_z^b)$$

$$\phi = \tan\left(\frac{a_y^b}{a_z^b}\right)^{-1}$$

$$\theta = \tan\left(\frac{-a_x^b}{\sqrt{(a_y^b)^2 + (a_z^b)^2}}\right)^{-1}$$

$$\psi = \tan\left(\frac{-m_x^b}{m_y^b}\right)^{-1} \pm D$$

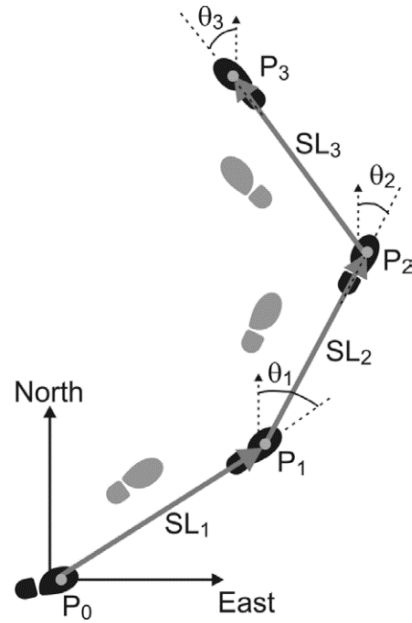
PDR algorithm:

$$P_k(\text{North}) = P_{k-1}(\text{North}) + SL_k \cdot \cos(\theta_{\text{stance}_k})$$

$$P_k(\text{East}) = P_{k-1}(\text{East}) + SL_k \cdot \sin(\theta_{\text{stance}_k})$$

Number of steps = k

θ : deviation of yaw angle



EKF measurement model:

$$SL = \sqrt{(p_k^N - p_{k-1}^N)^2 + (p_k^E - p_{k-1}^E)^2}$$

$$y = Hx$$

$$H = \text{Jacobian } (SL)$$

APPENDIX III

STRUCTURE OF LABORATORY REPORTS:

Laboratory reports must be presented in the form of an engineering technical report in accordance with the usual presentation standards. The evaluation scale of the laboratories is presented in this section. Without limitation, your report should at least consist of the following elements:

Introduction:

- Title page identifying the course (abbreviation and title), the professor, the laboratory supervisor, the laboratory (number and title), the name(s) of the collaborators and the date of submission.
- Introduction and background, briefly describing the purpose of this laboratory, its objectives, and the expected results.
- Detailed table of contents with page numbering (list of figures, tables, acronyms, glossary).

Theoretical study:

- Methodology used for the realization of the laboratory.
- Detail your approach and explain the reasons why you undertook it.
- Presentation of fundamental concepts related to the laboratory carried out. In this section, explain in your own words the theory behind the laboratory carried out without forgetting to cite your sources.
- Statement of all the equations useful for the realization of the laboratory and during the analysis of the results. In particular, support your achievements with the theoretical elements seen in class.
- Explain the development of the calculations used during the laboratory.

Experiments and results:

- Explain your experiment, hardware setup, configuration and data gathering process
- Detailed presentation of your approach. Explain how you got your results.

- Clear presentation of results using graphs and/or tables.

Analysis, conclusion, and discussions:

- Analysis of the results obtained including all relevant observations and comments.
- Discussion on the gap between the results obtained and those expected. Explain the difficulties encountered during the realization of the laboratory.
- Conclusion on the laboratory.
- Discuss the achievement of the objectives initially stated and the contributions of this laboratory on your personal journey (academic and/or professional).
- Possible improvements and/or modifications of the diagrams used and discussion of possible applications in connection with the techniques studied.

References:

- Presentation of bibliographical references without forgetting to list the addresses of your references on the internet.
- Any other document making it possible to clarify a statement or provide additional information may be added as an appendix.: