Machine Learning for Large-Scale Data Analysis and Decision Making (MATH80629A)
Fall 2021

Week #4 - **Summary**

Announcement

- **Hybrid classroom**: Mondays 8:30 am - 11:30 am
  Class room: Manuvie. This classroom is located on the 1st floor of Côte-Sainte-Catherine building.
  Zoom: Zoom link.

- **Hybrid office hour**: Mondays 11:30 am - 1 pm
  Office:4.834
  Zoom: Zoom link.

- **Lab session** on week #5 (September 27)
  Lab room: Laboratoire Lachute

Today

- **Second Quiz** on Gradescope!
- Summary of Machine learning fundamental
- Q&A
- Hands-on session

Quiz 1

Login to your Gradescope account

Models for supervised learning

- (Mostly) linear models
- Focus on **classification**

1. **Non-Probabilistic Models**
   - Nearest Neighbor, Support Vector Machines (SVMs)
2. **Probabilistic Models**
   - Naive Bayes

Supervised learning

Train Data

$$\underbrace{\begin{array}{c|ccccc|}
 & \text{Nb.bed.} & \text{Area} & \text{Neigh.} & \cdot & \cdot \\
x_0 & 1 & 0 & 0 & 0 & 0 \\
x_1 & 1 & 100 & 1 & .2 & .5 \\
x_2 & 3 & 200 & 0 & .1 & .2 \\
x_3 & 1 & 150 & 1 & .4 & .1 \\
x_4 & 2 & 210 & 2 & .5 & 1.1
\end{array}}_{X}
\quad
\underbrace{\begin{array}{c|c|}
 & \text{Sell-ability} \\
y_0 & 1 \\
y_1 & 2 \\
y_2 & 0 \\
y_3 & 2 \\
y_4 & 1
\end{array}}_{Y}$$

Task

$$f : \mathbb{R}^n \to \{0, 1, 2\}$$

Test Data

$$\underbrace{\begin{array}{c|ccccc|}
 & \text{Nb.bed.} & \text{Area} & \text{Neigh.} & \cdot & \cdot \\
x_0 & 1 & 0 & 0 & 0 & 0 \\
x_1 & 2 & 50 & 1 & .3 & .8 \\
x_2 & 1 & 100 & 1 & .5 & 1.4 \\
x_3 & 4 & 170 & 0 & .7 & .4 \\
x_4 & 1 & 120 & 3 & .9 & .5
\end{array}}_{X^{new}}
\quad
\underbrace{\begin{array}{c|c|}
 & \text{Sell-ability} \\
y_0 & ? \\
y_1 & ? \\
y_2 & ? \\
y_3 & ? \\
y_4 & ?
\end{array}}_{Y^{new}}$$

Supervised learning

Train Data

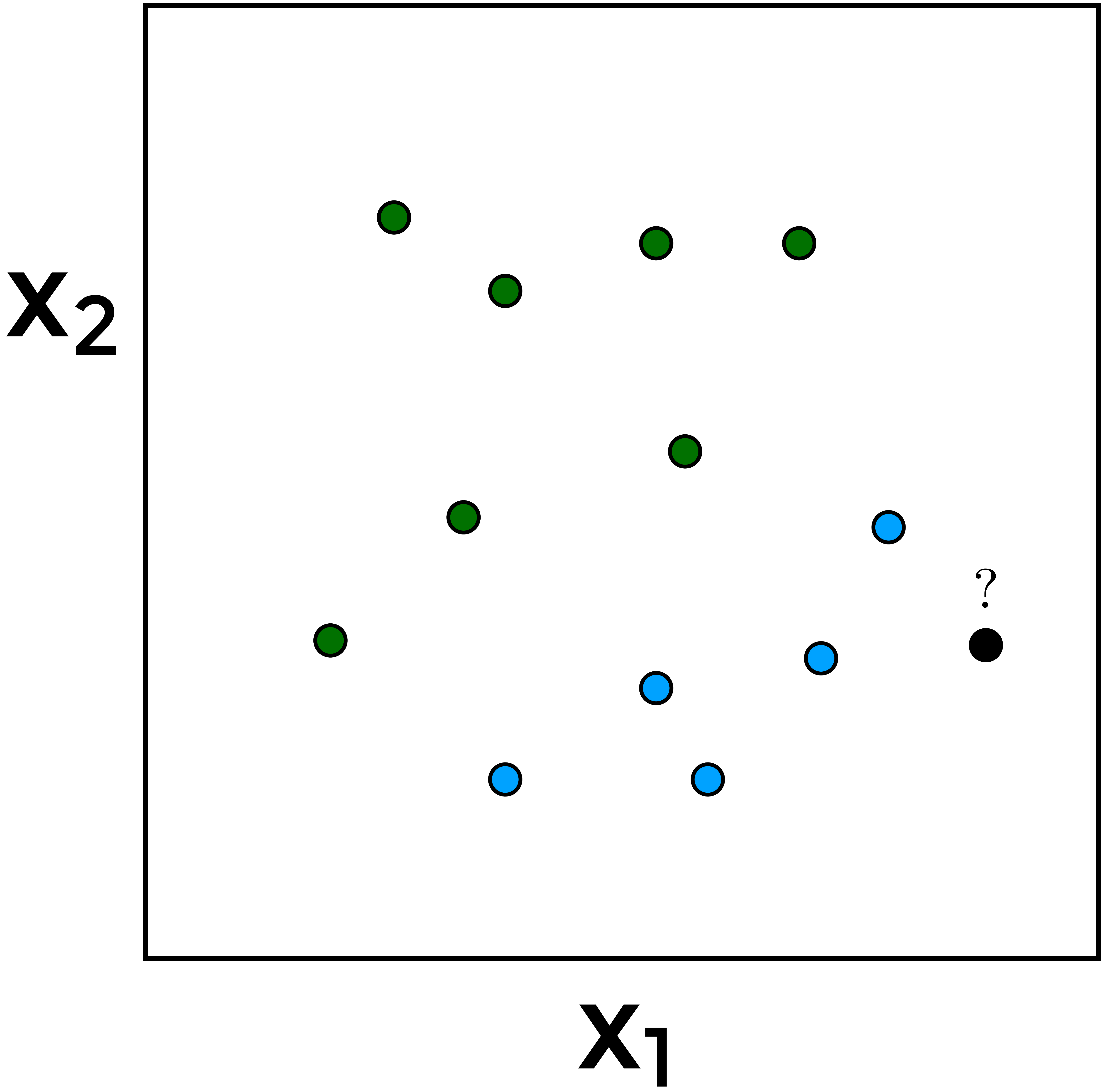
$$\underbrace{\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\begin{array}{c} \begin{array}{ccccc} \text{Nb.bed.} & \text{Area} & \text{Neigh.} & \cdot & \cdot \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 100 & 1 & .2 & .5 \\ 3 & 200 & 0 & .1 & .2 \\ 1 & 150 & 1 & .4 & .1 \\ 2 & 210 & 2 & .5 & 1.1 \end{bmatrix}\end{array}}_{X} \quad \underbrace{\begin{array}{c} \\ y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{array}\begin{array}{c} \text{Sell-ability} \\ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}\end{array}}_{Y}$$

Task

Models $\boxed{f}\ \mathbb{R}^n \rightarrow \{0, 1, 2\}$

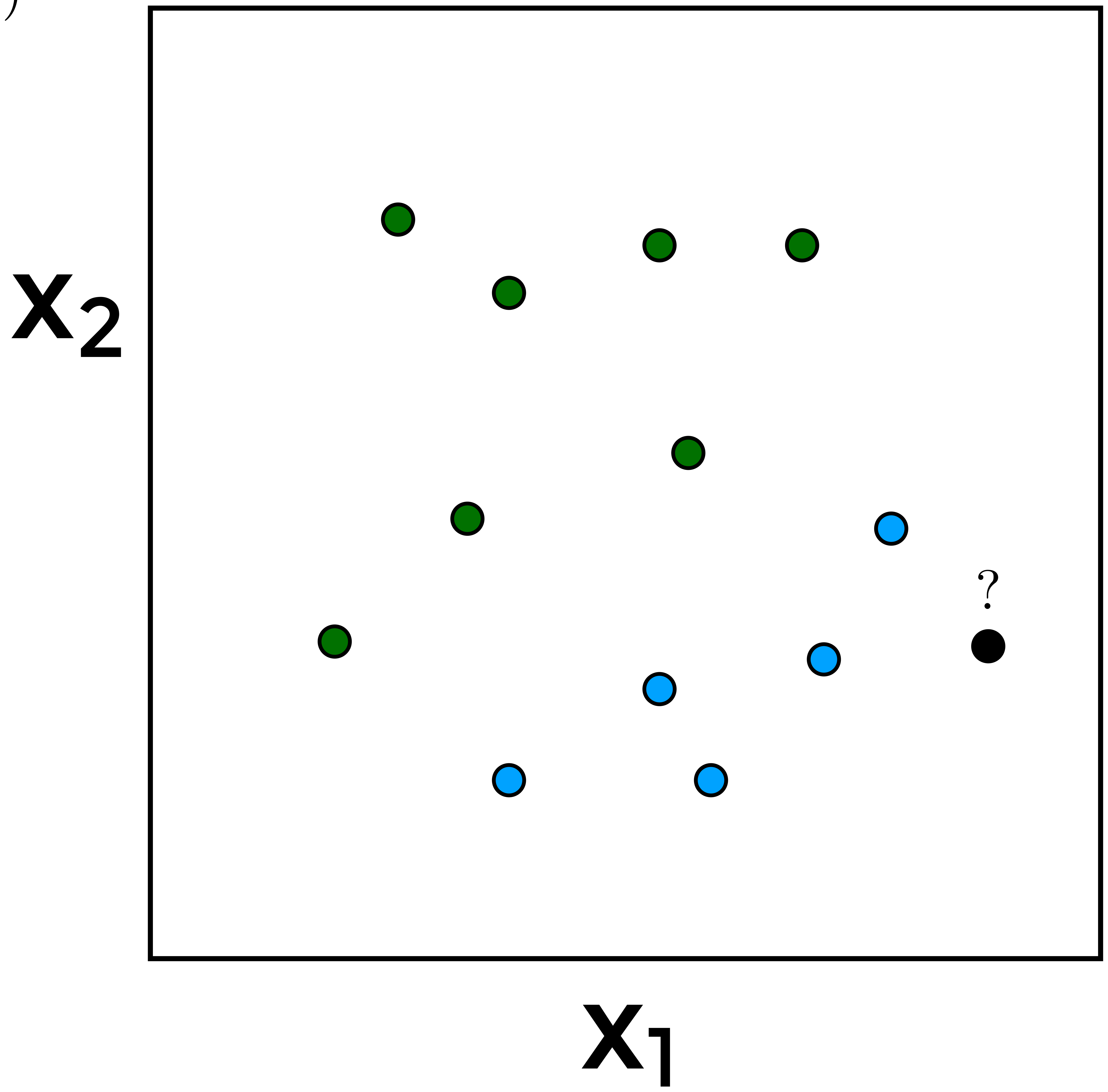
Test Data

$$\underbrace{\begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\begin{array}{c} \begin{array}{ccccc} \text{Nb.bed.} & \text{Area} & \text{Neigh.} & \cdot & \cdot \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 50 & 1 & .3 & .8 \\ 1 & 100 & 1 & .5 & 1.4 \\ 4 & 170 & 0 & .7 & .4 \\ 1 & 120 & 3 & .9 & .5 \end{bmatrix}\end{array}}_{X^{new}} \quad \underbrace{\begin{array}{c} \\ y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{array}\begin{array}{c} \text{Sell-ability} \\ \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}\end{array}}_{Y^{new}}$$



$$i' = \arg\min_i \mathsf{dist}(x_i, x_j)$$

$$y_j = y_{i'}$$

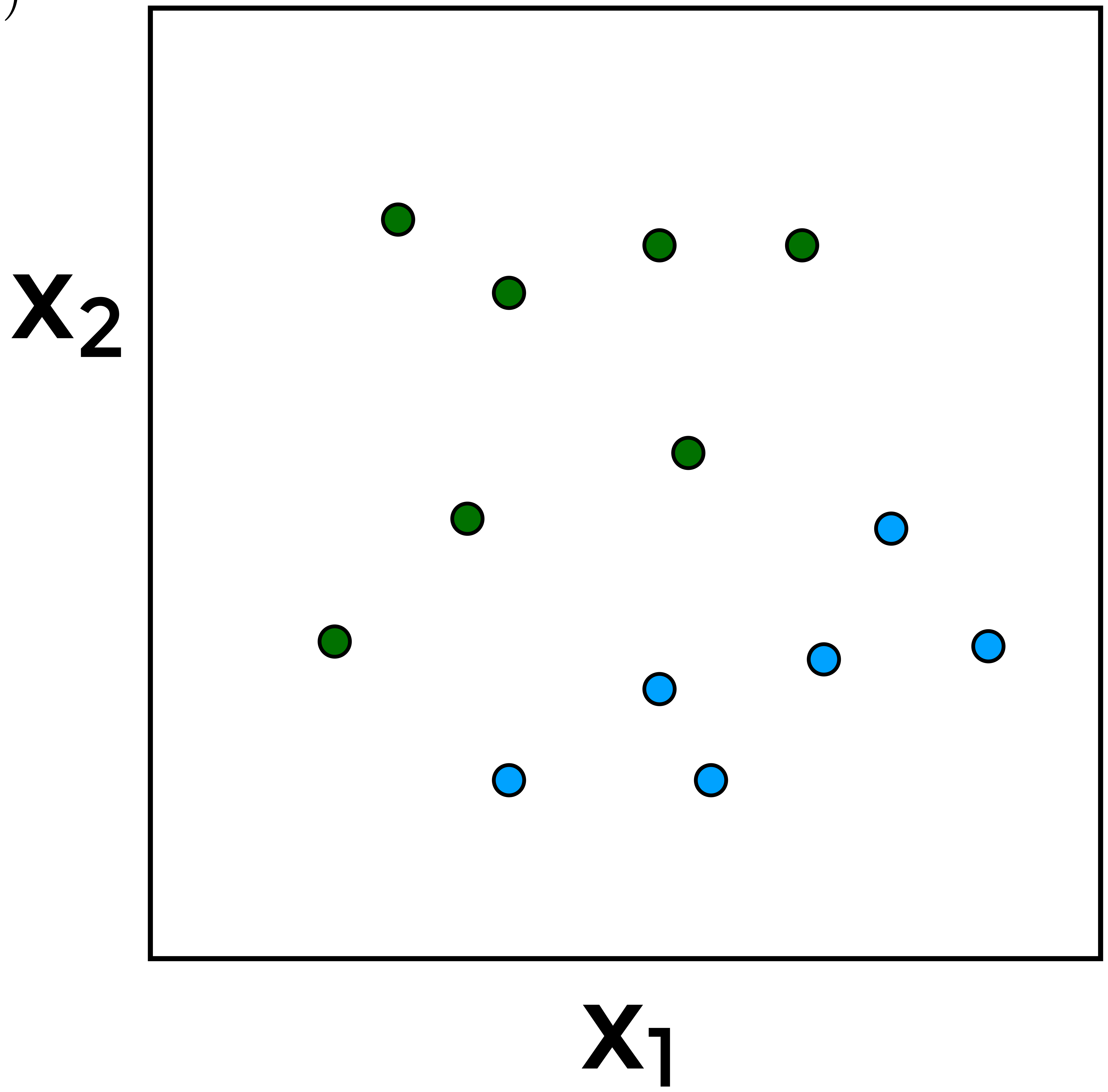


- **1-NN**

Instance classified according to its nearest neighbor

$$i' = \arg\min_i \mathsf{dist}(x_i, x_j)$$

$$y_j = y_{i'}$$



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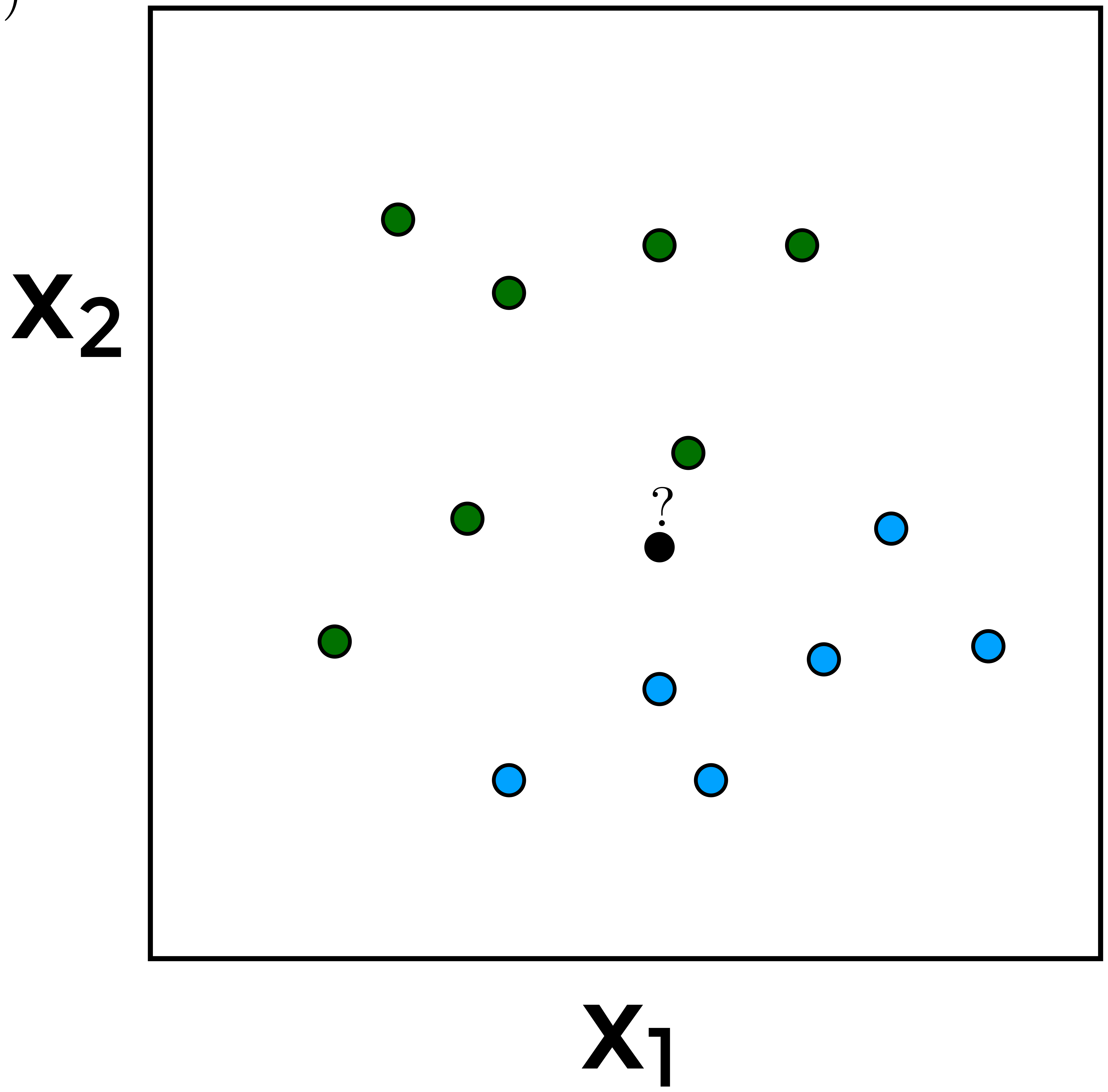


- **1-NN**

Instance classified according to its nearest neighbor

- **K-NN**

Instance classified according to the majority of its K nearest neighbors

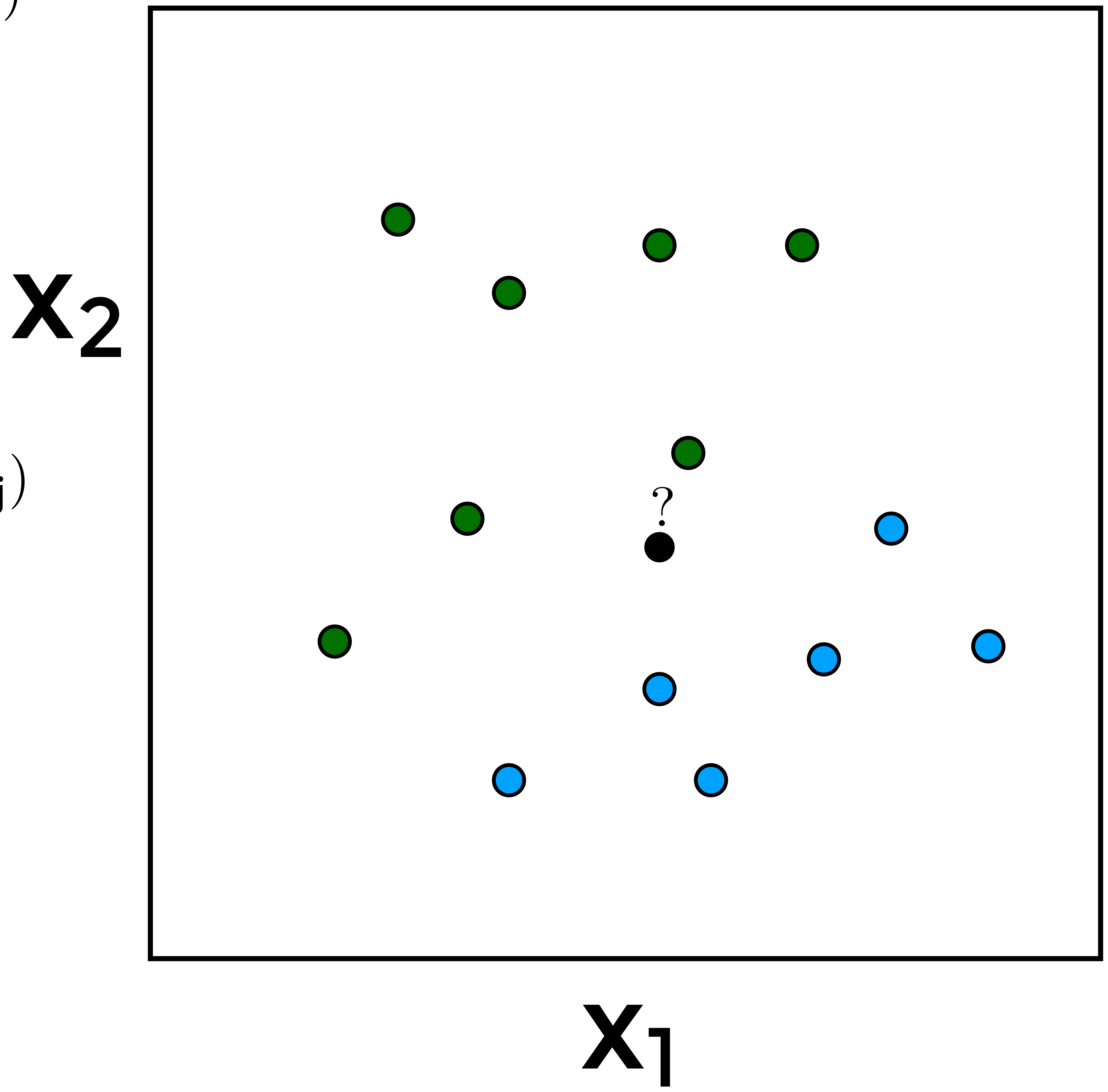
$$i' = \arg\min_i \text{dist}(x_i, x_j)$$

$$y_j = y_{i'}$$

$$k = 5 \text{ (assumption)}$$

$$i = \arg\text{sort}_i \, \text{dist}(x_i, x_j)$$

$$y_j = \text{majority}(i_{:5})$$



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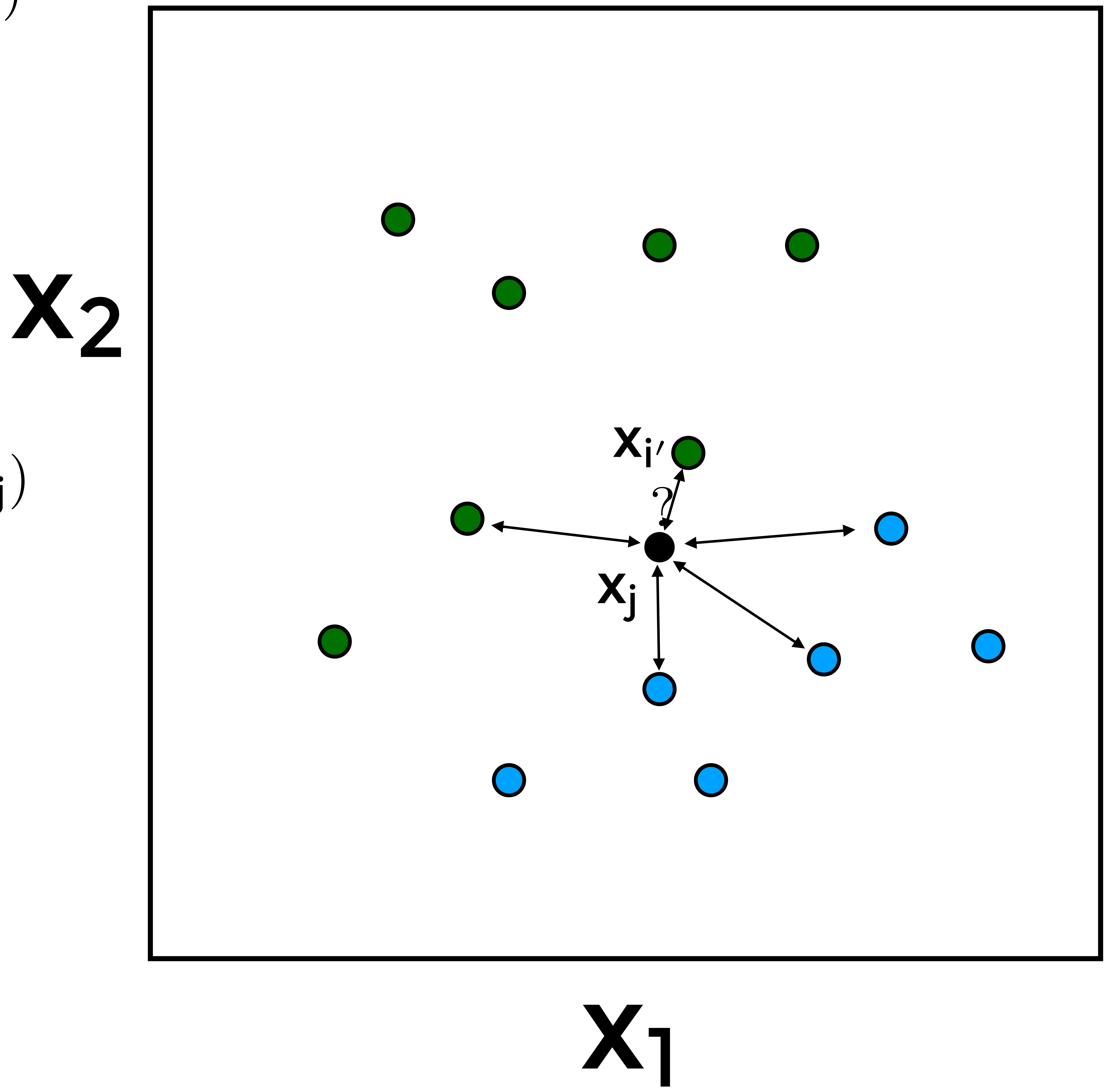

- **1-NN**

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- **K-NN**

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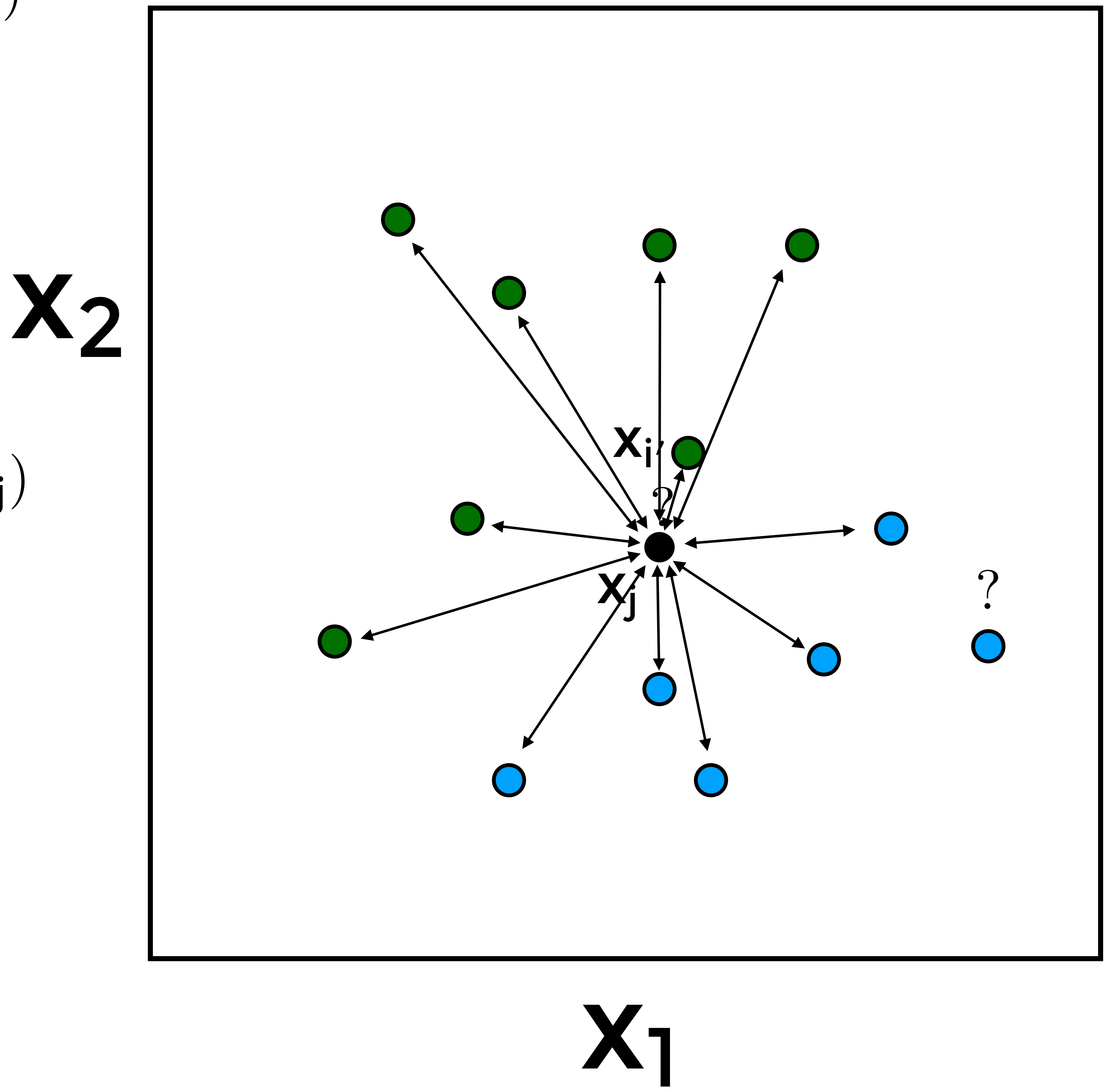


- **1-NN**

Instance classified according to its nearest neighbor

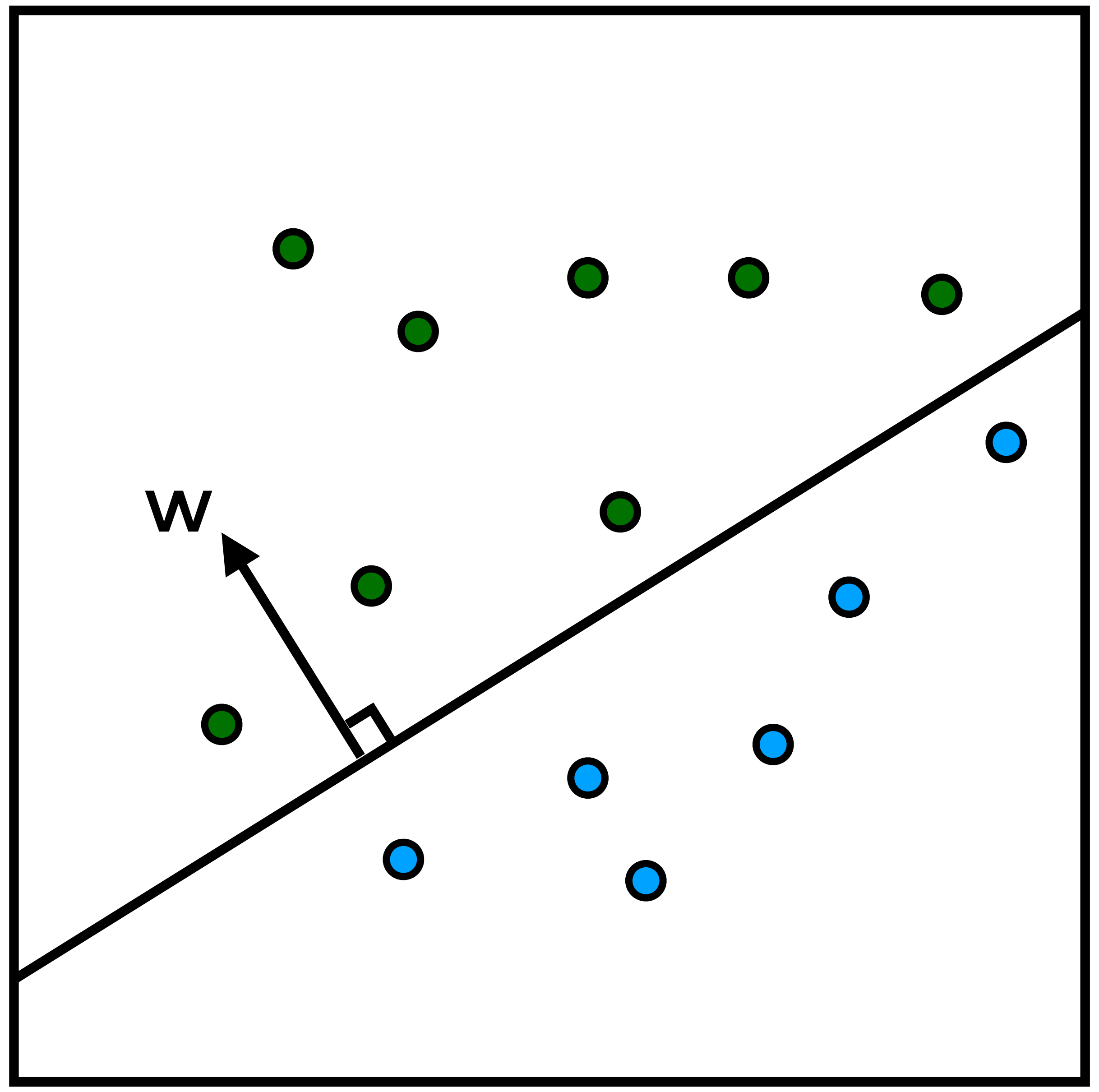
- **K-NN**

Instance classified according to the majority of its K nearest neighbors

- **weighted-NN**

Instance classified according to all neighbors. The contribution of each neighbor is weighted by its distance.

Linear Classification



decision boundary: $y(\mathbf{x}) = 0$

take two points on the boundary: $\mathbf{x}_a, \mathbf{x}_b$

then: $\mathbf{w}^\top \mathbf{x}_a + w_0 = \mathbf{w}^\top \mathbf{x}_b + w_0$

$\implies \mathbf{w}^\top (\mathbf{x}_a - \mathbf{x}_b) = 0$

$\implies$ w is perpendicular to the decision boundary

w represents the orientation of the decision boundary

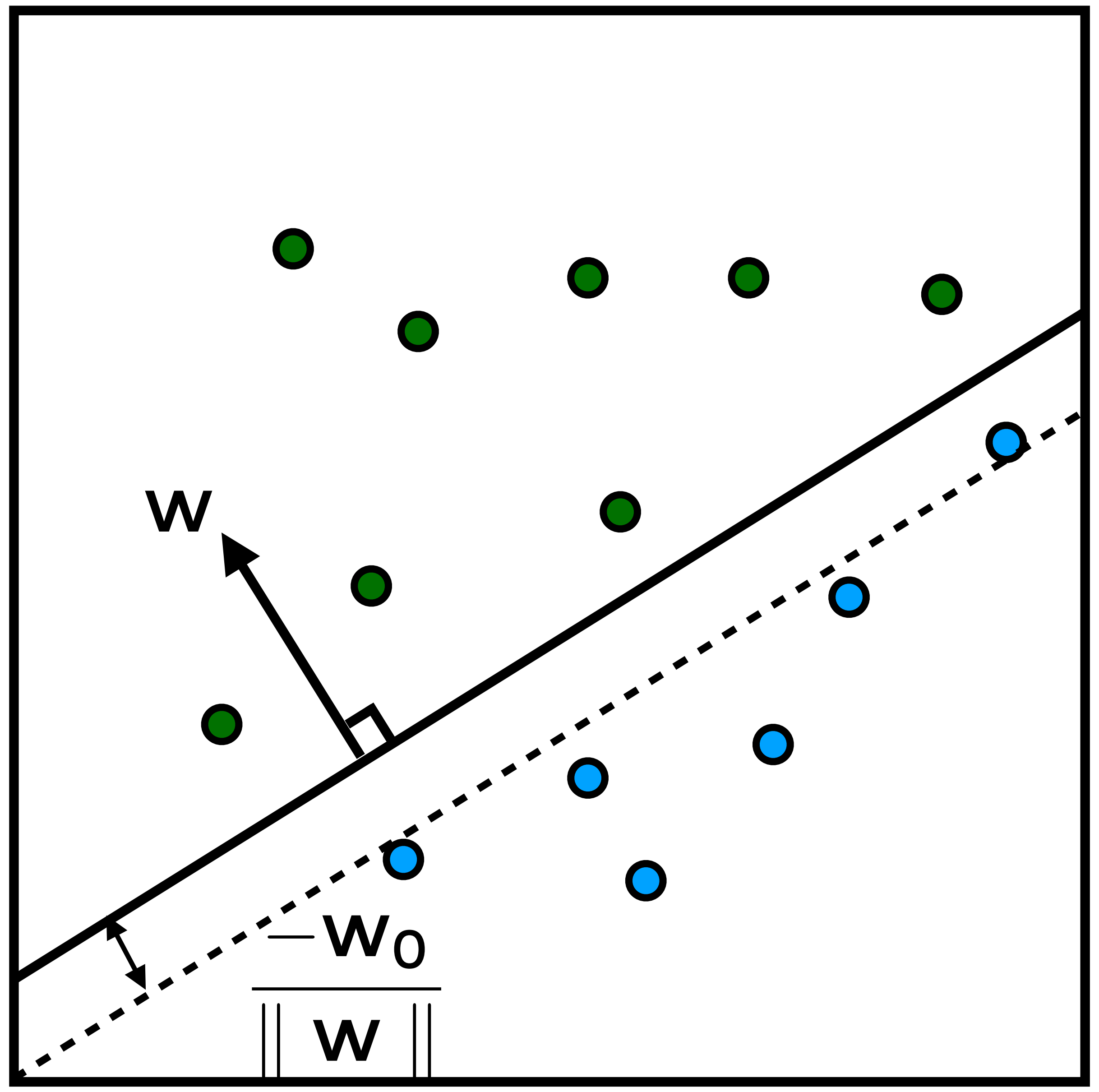
$$y(\mathbf{x}) = \mathbf{w}^\top \mathbf{x} + w_0$$

Decision

$$(\mathbf{w}^\top \mathbf{x} + w_0) > 0 \implies \text{green}$$

$$(\mathbf{w}^\top \mathbf{x} + w_0) < 0 \implies \text{blue}$$

Linear Classification



$$y(\mathbf{x}) = \mathbf{w}^\top \mathbf{x} + w_0$$

Decision

$$(\mathbf{w}^\top \mathbf{x} + w_0) > 0 \implies \text{green}$$

$$(\mathbf{w}^\top \mathbf{x} + w_0) < 0 \implies \text{blue}$$

w_0 is a scalar

you can think of it like an intercept

take $\mathbf{x}'$ as the closest point on the decision boundary to the origin

$\mathbf{x}' = \beta \mathbf{w}$

$$\implies y(\mathbf{x}') = \mathbf{w}^\top \mathbf{x}' + w_0$$

$$\implies y(\mathbf{x}') = \mathbf{w}^\top (\beta \mathbf{w}) + w_0$$

$$\implies 0 = \beta \| \mathbf{w} \|^2 + w_0$$

$$\implies \beta = \frac{-w_0}{\| \mathbf{w} \|^2}$$

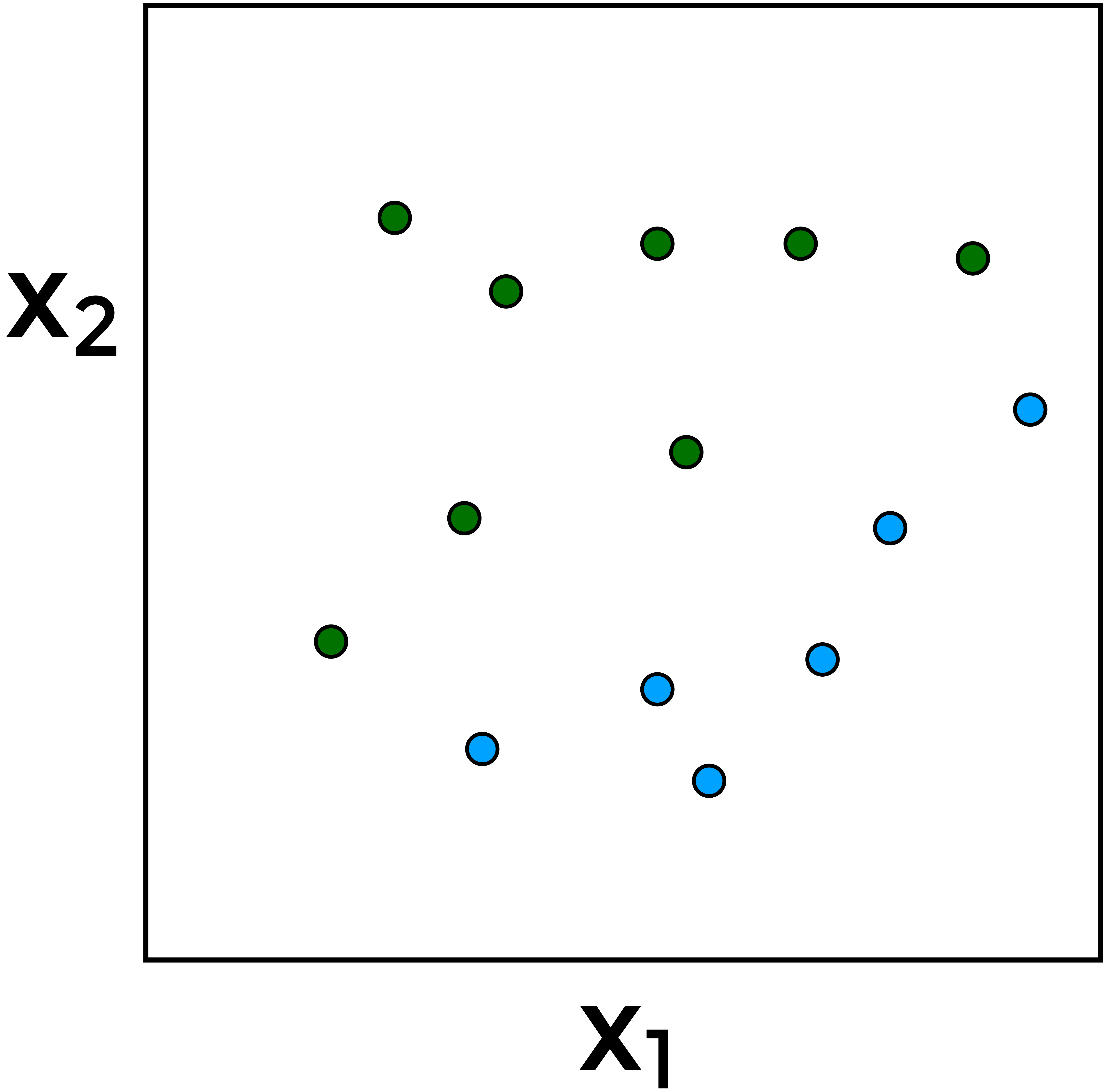
Then you know that the distance from the origin to $\mathbf{x}'$ is:

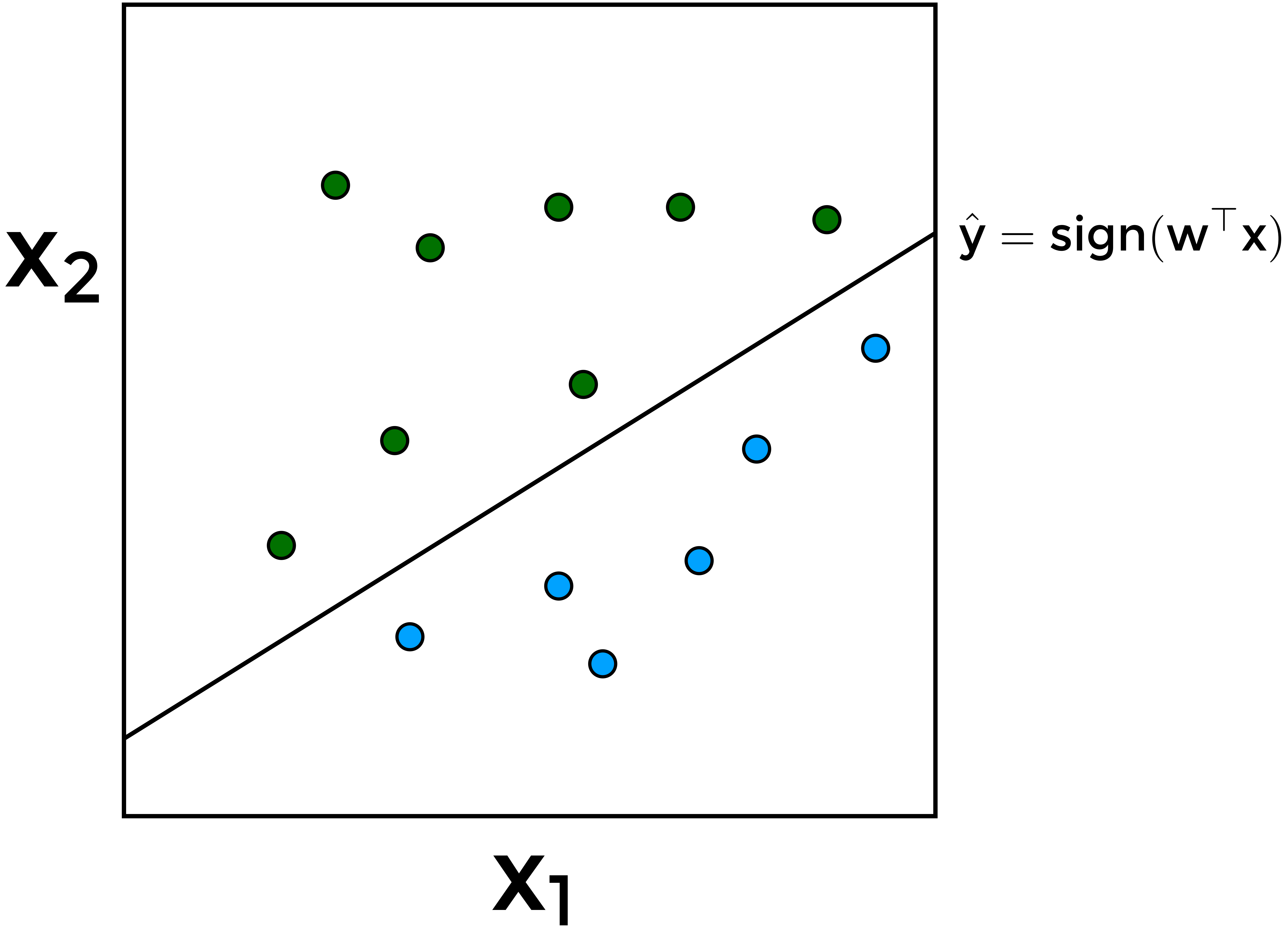
$\| \mathbf{x}' \| = \| \beta \mathbf{w} \|$

$$\implies \| \mathbf{x}' \| = \beta \| \mathbf{w} \|$$

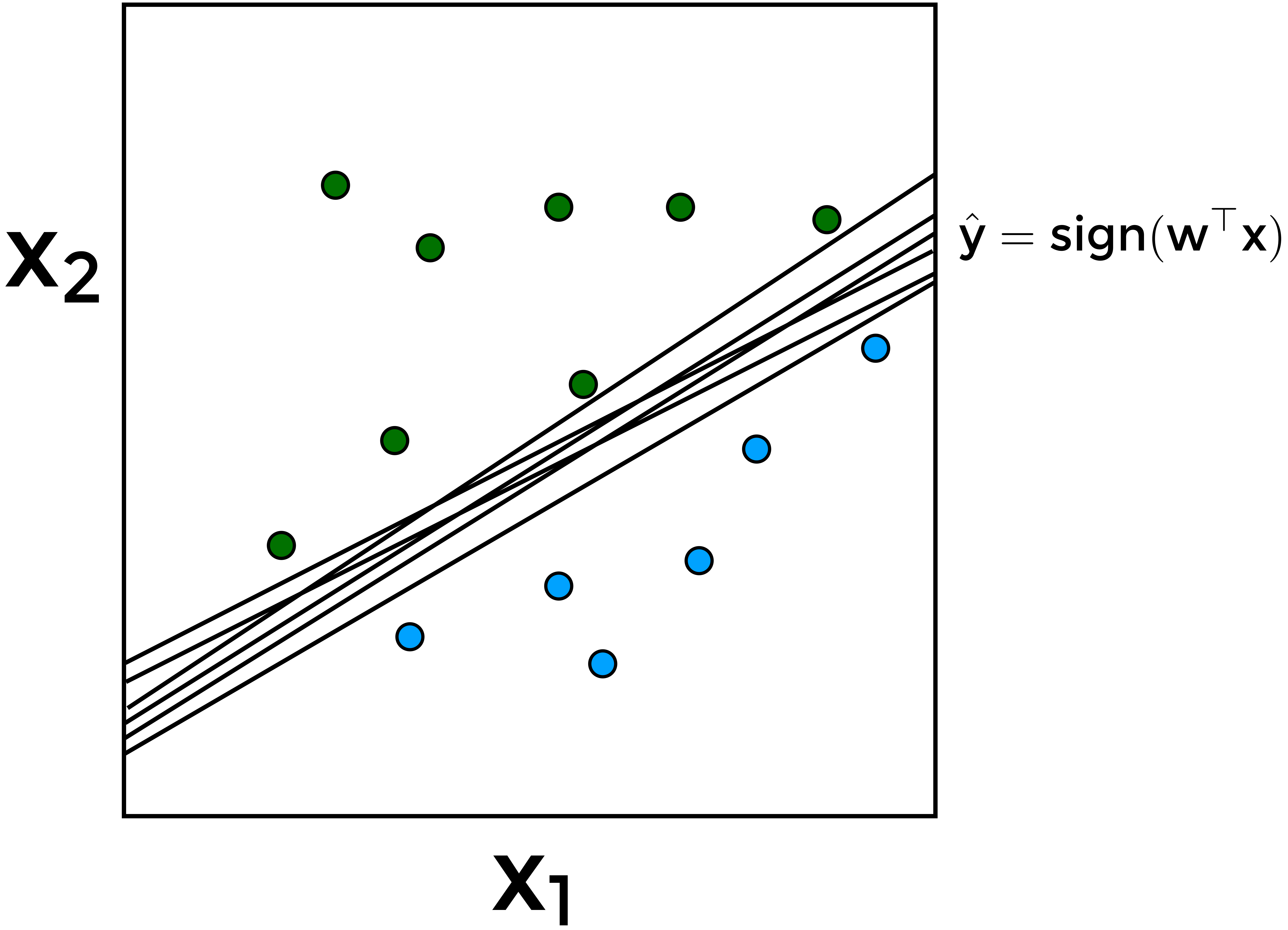
$$\implies \| \mathbf{x}' \| = \frac{-w_0}{\| \mathbf{w} \|^2} \| \mathbf{w} \|$$

$$\implies \| \mathbf{x}' \| = \frac{-w_0}{\| \mathbf{w} \|}$$

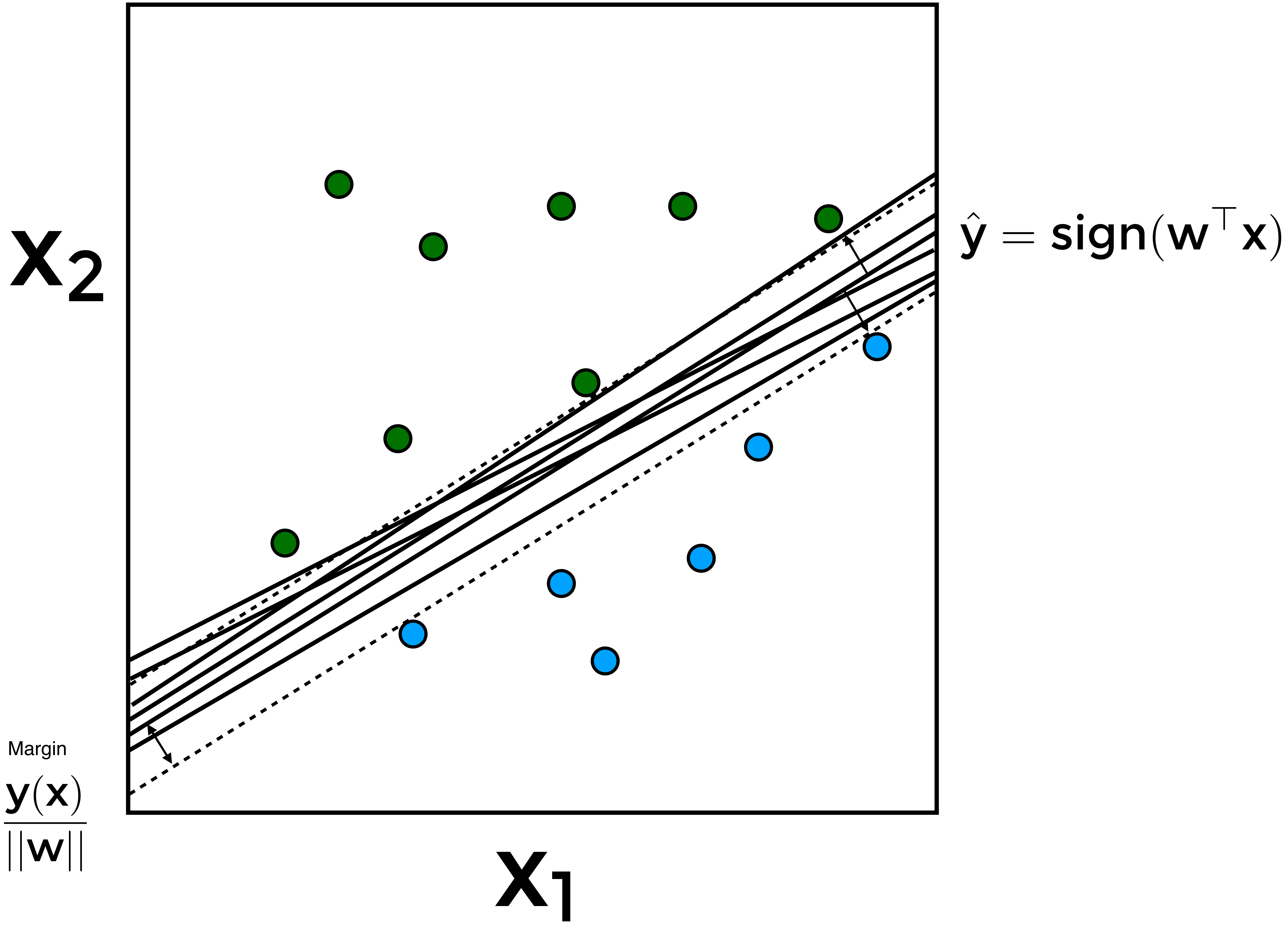

x_2
x_1



$\mathbf{x}_2$
$\hat{y} = \text{sign}(\mathbf{w}^\top \mathbf{x})$
$\mathbf{x}_1$


x_2
$\hat{y} = \text{sign}(w^\top x)$
x_1

The objective is to find the separating boundary that maximizes the margin



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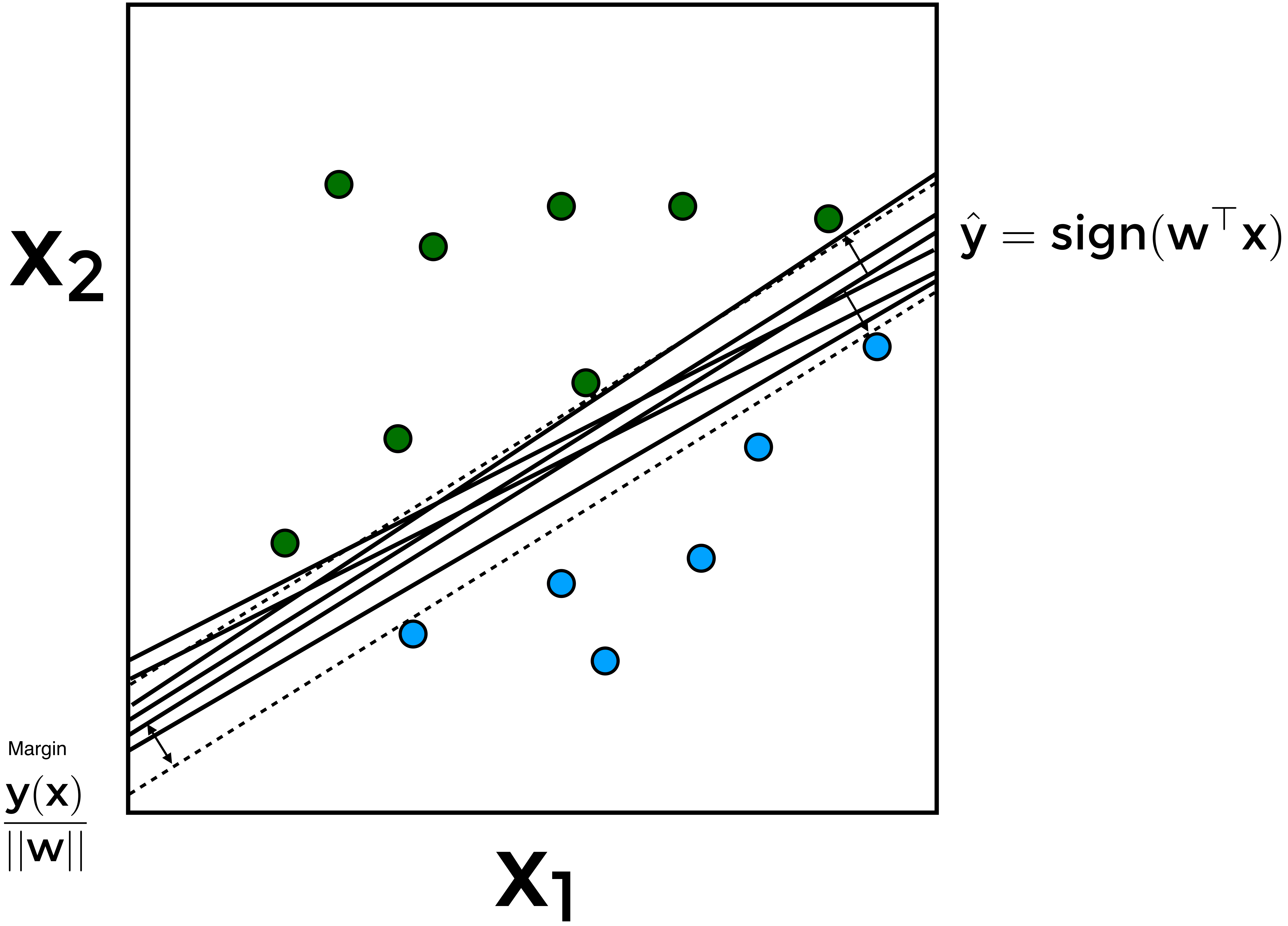

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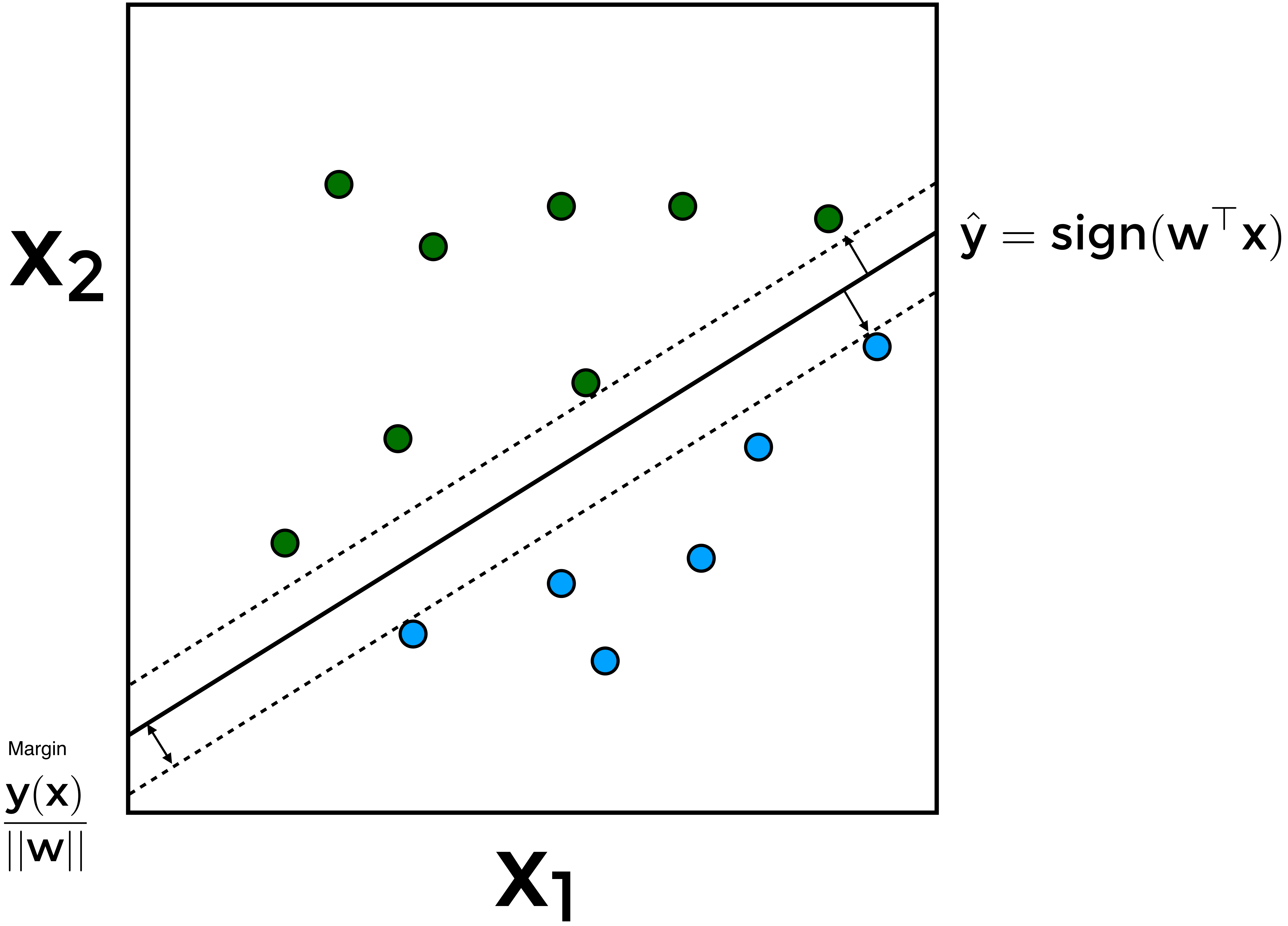
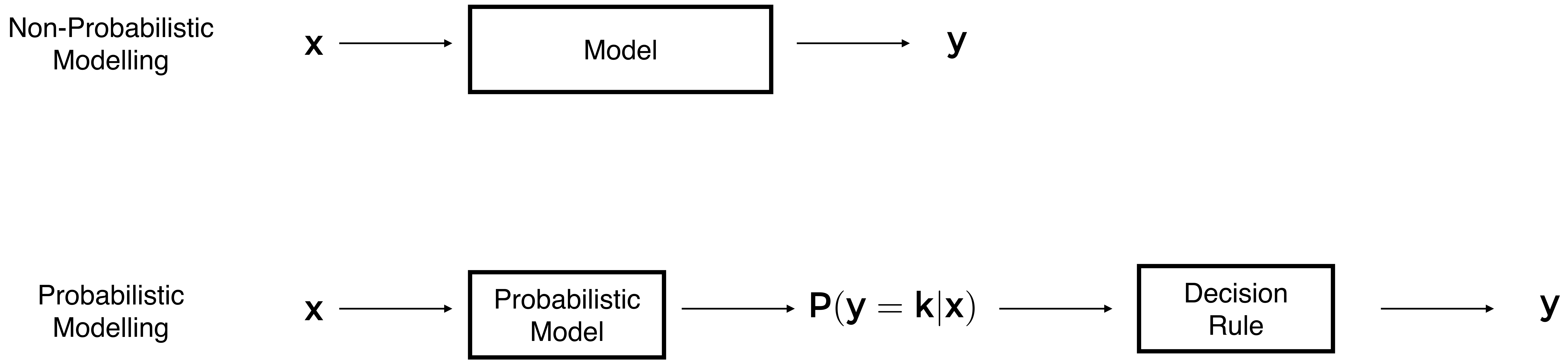


Probabilistic Models for Classification

Probabilistic Models separate Decision and Inference



Probabilistic models

1. **Model the conditional directly:**

$$P(y = k|x)$$

2. **Model the joint (or the prior and the class conditionals):**

$$\underbrace{P(y = k|x)}_{\text{posterior}} \propto \underbrace{P(y = k, x)}_{\text{joint}}$$

$$= \underbrace{P(x \mid y = k)}_{\text{class conditional densities}} \underbrace{P(y = k)}_{\text{class prior}}$$

Bayes' Theorem

Probabilistic Modelling

Often
inteertwinned

1. **Posit a model: P(X, Y)**
    - How the data is generated
2. **Parametrize the distributions: P(X, Y | Parameters)**
3. **Set the objective (e.g., MLE)**
4. **Learn the parameters of the model:**
    - E.g., Naive Bayes: learn the parameters of the class conditional P(X | Y) and of the prior P(Y)
5. **Use the model (e.g., for predictions)**