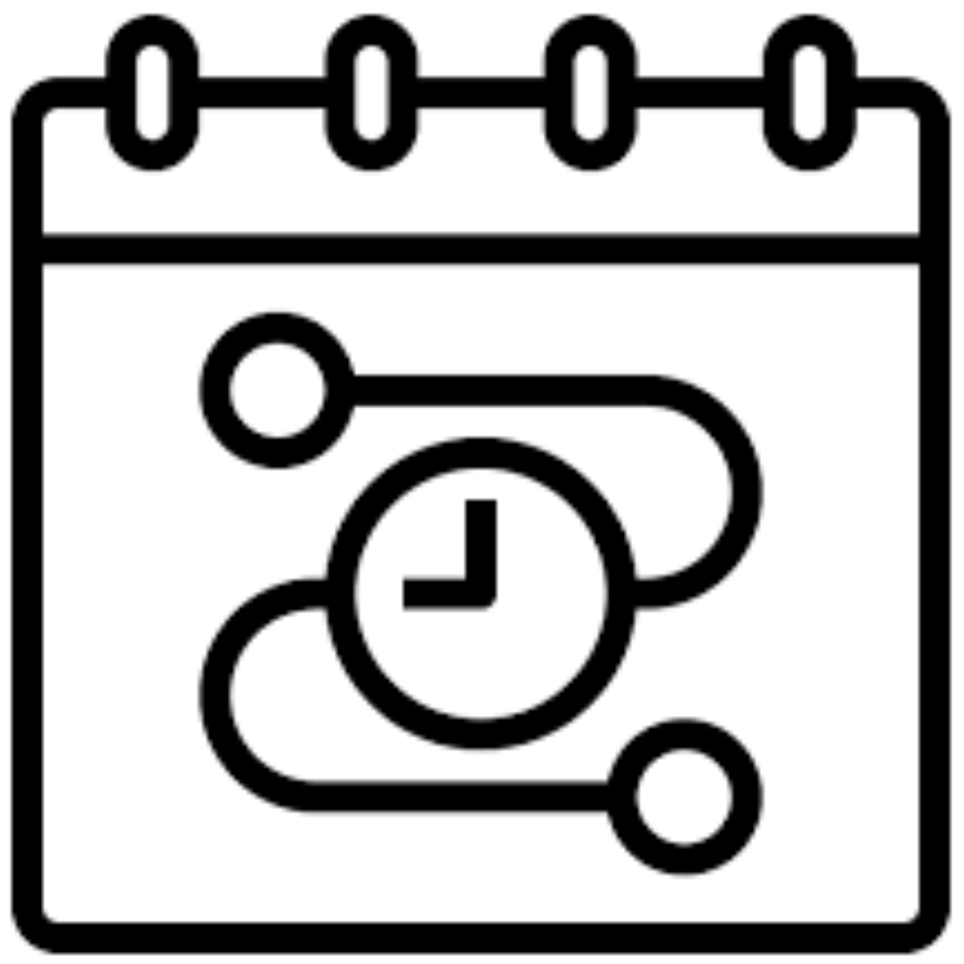
Machine Learning for Large-Scale Data Analysis and Decision Making (MATH80629A) Winter 2022

Week #7 - **Summary**

Announcement

- **Study Plan:** due **February 27, 2022**
- **Project Meeting: March 2, 2022**
- **Homework 1:** due **March 9, 2022**
- **Homework 2:** due **March 14, 2022**



Today

- **Fifth Quiz** on Gradescope!
- Summary of Unsupervised Learning
- Q&A
- Hands-on session

Quiz 4

Login to your Gradescope account

Recall

Experience (E)

- What data does f experience?
  - (Focus on algorithms that experience whole datasets)
  - Unsupervised. Examples alone $\{x_i\}_{i=0}^{n}$
  - Supervised. Examples come with labels $\{(x_i, y_i)\}_{i=0}^{n}$

Recall

Unsupervised

$$\{x_i\}_{i=0}^{n}$$

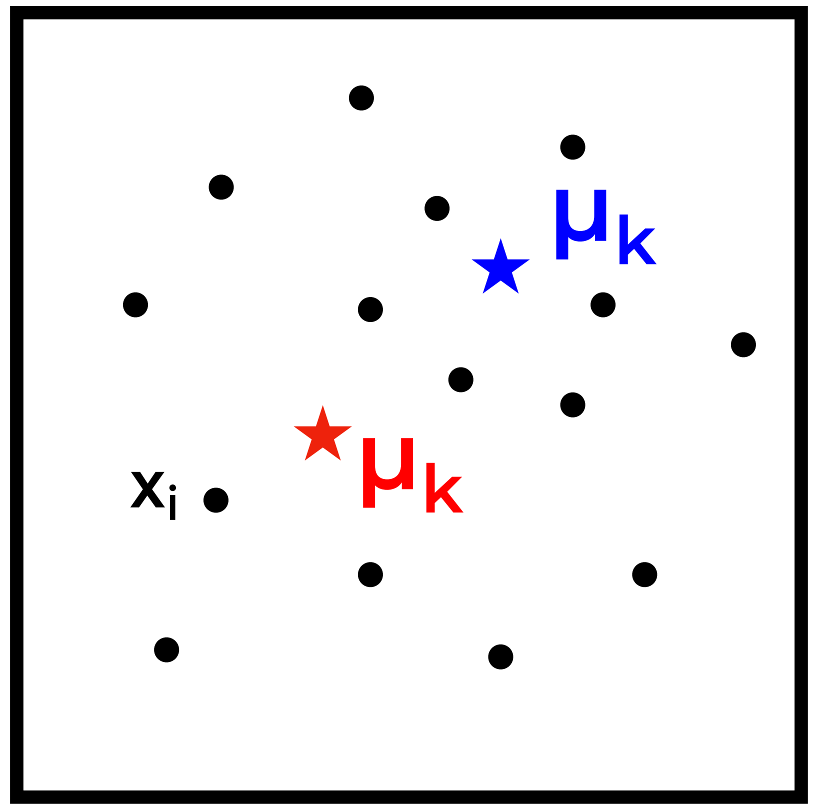
- Experience examples alone
- Learn “useful properties of the structure of the data”
  - E.g., clustering, density modeling (p(x)), PCA, FA.

Different tasks

- **Finding patterns**
  - **Clustering** $f: X \to \{1, 2, \ldots, K\}$ (K clusters)
  - **Dimensionality reduction** $f: X^p \to X^k, k << p$
  - **Density modelling** $f: X \to [0, 1]$
  - **…**

K-means clustering

- A particular clustering model (and accompanying algorithm)
  - There are K clusters. Each point belongs to a cluster. Clusters have centers: μ
- Objective: Find cluster centers μ_k that minimize the within cluster distance



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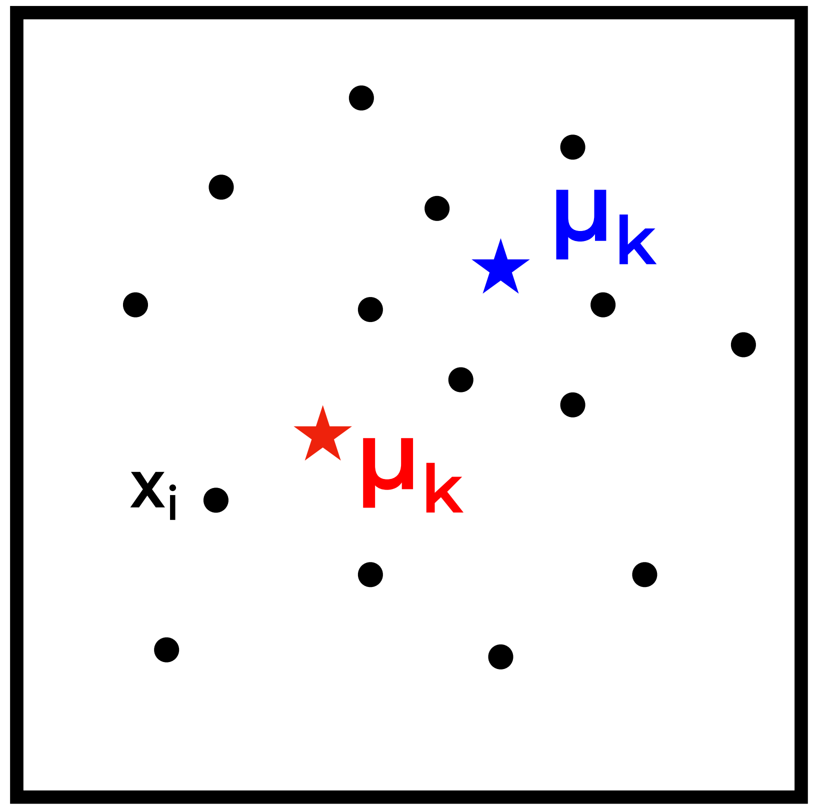
$$\text{Objective} := \sum_{i=1}^{N} \sum_{k=1}^{K} r_{ik} ||x_i - \mu_k||^2$$

$$r = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix}_{N \times 2}$$



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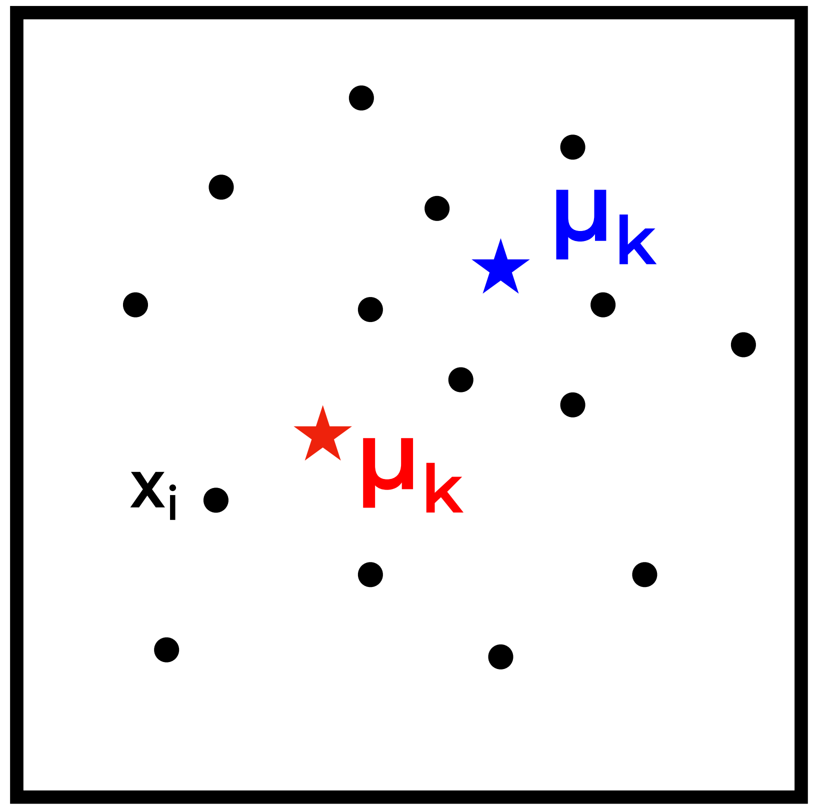

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- Algorithm to minimize the objective:
  - Initialize the cluster centers
  - Until convergence:
    1. Update responsibilities: r
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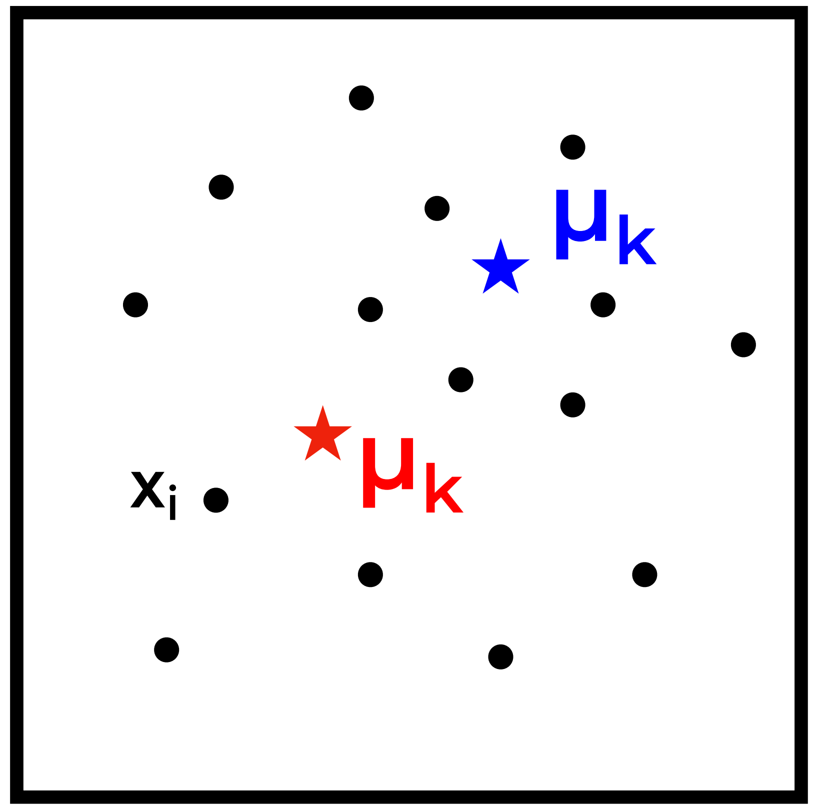

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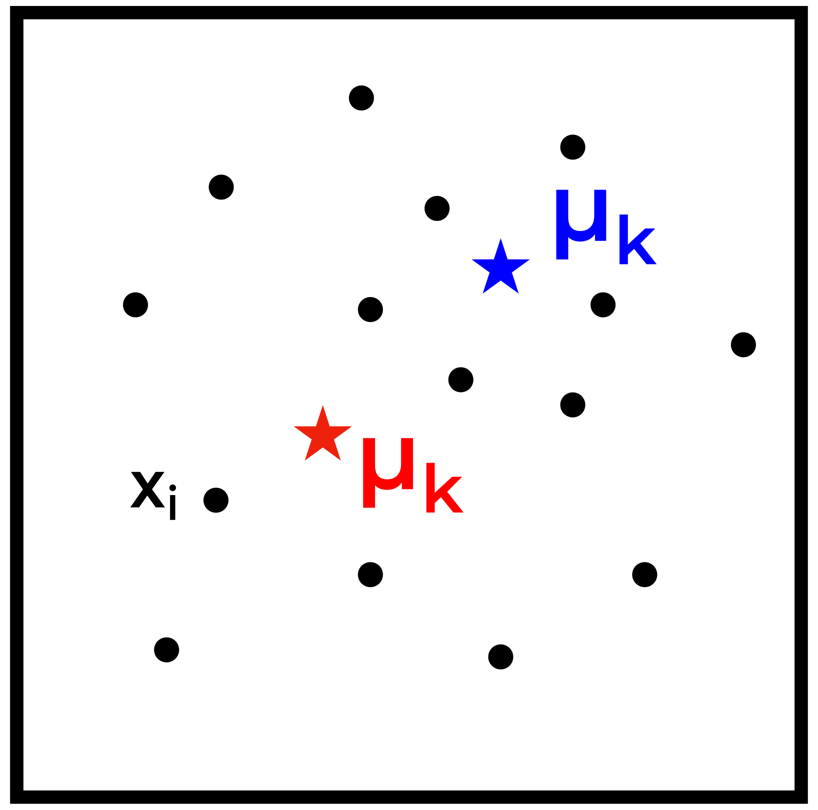

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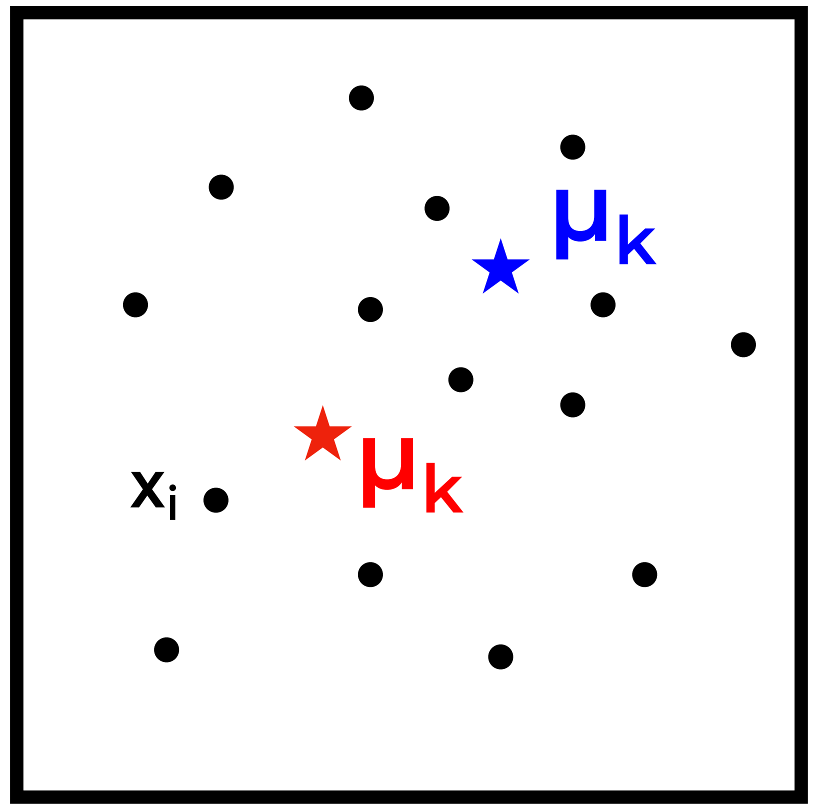

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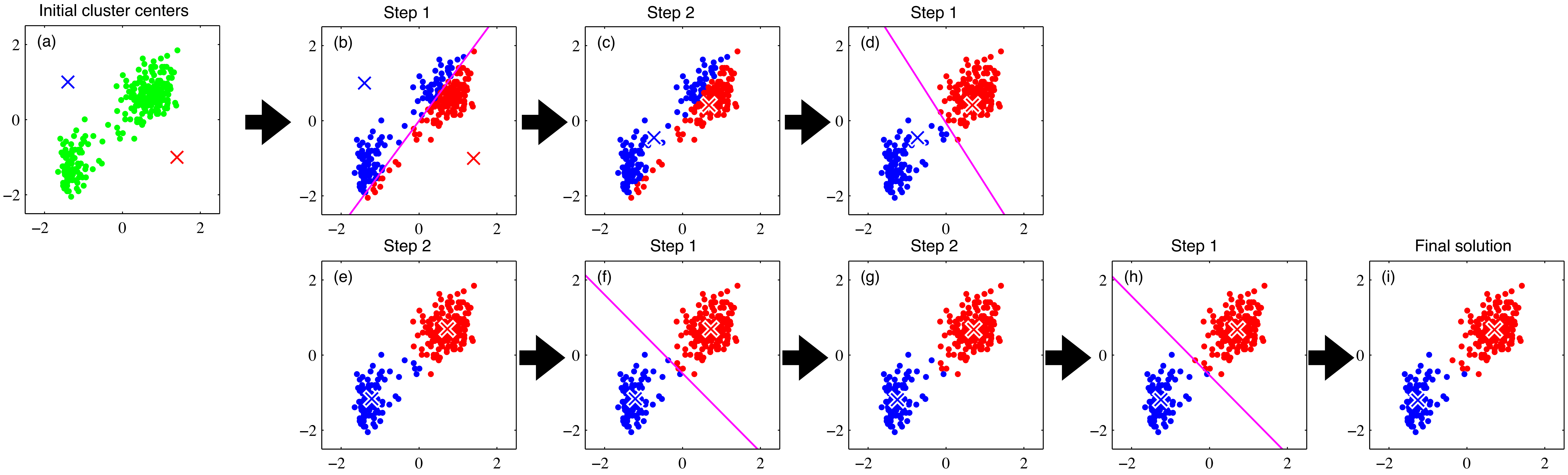
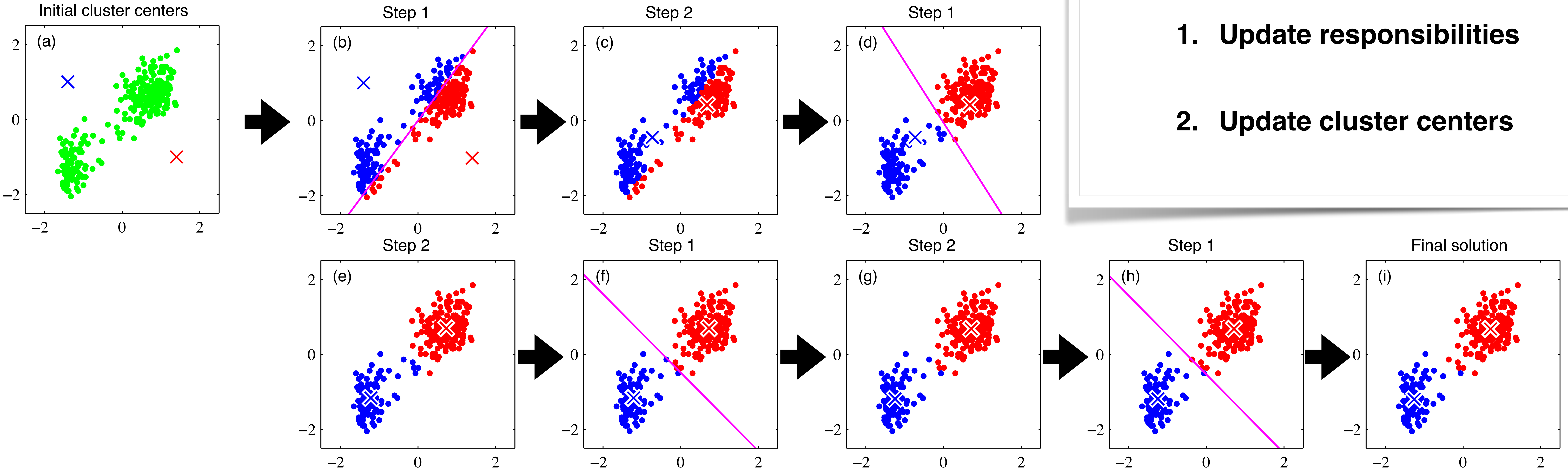


[Figure 9.1 from PRML]

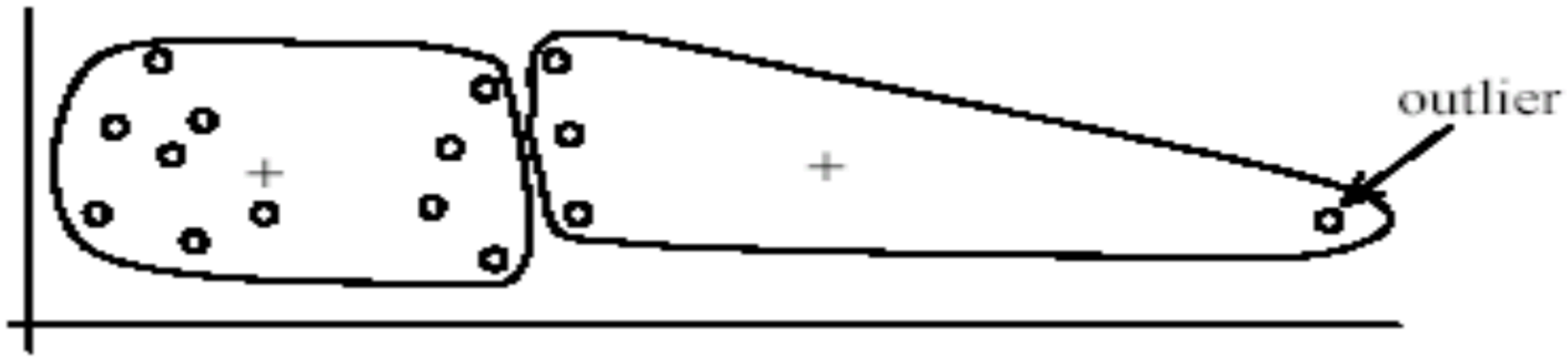
Algorithm

- **Initialize the cluster centers**
- **Until convergence:**
  1. **Update responsibilities**
  2. **Update cluster centers**

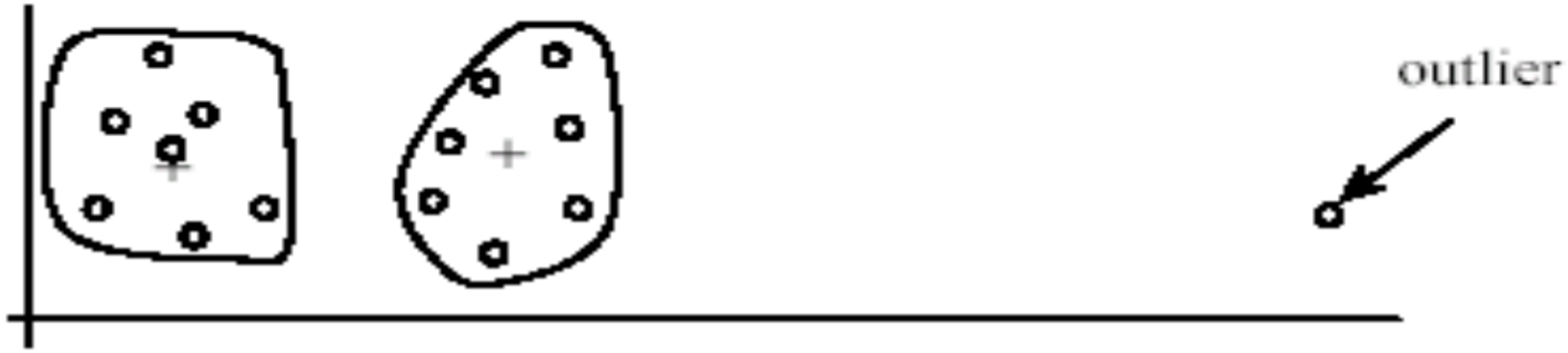


[Figure 9.1 from PRML]

K-means and sensitivity to outliers

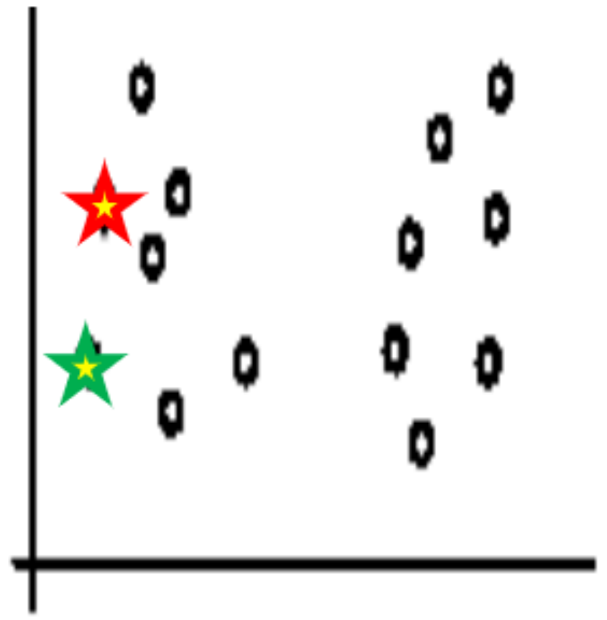


(A): Undesirable clusters

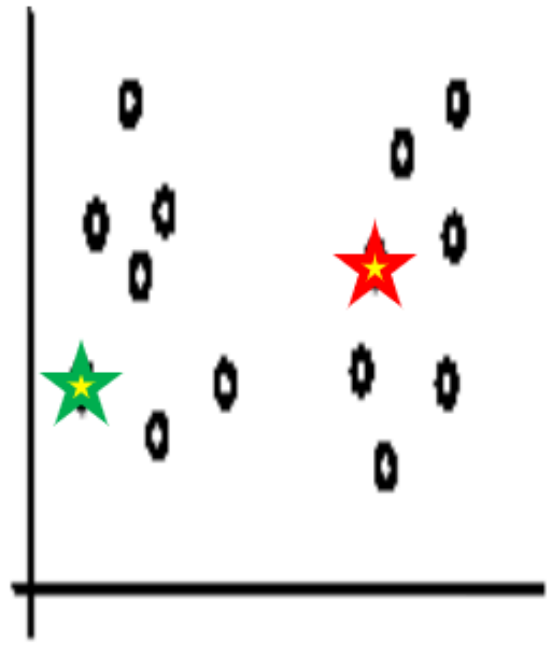


(B): Ideal clusters

K-means and sensitivity to initial seeds



Random selection of seeds (centroids)



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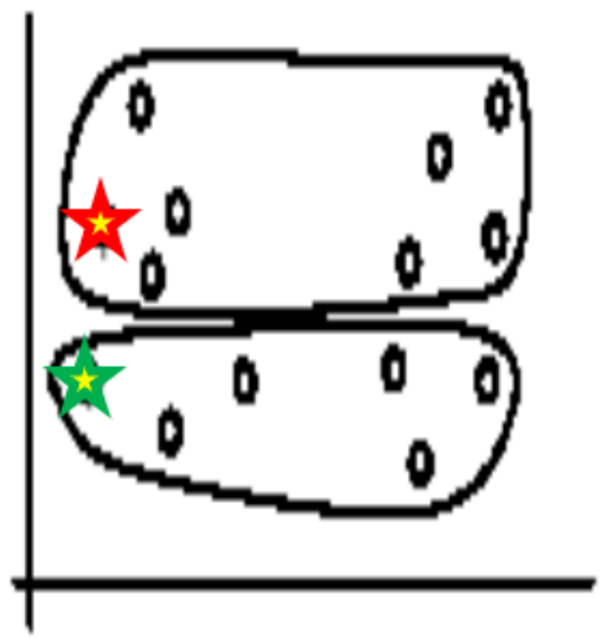
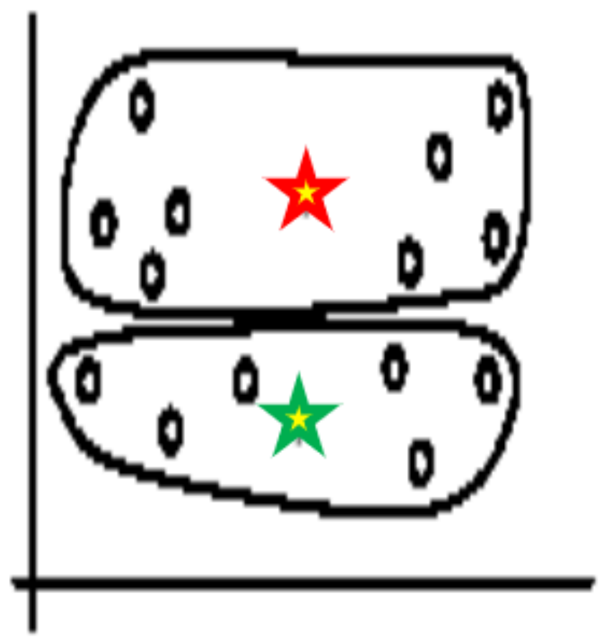
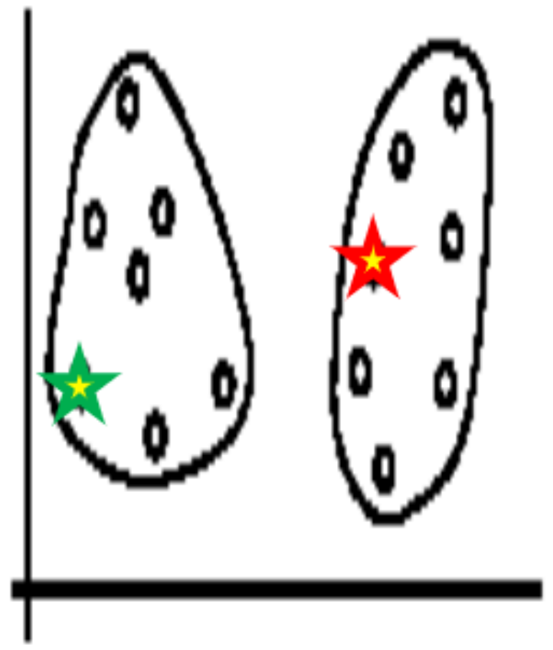


Iteration 1



Iteration 2



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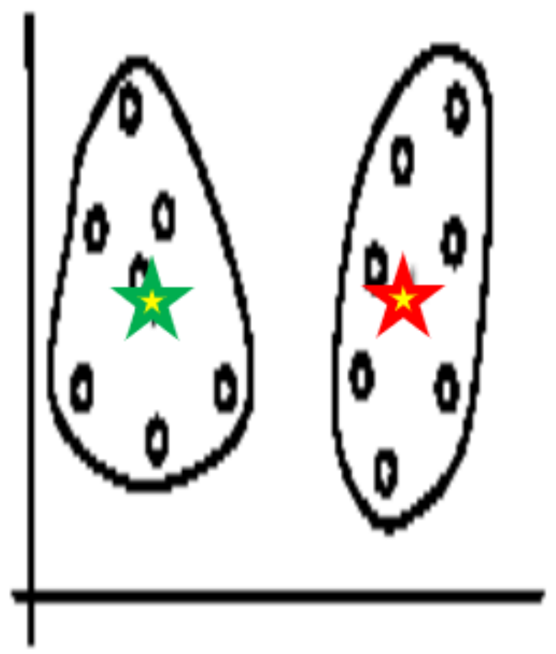
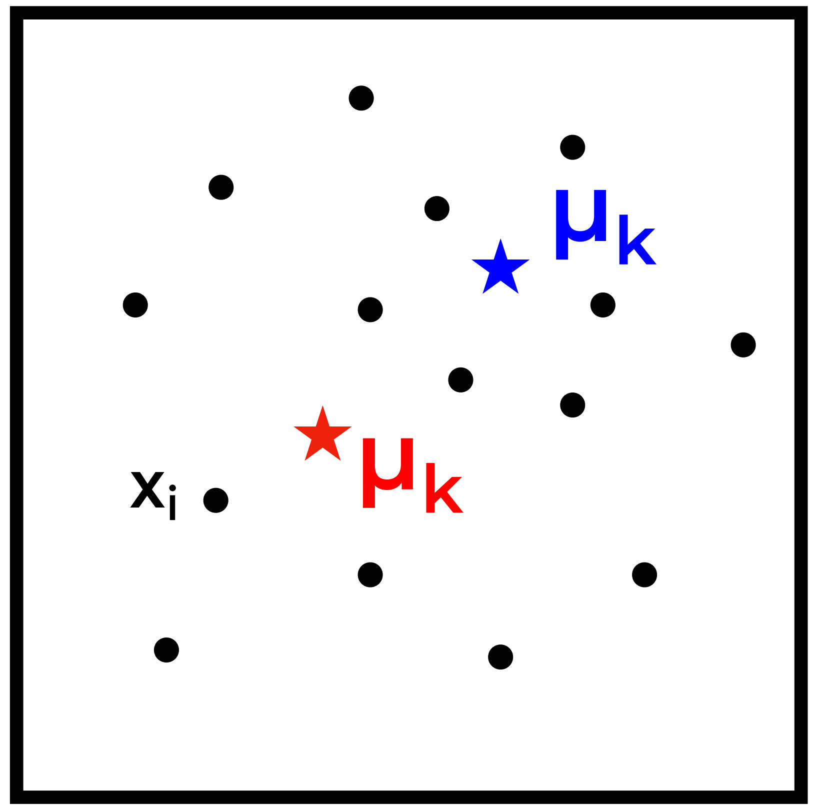


Iteration 2

A probabilistic approach to k-means clustering

K-means Clustering



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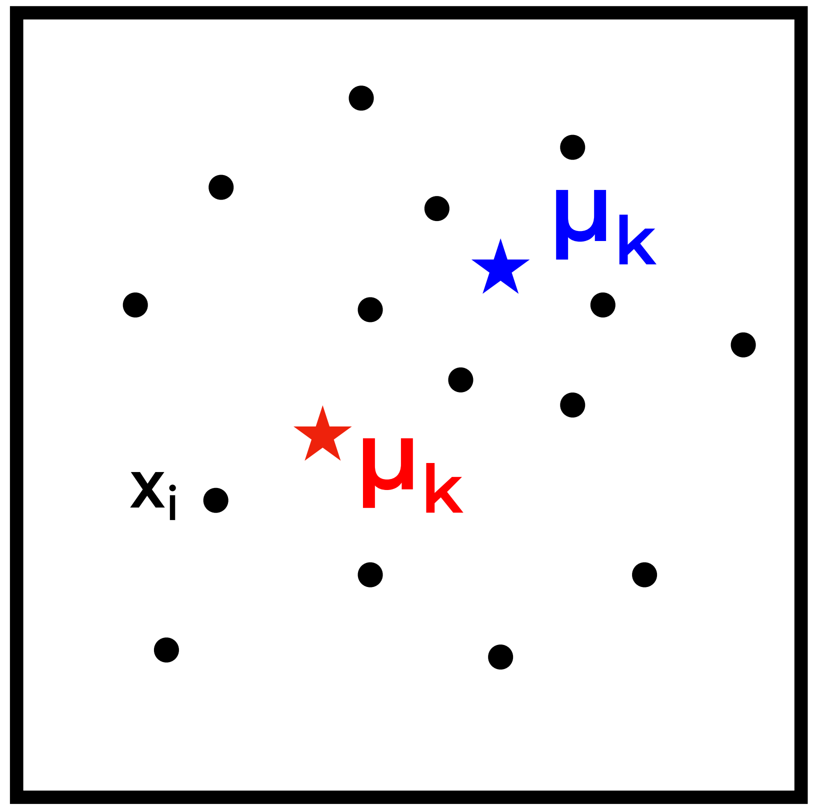

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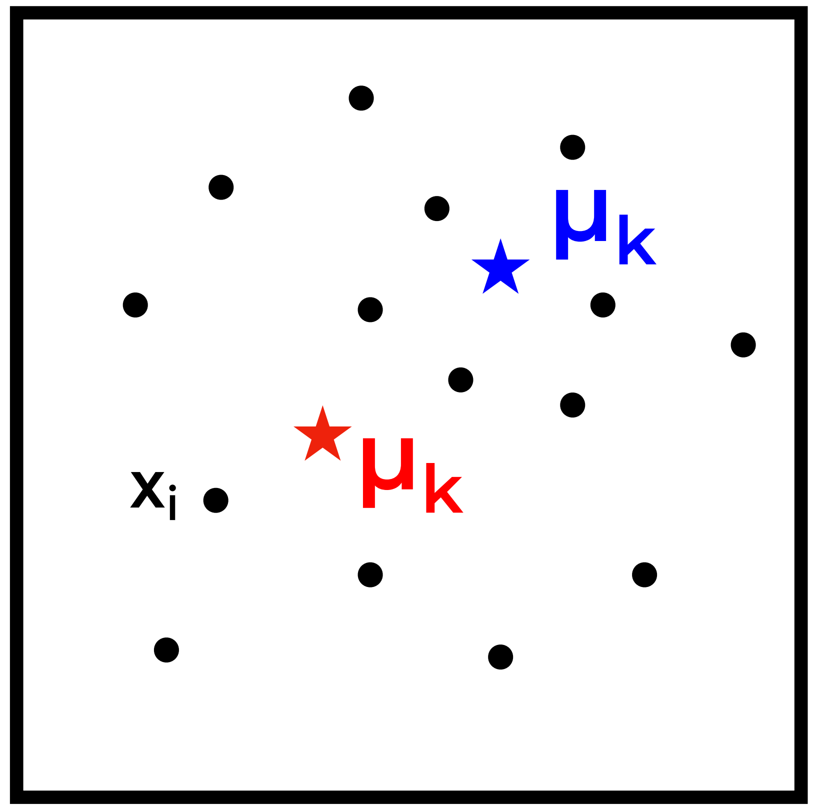
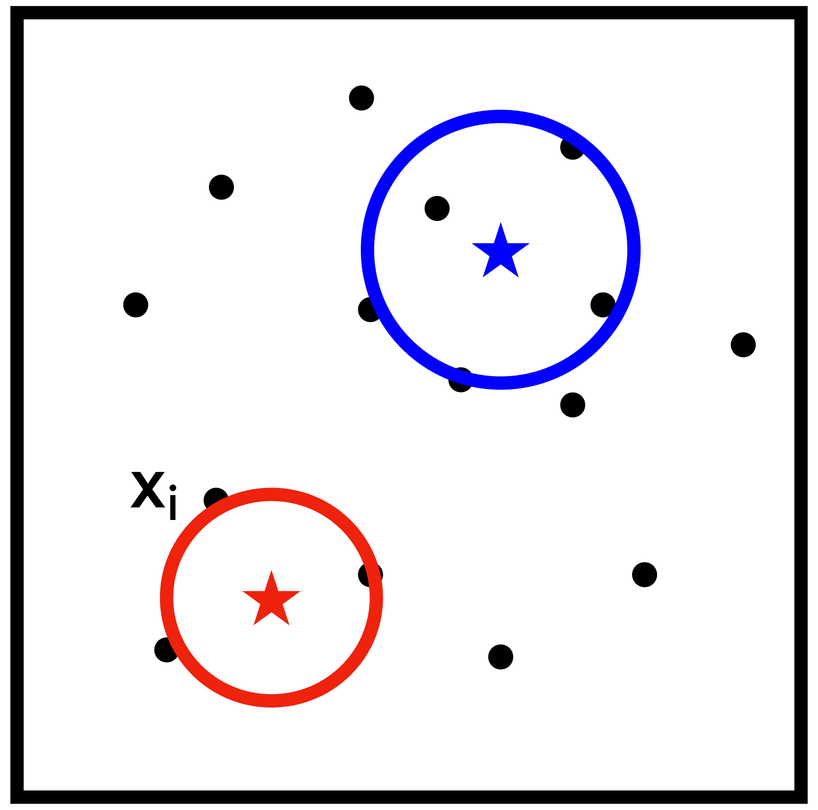
A probabilistic approach to k-means clustering

K-means Clustering



Soft K-means Clustering



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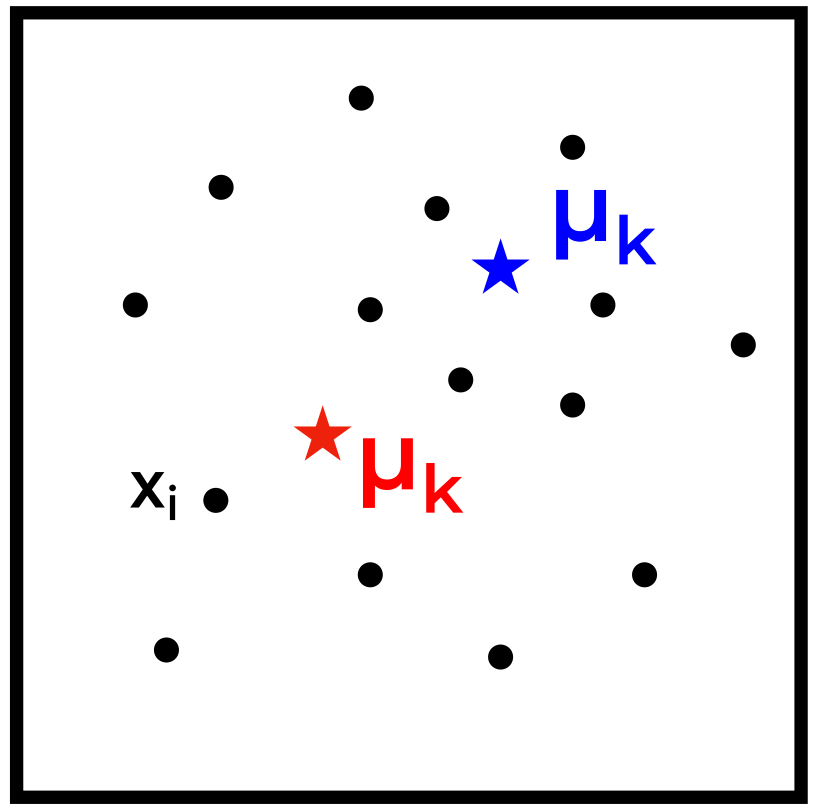
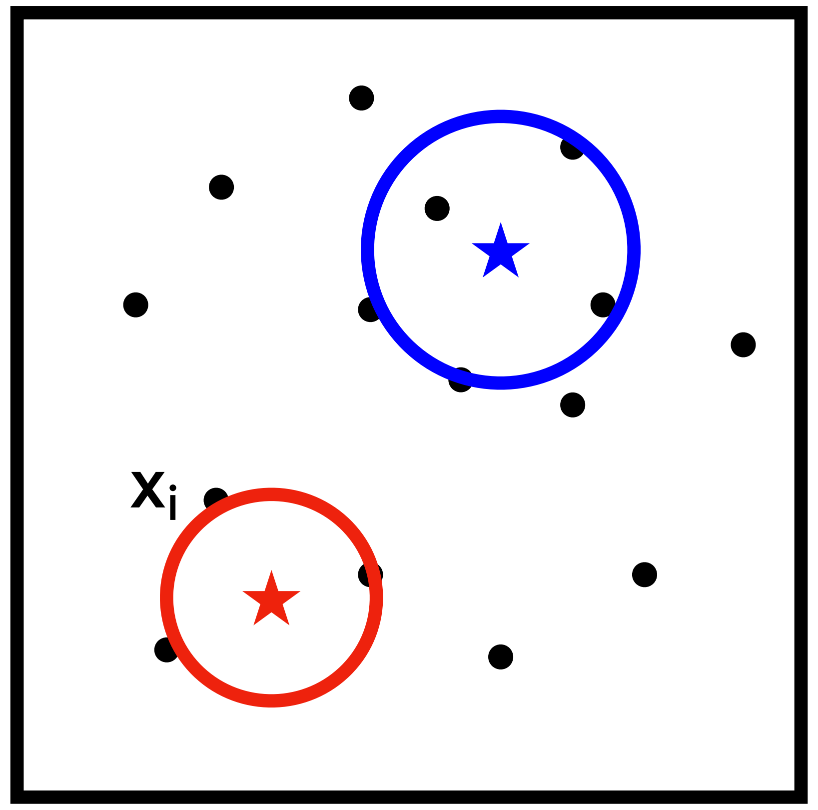


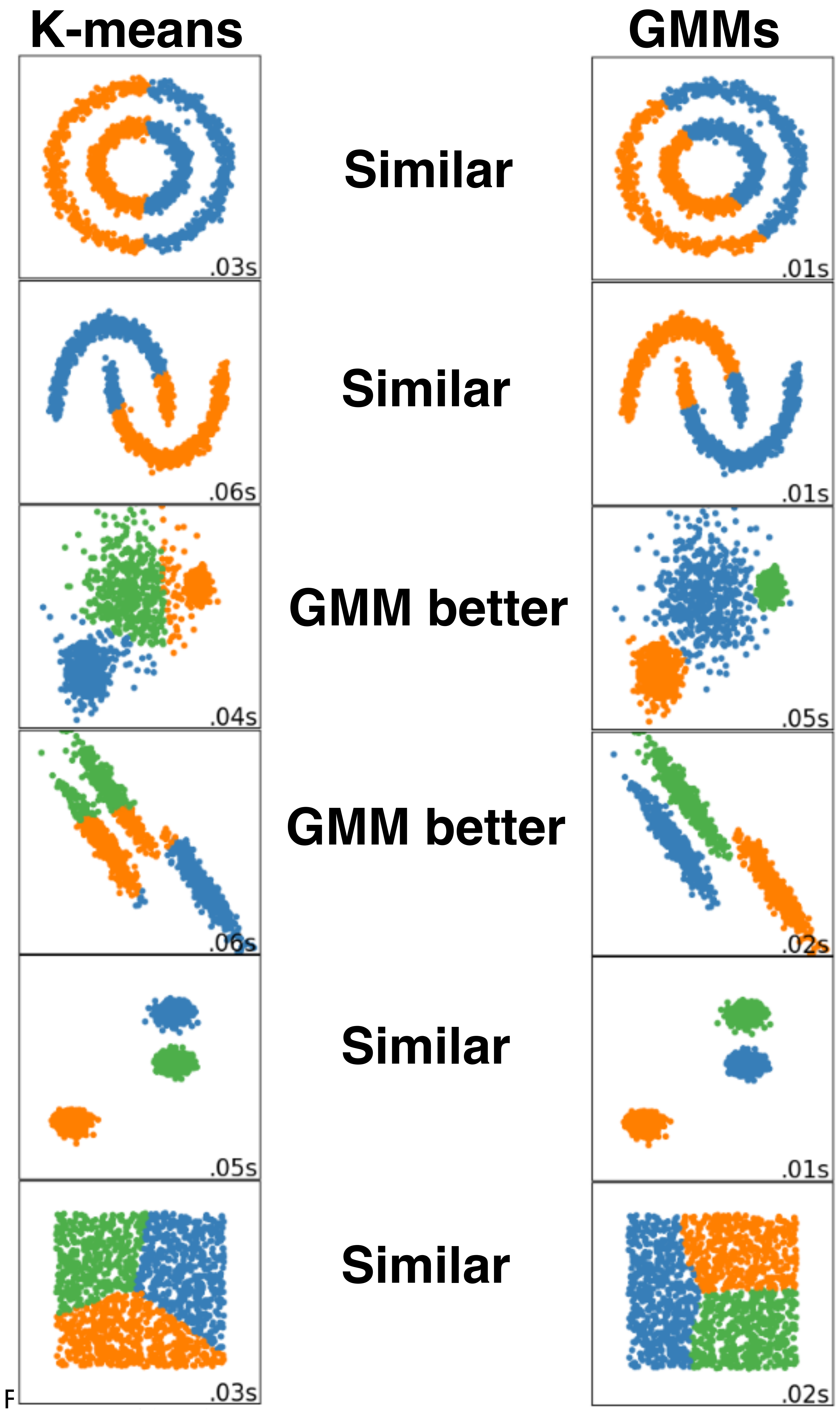
Responsibility   center

$$\text{Objective} := \sum_{i=1}^{N} \sum_{k=1}^{K} \boxed{r_{ik}} \, \| \, x_i - \boxed{\mu_k} \, \|^2$$

Soft K-means Clustering



- Responsibilities are continuous [0, 1]
  - Each cluster has a responsibility: π_k
- Each cluster models data using a Gaussian: $\mathcal{N}(x_i \mid \mu_k, \Sigma_k)$


K-means
GMMs
.03s
Similar
.01s
.06s
Similar
.01s
.04s
GMM better
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.05s
Similar
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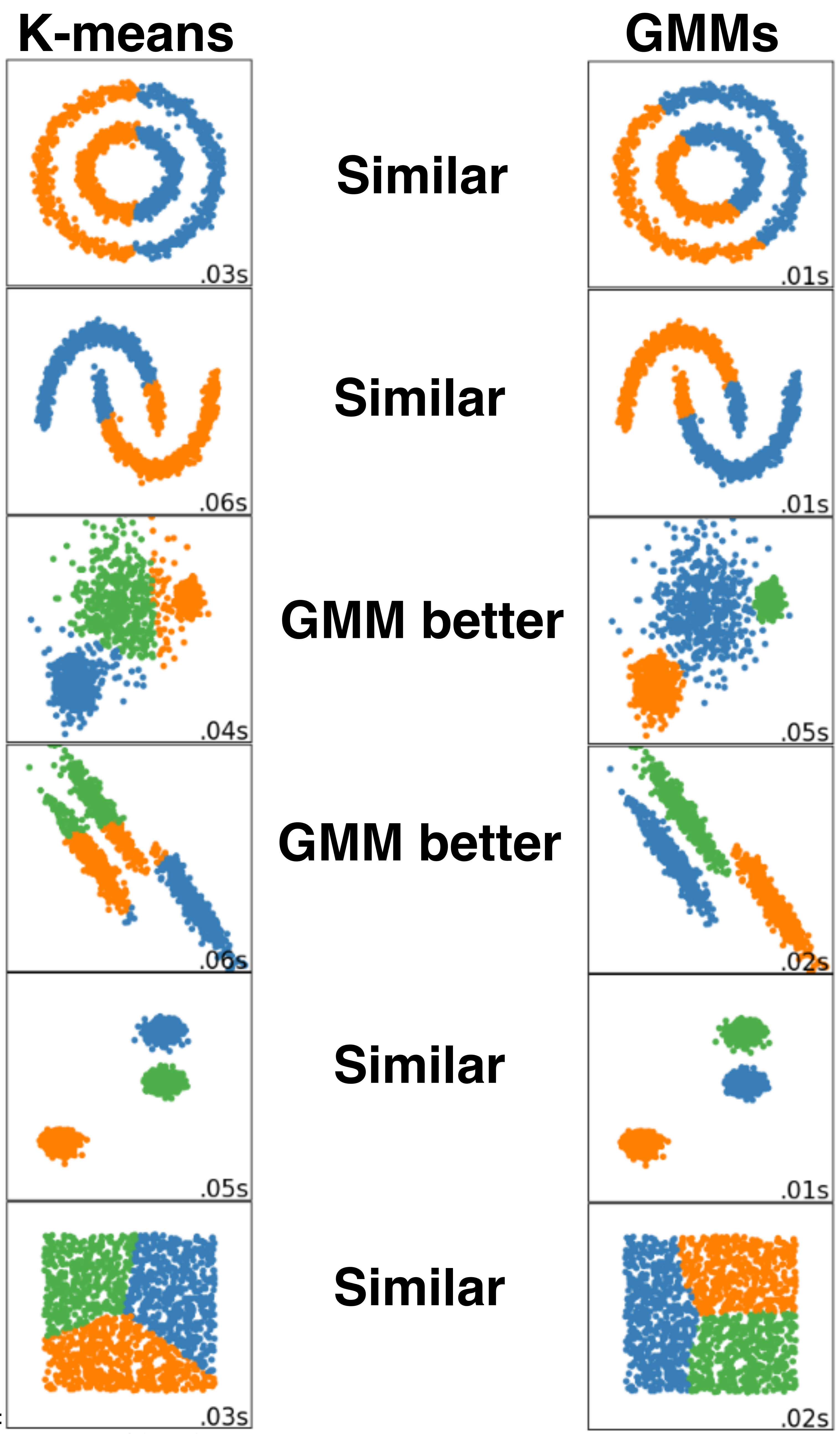
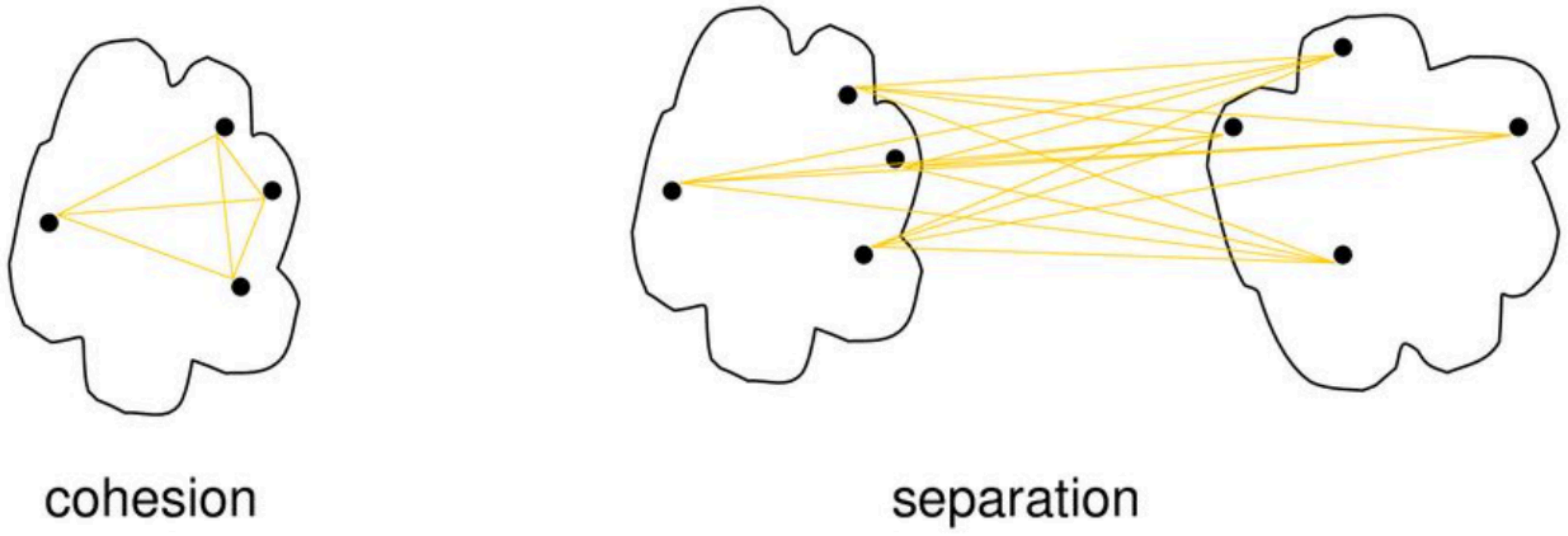


Comparing K-means to GMMs

- GMMs learns covariance matrix
  - Per cluster variance
  - Covariance terms
- GMMs has many more parameters
  - Covariance matrix (MxM)

Additional info

- **Evaluation**
  - Comparing different clustering algorithms is a difficult task. No one knows the correct clusters!
  - Internal evaluation and external evaluation
  - E.g., evaluation: Cohesion and Separation



Additional info

- **Cluster Cohesion**: how closely are the objects within the cluster

WSS (Within Clusters Sum of Squares)

We want this to be small

$$WSS = \sum_{i} \sum_{x \in C_i} (x - C_i)^2$$

- **Cluster Separation**: measure how distinct clusters are wrt each other

BSS (Between Clusters Sum of Squares)

We want this to be large

$$BSS = \sum_{i} m_i (C - C_i)^2$$

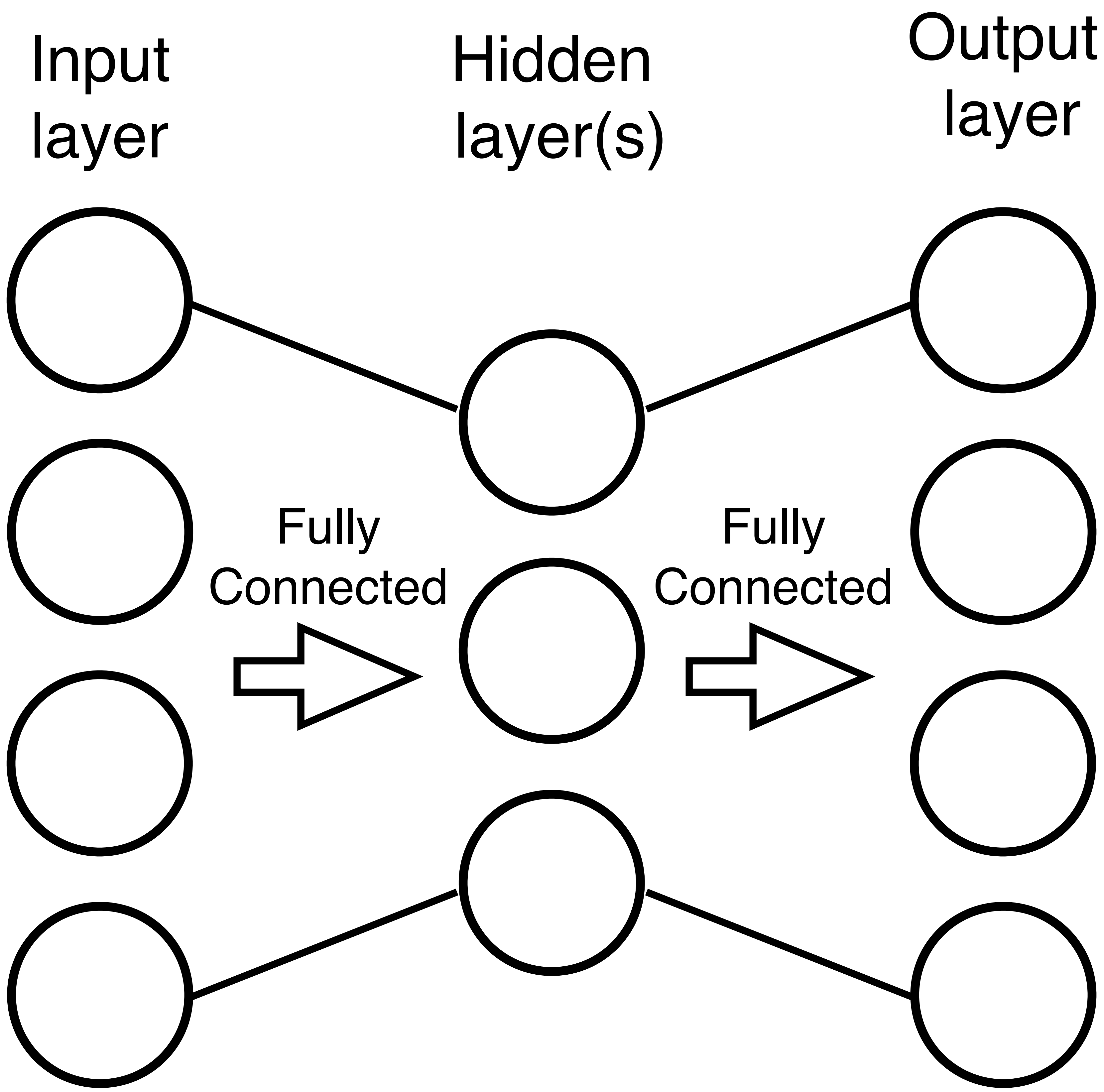
$$\boxed{BSS + WSS = CONSTANT}$$

Additional info

- **Evaluation**
  - Comparing different clustering algorithms is a difficult task. No one knows the correct clusters!
  - Internal evaluation and external evaluation
  - E.g., internal evaluation: Cohesion and Separation
- For some data, **hierarchical Clustering** is more appropriate, e.g., **biological taxonomy**

Autoencoders

- **A neural network architecture for unsupervised learning**



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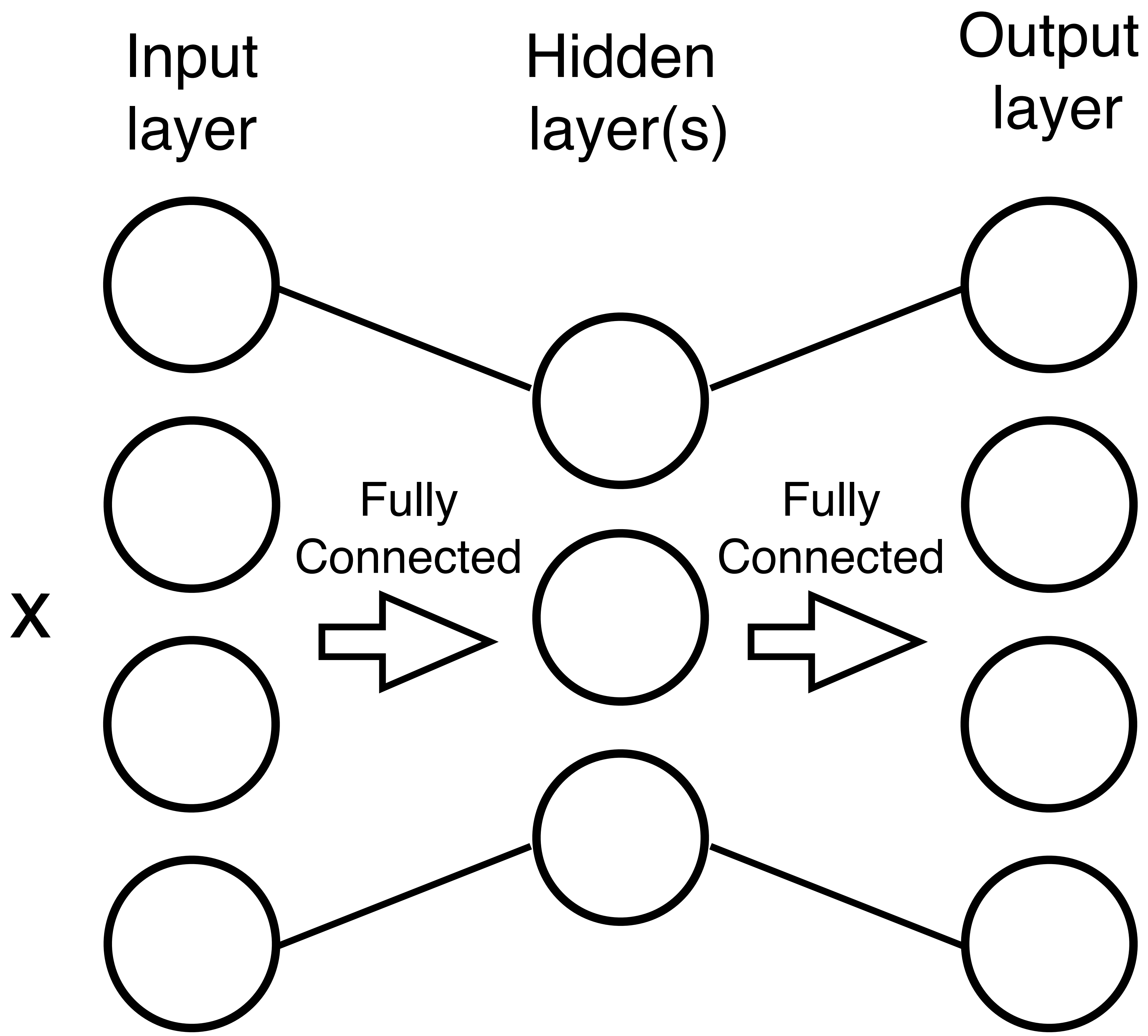

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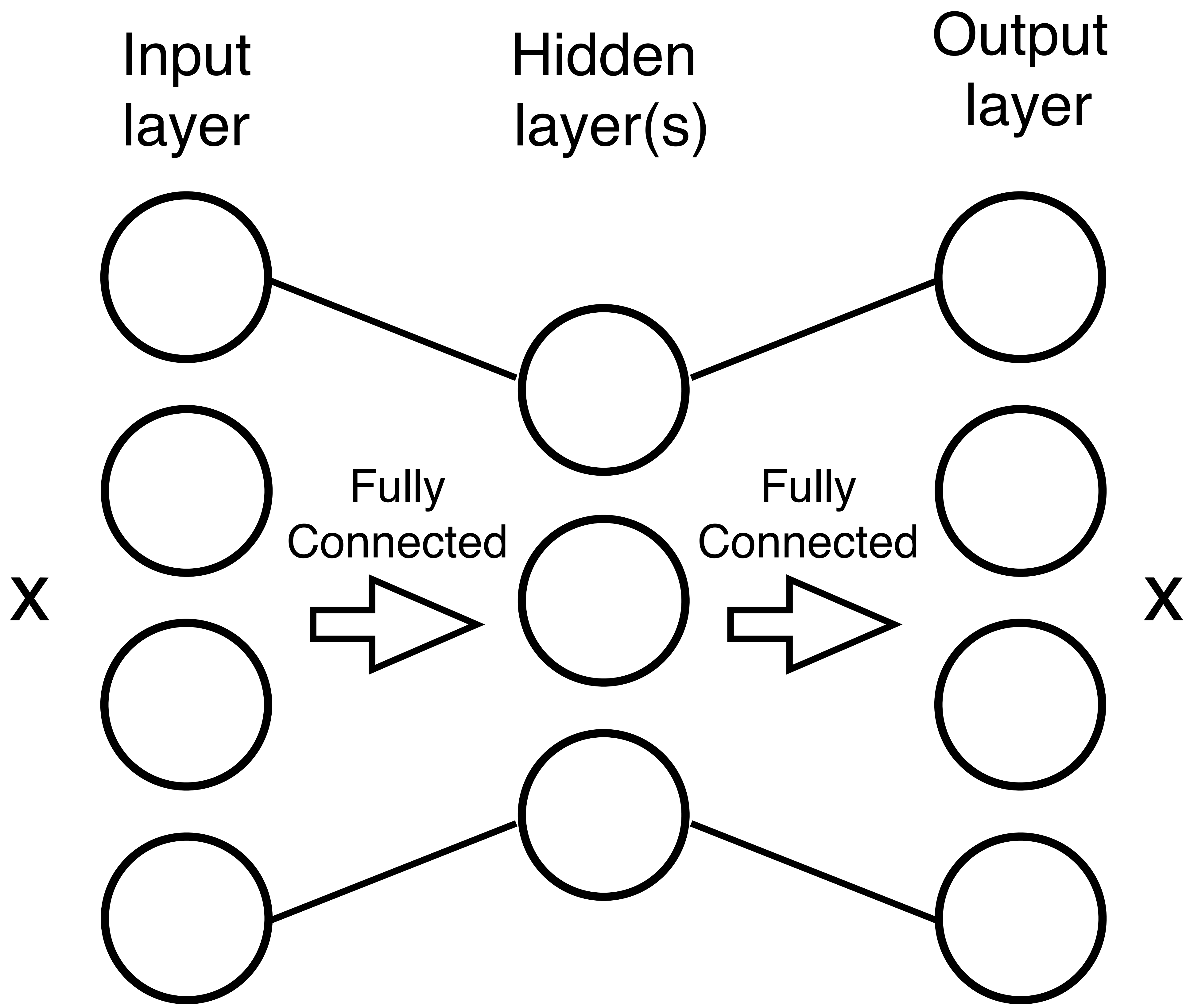

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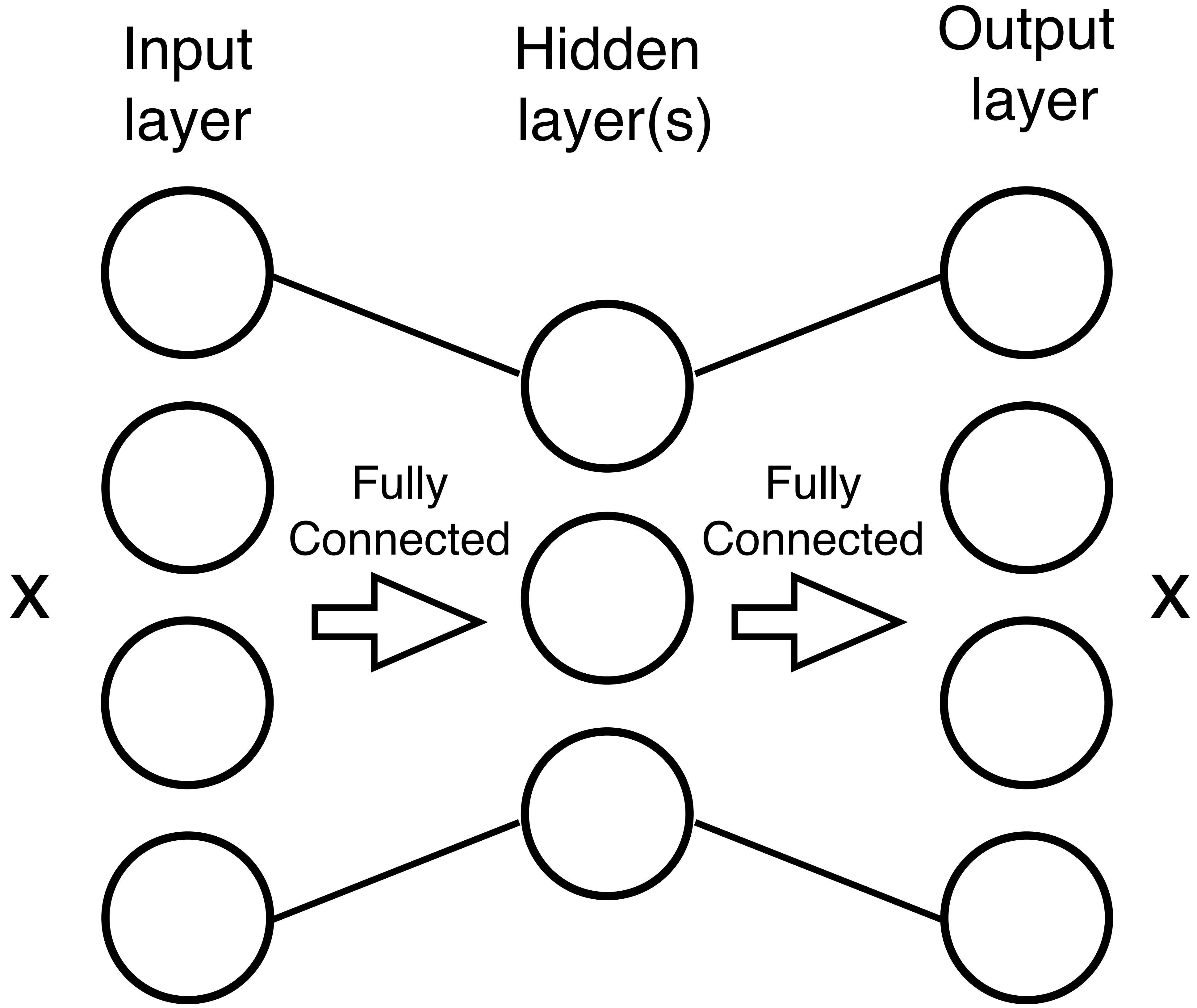

Objective:
How well the network predicts X ?

$$\text{Loss} := \sum_{i=1}^{N} (x_i - \hat{x}_i)^2$$
$$= \sum_{i=1}^{N} (x_i - f_2(f_1(x)))^2$$

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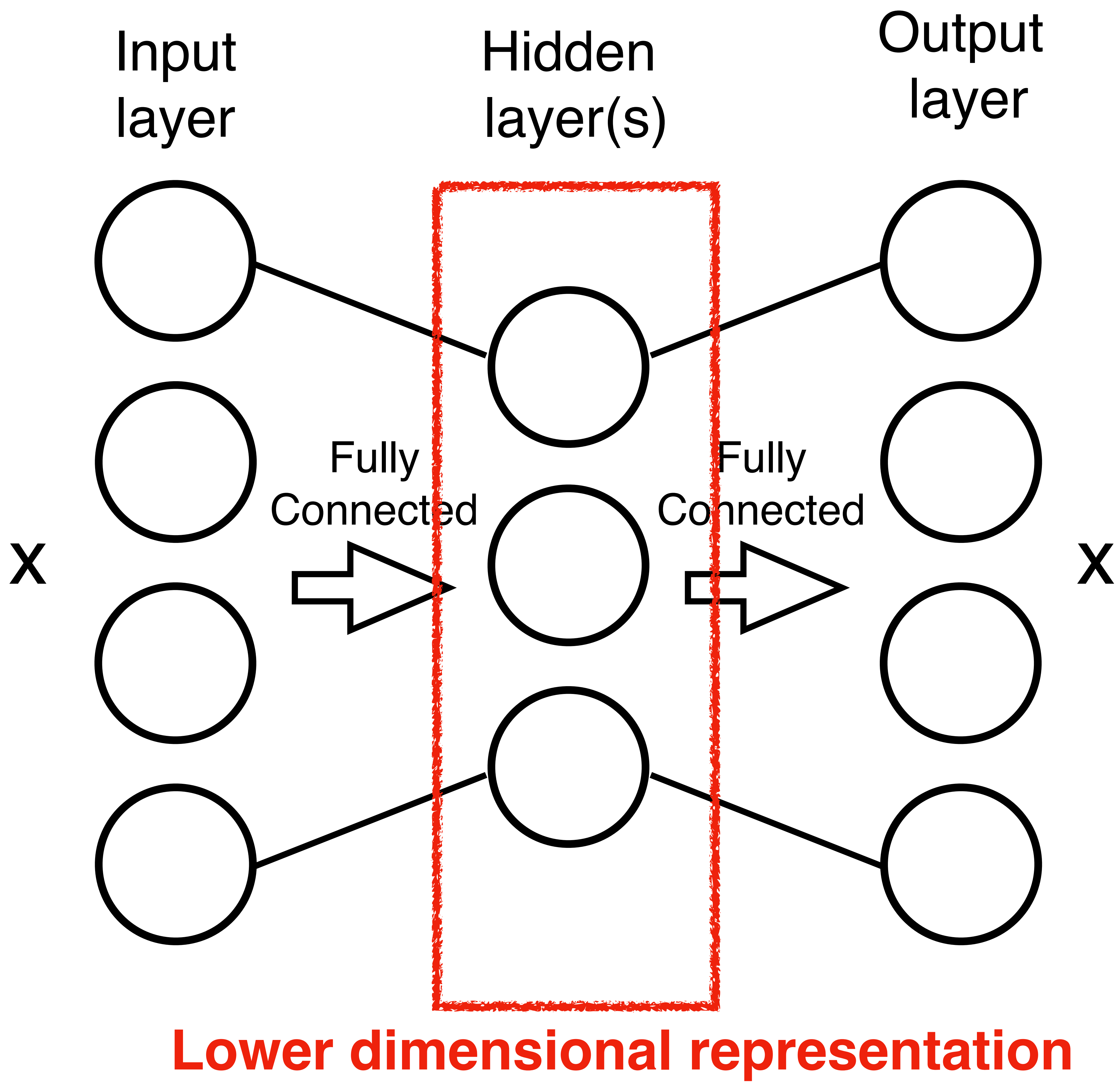

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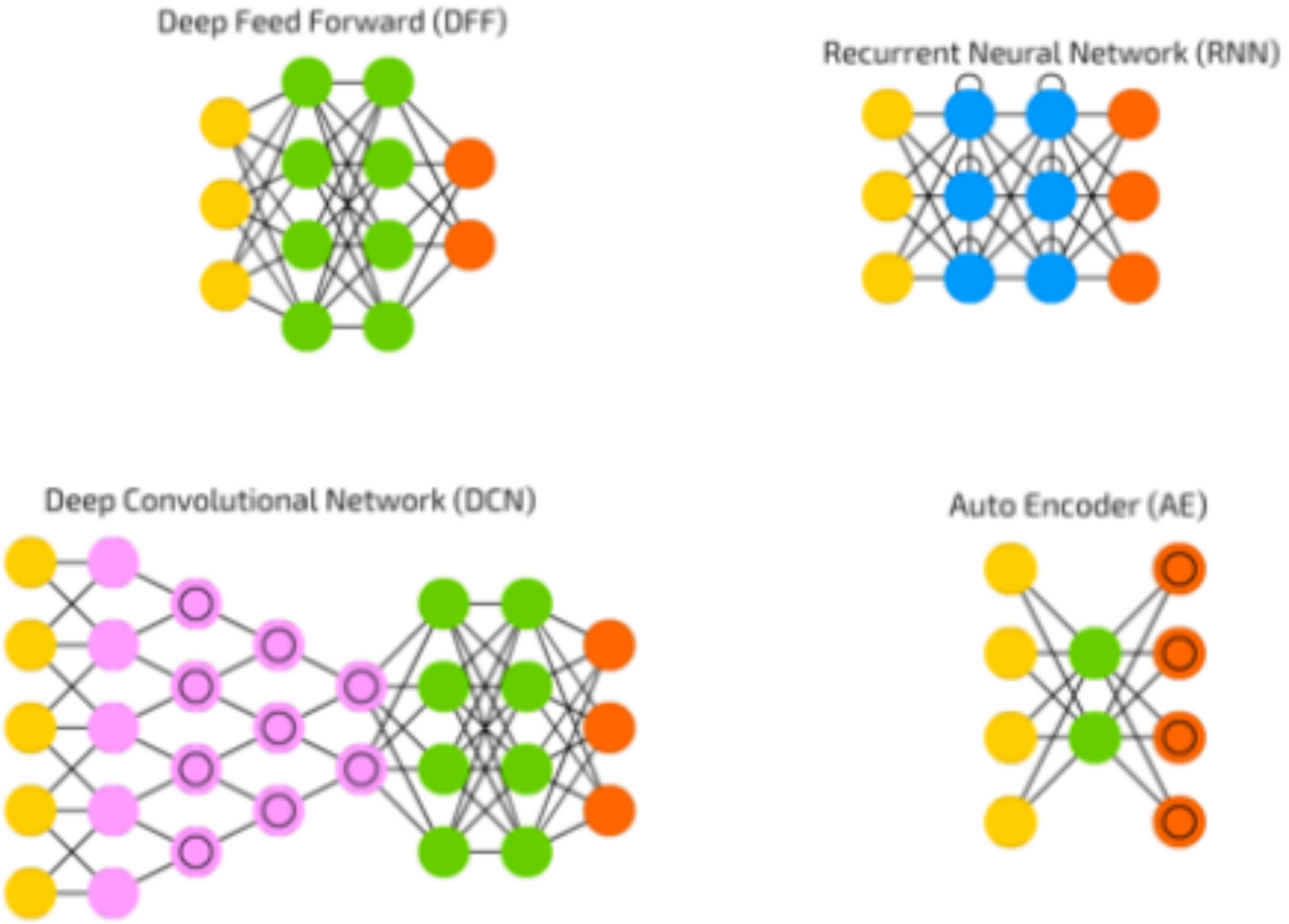
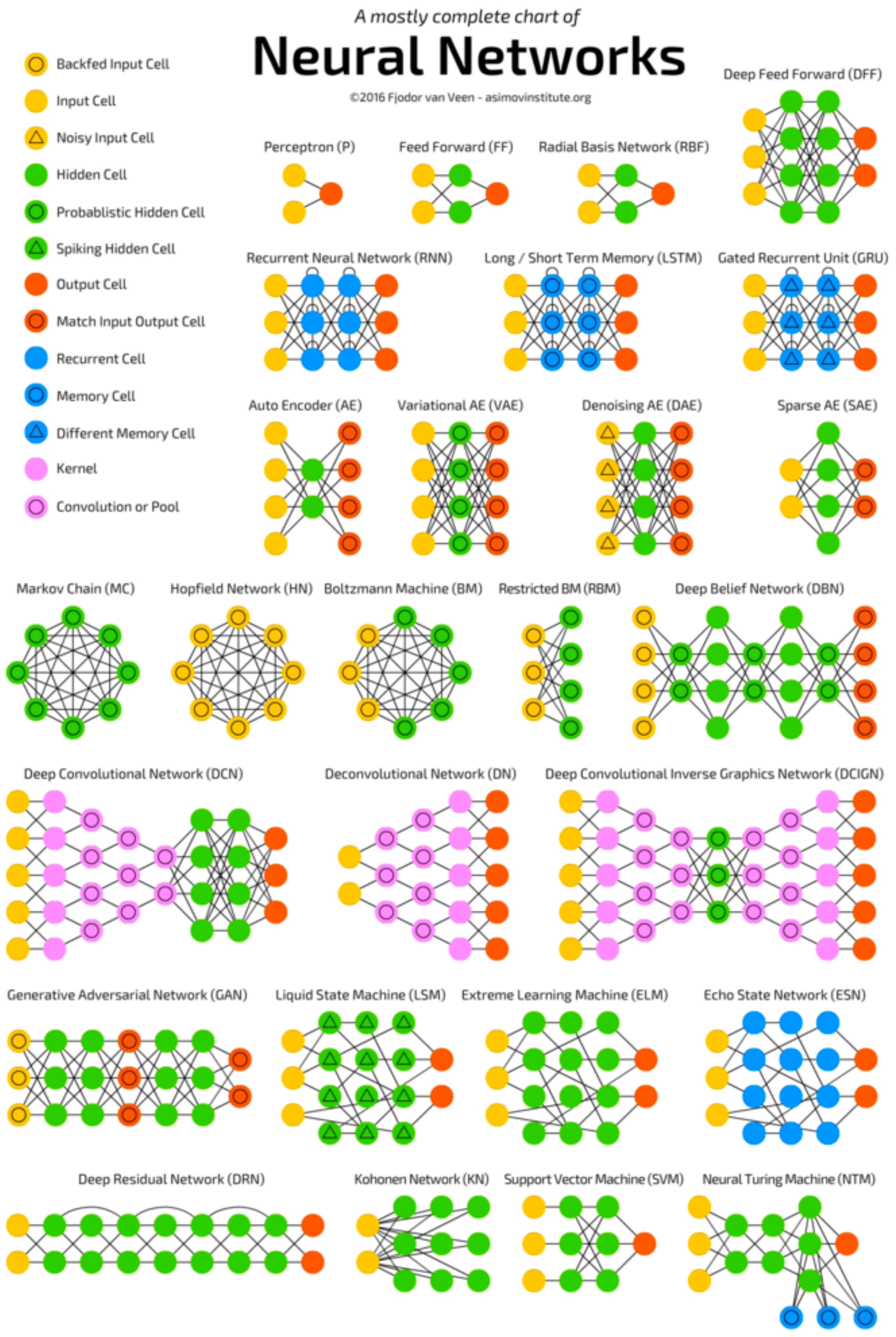
Various Architectures





Leijnen, Stefan, and Fjodor van Veen. "The neural network zoo." *Multidisciplinary Digital Publishing Institute Proceedings*. Vol. 47. No. 1. 2020.