



Vavuniya Campus of the University of Jaffna
First Examination in Information and Communication
Technology - 2018

First Semester - September/October 2019

IT1122 Foundation of Mathematics

Answer Four Questions Only

Time Allowed : Two hours

1. (a) Write down the elements in each of the following sets:

i. $A = \{x \in \mathbb{N} \mid x \text{ is even, } x < 11\};$

ii. $B = \{x \in \mathbb{N} \mid 3 < x < 9\};$

iii. $C = \{x \in \mathbb{N} \mid 4 + x = 3\};$

iv. $D = \{x \in \mathbb{R} \mid x^2 = 4 \text{ OR } x^2 = 9\}.$

[20%]

(b) Suppose A is the set of distinct letters in the word *ELEPHANT*, B is the set of distinct letters in the word *SYCOPHANT*, C is the set of distinct letters in the word *FANTASTIC*, and D is the set of distinct letters in the word *STUDENT*. The universe U is the set of *English alphabet*. Find:

i. $A \cup B;$

ii. $A \cap B;$

iii. $A \cap (C \cup D);$

iv. $(A \cup B \cup C \cup D)^c.$

[30%]

[This question is continued on the next page]

(c) If $A = \{ 0, 1 \}$, $B = \{ 1, 2 \}$ and $C = \{ 0, 1, 2 \}$ find:

- i. $P(C)$, the power set of C ;
- ii. $A \times C$;
- iii. $A \times B \times C$.

[30%]

(d) Find A^2 if:

- i. $A = \{ 0, 1, 3 \}$
- ii. $A = \{ 1, 2, a, b \}$

[20%]

2. (a) In an examination, 73% of the candidates passed in mathematics, 70% passed in physics, 64% passed in both subjects and 63 candidates are failed in both subjects. Use a Venn diagram to find the total number of candidates who appeared at the examination.

[30%]

(b) Using a membership table to show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

[20%]

(c) Using the set identities prove the following:

- i. $(A^c \cup B)^c \cap A^c = \phi$
- ii. $(A \cup B) \cap (A^c \cap B)^c = A$
- iii. $(A \cap B) \cup (A \cup B^c)^c = B$

[30%]

- (d) A genetics experiments classifies fruit flies according to the following criteria:
Sex: m-male, f-female; Wing Span: s-short winged, m-medium winged, l-Long winged. List all the categories in this classification scheme.

[20%]

3. (a) Define the following function:

- i. *Injective* function;
- ii. *Surjective* function;
- iii. *Bijective* function.

[30%]

[This question is continued on the next page]

(b) Determine whether each of the following functions is a bijection from \mathbb{R} to \mathbb{R} .

Justify your answer:

i. $f(x) = 2x + 1$

ii. $f(x) = x^2 + 1$

iii. $f(x) = x^3$

iv. $f(x) = (x^2 + 1)/(x^2 + 2)$

[40%]

(c) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$. Find $f \circ g$ and $g \circ f$, if :

i. $f(x) = 2x$ and $g(x) = x^2$

ii. $f(x) = 4x - 3$ and $g(x) = x^2 + 2$

[30%]

4. (a) Define the terms *arithmetic progression* and *geometric progression*.

[20%]

(b) Decide whether the each of the following scenarios describes an arithmetic or geometric sequence. Then, write the formula for the sequence.

i. Round 1 of a tennis tournament starts with 128 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 64 players, in round 3 there are 32 players and so on.

ii. Paul has Rs 6800.00 in a saving account. He makes a deposit after he receives each paycheck. After one month he has Rs 7580.00 in the account. The next month the balance is Rs 8360.00. The balance after the third month is Rs 9140.00.

[30%]

(c) Find the sum of the first six terms in the sequence of $\{a_n\}$, $n > 2$

where $a_n = 2a_{n-1} + a_{n-2}$, $a_1 = 1$, and $a_2 = 1$.

[20%]

(d) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation

$$a_n = 8a_{n-1} - 16a_{n-2}, n > 2 \text{ if}$$

i. $a_n = 4^n$

ii. $a_n = 2 \cdot 4^n + 3n \cdot 4^n$

[30%]

5. (a) Find the product AB, where

$$\text{i. } A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix};$$

$$\text{ii. } A = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 0 & 3 & -1 \\ -3 & -2 & 0 & 2 \end{bmatrix};$$

$$\text{iii. } A = \begin{bmatrix} 0 & -1 \\ 7 & 2 \\ -4 & -3 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 & 2 & 3 & 0 \\ -2 & 0 & 3 & 4 & 1 \end{bmatrix}.$$

[30%]

(b) Let A be the 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\text{i. Show that if } ad - bc \neq 0, \text{ then } A^{-1} = \begin{bmatrix} \frac{d}{ad - bc} & \frac{-b}{ad - bc} \\ \frac{-c}{ad - bc} & \frac{a}{ad - bc} \end{bmatrix};$$

[30%]

$$\text{ii. if } A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}; \text{ Find } A^{-1}.$$

[15%]

(c) Write an algorithm for swapping two numbers a and b.

[25%]