University of Vavuniya

First Examination in Information Technology - 2020

First Semester - Apr/May 2022

IT1122 Foundation of Mathematics

Answer Four Questions Only

Time Allowed : **Two hours**

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2 3 4 5   3 4 5 6 7 8

1. (a) Let $A = \{x \in \mathbb{N} | 1 < x \leq 5\}$ and $B = \{x \in \mathbb{N} | 3 \leq x \leq 8\}$.

   Find each of the following:

   i. $A \cup B$

   ii. $A \cap B$

   iii. $A - B$

   iv. $B - A$

   [20%]

   (b) Find the sets A and B, if $A - B = \{1,5,7,8\}$, $B - A = \{2,10\}$ and $A \cap B = \{3,6,9\}$. [10%]

   (c) In an examination, 70% of the candidates passed in Mathematics, 73% passed in Computer Science, 64% passed in both subjects and 63 candidates are failed in both subjects.

   Use a Venn diagram to find the total number of candidates who appeared at the examination. [15%]

[This question is continued on the next page]

(d) Let A and B be two non empty sets.

Using the Set Identities show that $(A^c \cup B)^c \cap A^c = \emptyset$. [10

(e) Let A and B be two non empty sets.

Using the Set Identities simplify the following expression: $(A \cap B) \cup (A \cup B^c)^c$ [10

(f) Let $A = \{x, y\}$, $B = \{1, 2, 3\}$ and $C = \{a, b\}$. Find each of the following:

i. $P(B)$

ii. $A \times B \times C$ [2

20 − 0.6

(g) A school basketball team plays 20 matches each year. The probability for winning any match is 0.6. Find each of the following:

i. The probability that the school losing a match. 1 − 0.6

ii. The number of matches that the school may expect to win each year. [15%]

2. (a) Define each of the following functions with the aid of an example:

i. One-to-one function

ii. Onto function

iii. Bijective function [30%]

(b) Determine whether each of the following function is a ***bijection*** from $\mathbb{R}$ to $\mathbb{R}$. Justify your answer.

i. $f(x) = 2x + 1$

ii. $f(x) = x^2 + 1$

iii. $f(x) = x^3$ [30%]

(c) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = 3x + 1$. Determine whether the function $f(x)$ invertible. If so, find the inverse function, f^{-1}. [20%]

(d) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = 4x - 3$ and $g(x) = x^2 + 2$ for all x$\in \mathbb{R}$. Find $(g \circ f)(x)$ and $(f \circ g)(x)$. [20%]

3. (a) Find the *domain* and *range* of each of the relations R on the sets A to B, where R is defined as given below:

   i. $A = \{1,2,3,4,5\}$, $B = \{1,2,3,10\}$, aRb if and only if $2a = b$.

   ii. $A = B = \{1,2,3,4\}$, aRb if and only if $a + b = 5$. [20%]

(b) Let $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$. The relation $R_1 = \{(1,1),(2,2),(3,3)\}$ and $R_2 = \{(1,1),(1,2),(1,3),(1,4)\}$. Find the each of the following:

   i. $R_1 \cup R_2$

   ii. $R_1 \cap R_2$

   iii. $R_1 - R_2$

   iv. $R_2 - R_1$ [20%]

(c) Draw a directed graph representation for each of the following relations on the set {1,2,3,4}. Determine whether each of the following relations is *reflexive*, *symmetric* or *transitive*:

   i. $R_1 = \{(1,3),(1,4),(2,2),(3,1),(3,3),(4,1),(4,4)\}$

   ii. $R_2 = \{(1,1),(2,2),(3,3),(4,4)\}$

   iii. $R_3 = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,3),(1,3),(3,2)\}$

[30%]

(d) Consider the relation $R = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} | x - y \text{ is divisible by } 3\}$. Prove that R is an equivalence relation. [30%]

4. (a) Show that the compound propositions $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. [10%]

(b) Prove each of the following using truth table:

   i. $(p \longrightarrow q) \longleftrightarrow (\neg p \vee q)$ is a tautology.

   ii. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ [20%]

[This question is continued on the next page]

(c) Find the dual of each of the following Boolean expressions:

i. $x \cdot \overline{z} + x \cdot 0 + \overline{x} \cdot 1$

ii. $\overline{x} \cdot y \cdot 1 + x \cdot \overline{y} \cdot 1 \cdot (x + y \cdot 0)$

[10%]

(d) Using the properties of Boolean algebra, simplify each of the following Boolean functions:

i. $F(x, y) = y + \overline{x}\overline{y}$

ii. $F(x, y) = \overline{x}\overline{y}(\overline{x} + y)(\overline{y} + y)$

[20%]

(e) Construct circuits using the basic gates that produce the following outputs:

i. $\overline{x} \cdot \overline{(y + \overline{z})}$

ii. $(x + y + z) \cdot (\overline{x} \cdot \overline{y} \cdot \overline{z})$

[20%]

(f) Use Karnaugh-Map to simplify the following expression:

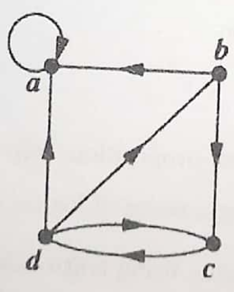
$x \cdot y \cdot z + x \cdot \overline{y} \cdot z + \overline{x} \cdot y \cdot z + x \cdot \overline{y} \cdot \overline{z} + \overline{x} \cdot \overline{y} \cdot \overline{z}$

111 101 011 100 000

[20%]

5. (a) Define a **Graph** and a **Tree** in Discrete Mathematics and describe their properties. [20%]

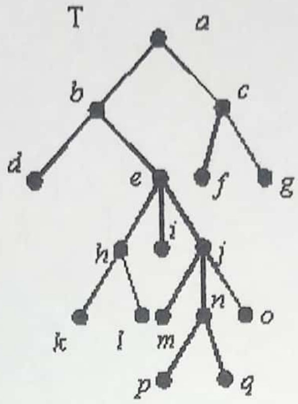
(b) Consider the following directed graph G:



Find the `in-degree` and `out-degree` of each of the vertices of G. Represent G as an adjacency matrix. Verify the `Handshaking Theorem` for the graph G. [25%]

[This question is continued on the next page]

(c) Consider the following Tree T:



i. Find the height of T.

ii. Is T balanced? Justify your answer.

iii. Determine the order of vertices of T using `pre-order`, `in-order` and `post-order` traversals.

[30%]

(d) Consider the finite-state machine M defined by the following state transition table shown below:

State	Input 0	Input 1
S_0	S_1	S_2
S_1	S_1	S_2
S_2	S_3	S_4
S_3	S_1	S_2
S_4	S_3	S_4

Draw the state diagram for the finite state machine M. [15%]

(e) Describe the characteristics of Turing machines. [10%]