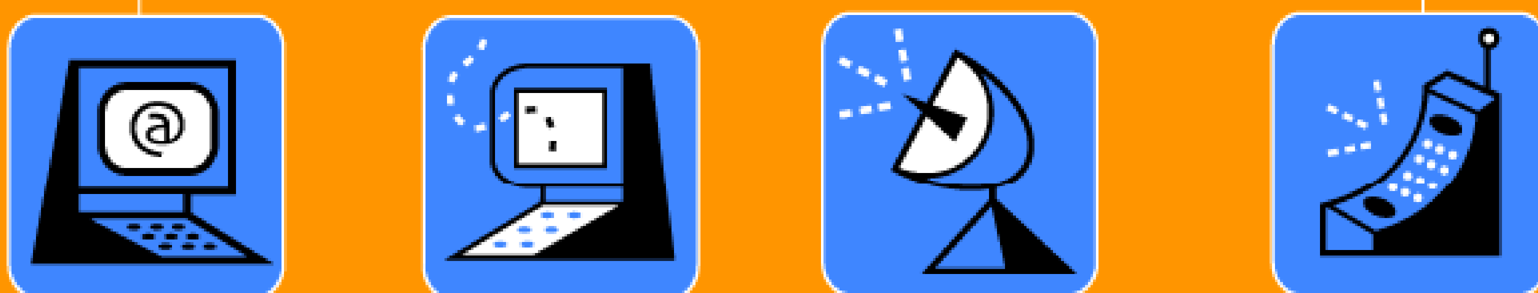
Verification of Chess TDMA for a Simple Fully Connected Wireless Sensor Network



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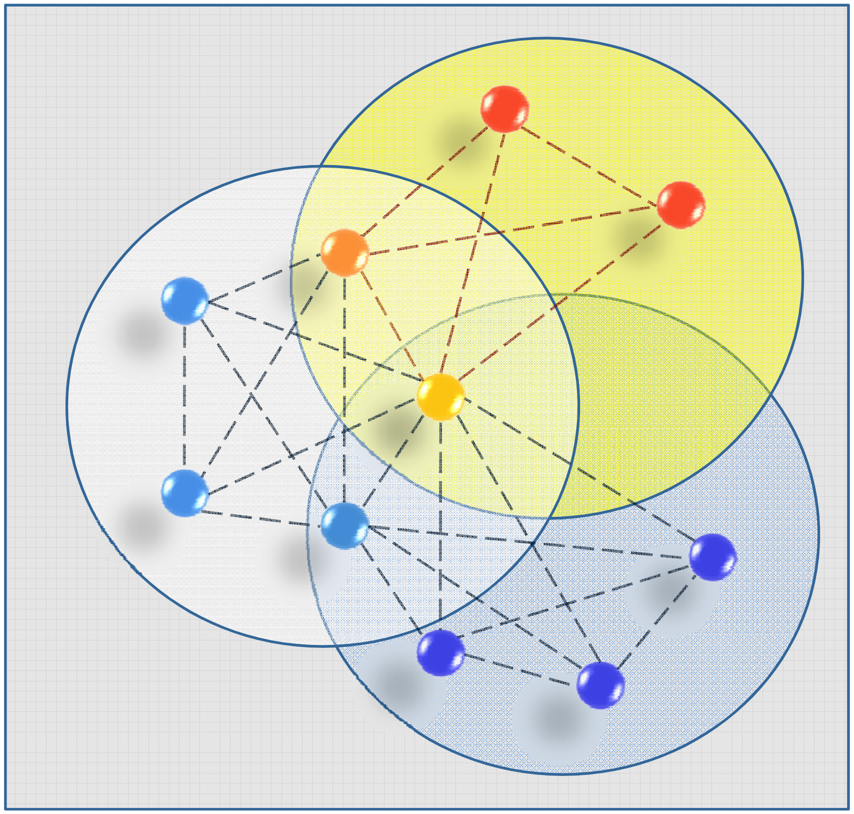
Introduction

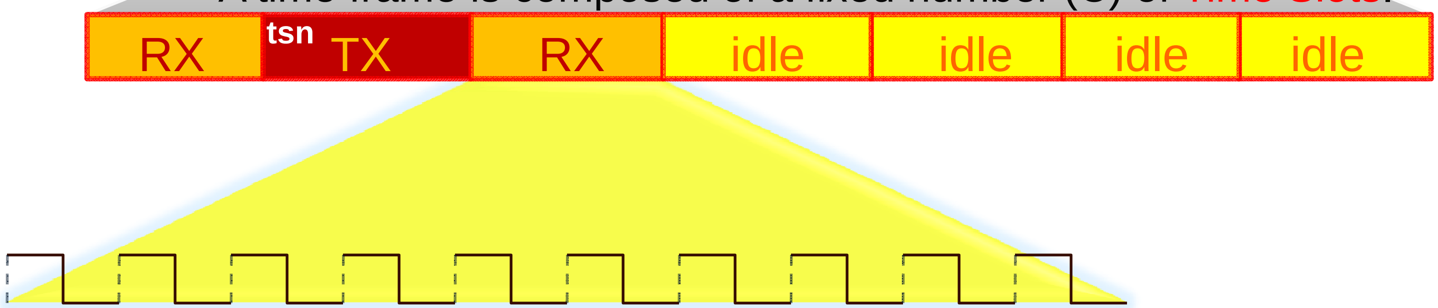
A **Wireless Sensor Network (WSN)** is a wireless network consisting of spatially distributed autonomous devices using sensors to cooperatively monitor physical or environmental conditions, such as temperature, sound, vibration, pressure, motion or pollutants, at different locations.



MAC Model

Time is considered as a sequence of Time Frames.

A time frame is composed of a fixed number (C) of Time Slots.



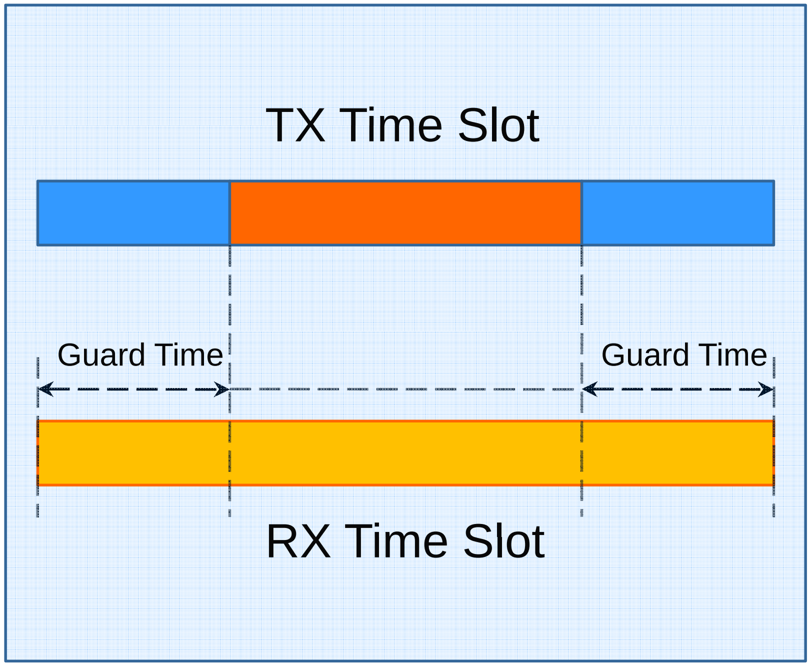
In a time slot the hardware clock of the sensor node ticks a fixed number (k_0) of times.

Synchronization

- When the nodes are powered on, they are all unsynchronized. In order to get synchronized, a node needs information about the relative time frame start, called T_0 moment.
- The clock synchronization is requested each time a node receives a message, keeping the internal oscillator to not drift from the given T_0 moment.
- Whenever a message is received, the transmitter T_0 moment is adopted by the receiver nodes.

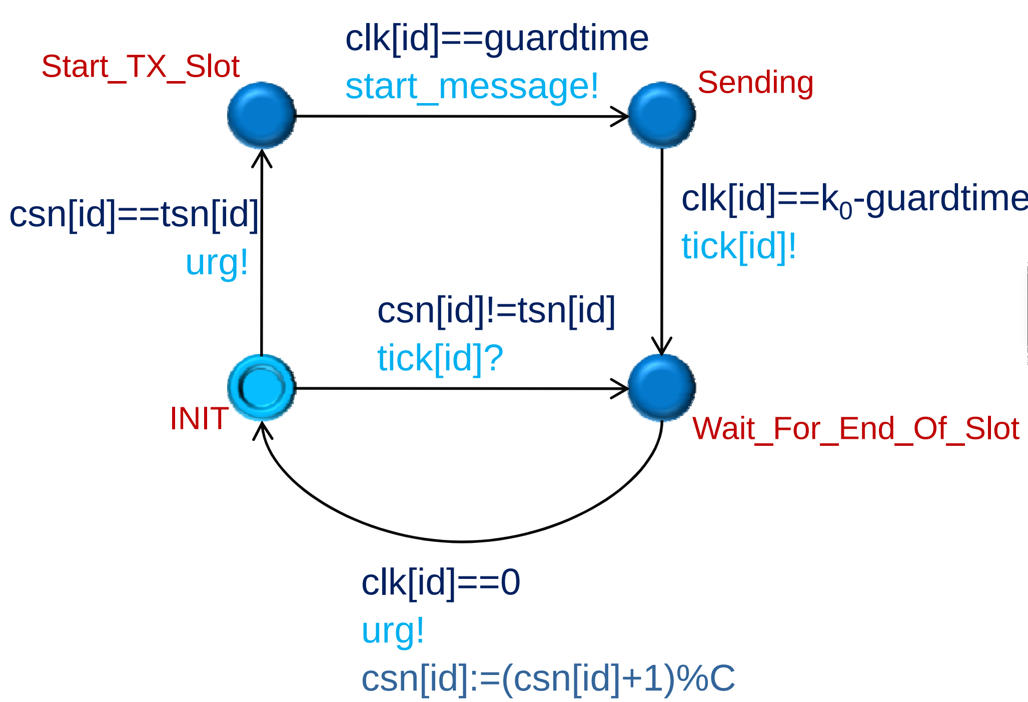
Guard Time

A Guard Time is allocated at the beginning of each TX slot to accommodate timing skew among the oscillators of the nodes. It is also required after the message to allow the transmitter to return to a quiescent state.

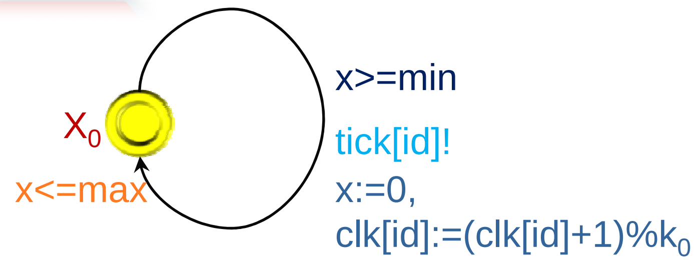


UPPAAL Model

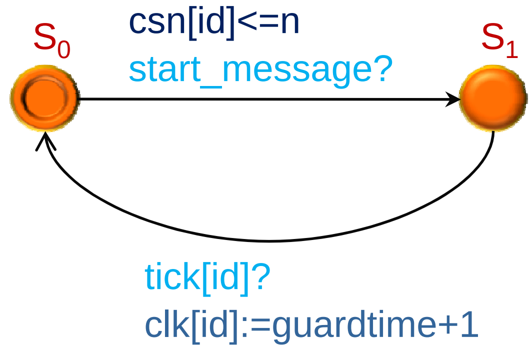
A Wireless Sensor Node



Clock

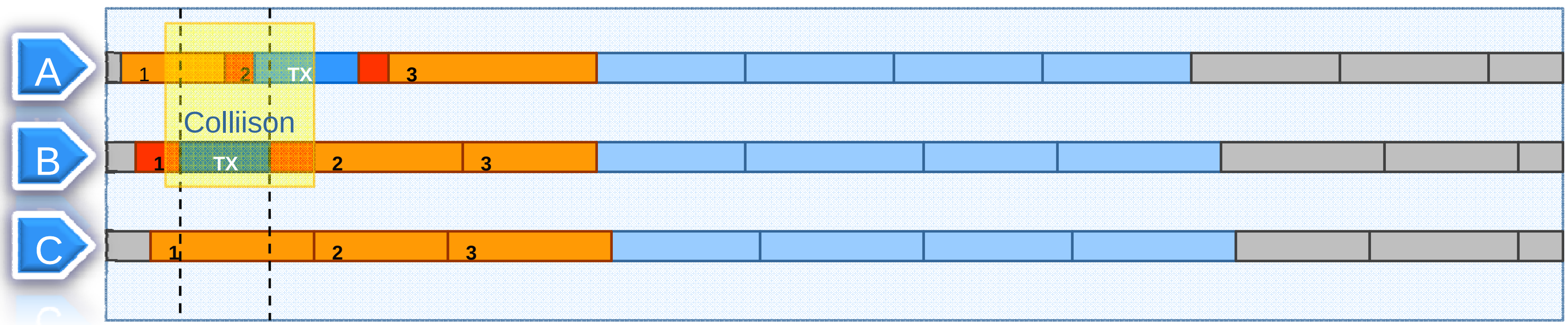


Synchronizer



A Collision- Free Network

A wireless sensor network is Collision-Free if no two nodes transmit, simultaneously and the receiver nodes are able to get the message of the transmitter in RX slots with the same slot number as the transmitter.



Definition. A fully-connected wireless sensor network is *collision-free* if and only if:

$$(\forall i, j \in Nodes)(WSN(i).Sending \rightarrow csn[i] = csn[j])$$

Numerical Results

Nodes	2			3			4		
C	6	8	10	6	8	10	6	8	10
n	4								
k_0	10								
g	2								
min	49	69	89	39	59	79	29	49	69
max	50	70	90	40	60	80	30	50	70

Nodes	2			3			4		
C	6								
n	4								
k_0	10								
g	2	3	4	2	3	4	2	3	4
min	49	24	16	39	19	13	29	14	9
max	50	25	17	40	20	14	30	15	10

Nodes	2				3				4			
C	6											
n	4											
k_0	5	10	15	20	5	10	15	20	5	10	15	20
g	2											
min	24	49	74	99	19	39	59	79	14	29	44	59
max	25	50	75	100	20	40	60	80	15	30	45	60

Theorem

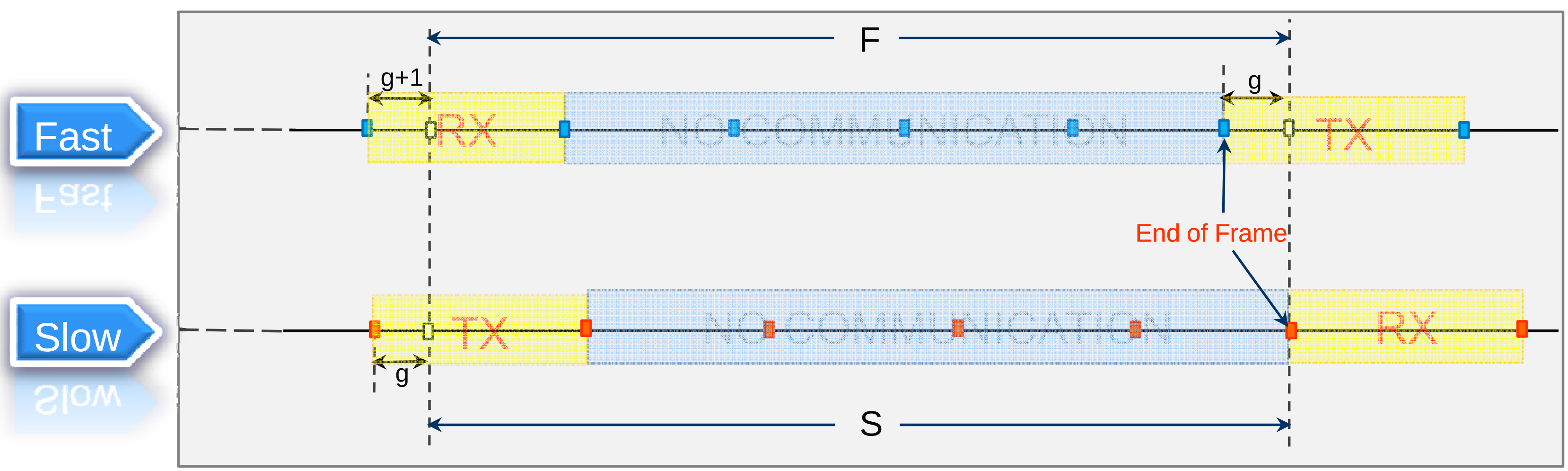
- A fully-connected wireless sensor network is collision-free if and only if:

$$\frac{(C-d)k_0 - g}{(C-d)k_0 - 1} < \frac{\min}{\max} \qquad (d = tsn_{\max} - tsn_{\min})$$

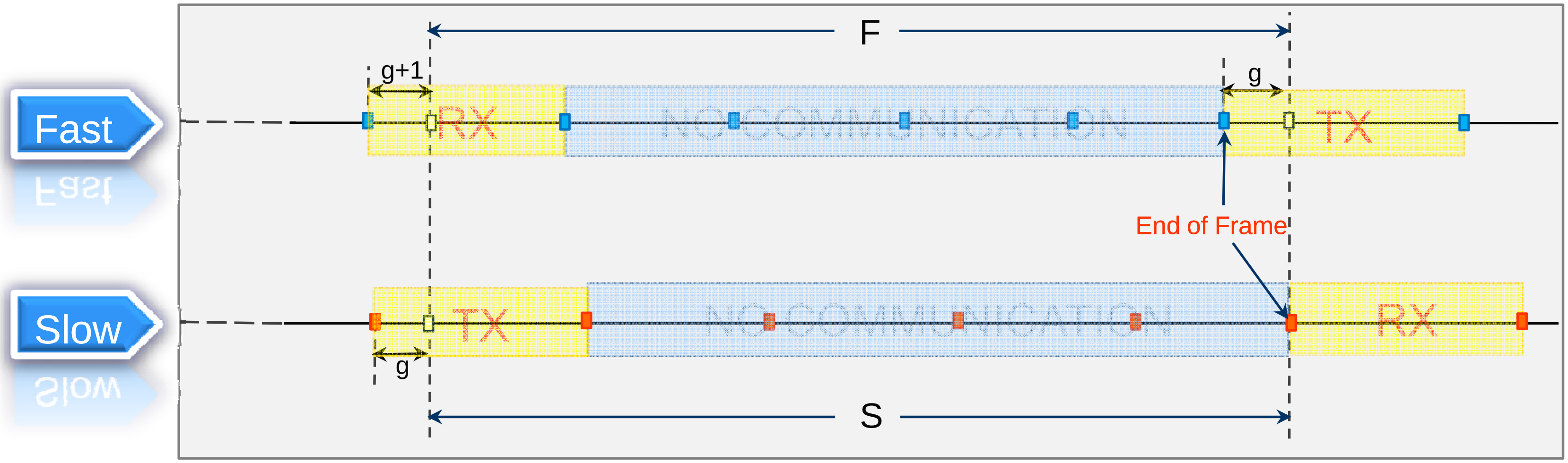
Necessity Proof by Counter Example

- Imagine: $\dfrac{(C-d).k_0 - g}{(C-d).k_0 - 1} \geq \dfrac{\min}{\max}$

- Suppose a wireless sensor network in which there exist one very slow sensor node (clock tick length $\approx$ max) and one very fast node (clock tick length $\approx$ min):



Necessity Proof



$$F = \min.\left((C-d).k_0 - 1\right)$$

$$S = \max.\left((C-d).k_0 - g\right)$$

$$\frac{(C-d).k_0 - g}{(C-d).k_0 - 1} \geq \frac{\min}{\max} \Rightarrow \max.\left((C-d).k_0 - g\right) \geq \min.\left((C-d).k_0 - 1\right) \Rightarrow \boxed{S \geq F}$$

Collision 😐

Sufficiency Proof using Invariants

☺ We started proving the necessity part of the theorem by describing the network of timed automata by primed formulas.

☺ We proved if $\frac{(C-d)k_0 - g}{(C-d)k_0 - 1} < \frac{\min}{\max}$ holds, then

$$\boxed{(\forall i, j)(WSN[i].Sending \rightarrow csn[i] = csn[j])}$$

is an Invariant.

System Invariants

1. $0 \le x[i] \le \max$

2. $0 \le clk[i] \le k_0$

3. $0 \le csn[i] \le C$

4. $WSN[i].Sending \rightarrow \left(csn[i] = tsn[i] \wedge csn[i] \le n \right)$

5. $WSN[i].Sending \rightarrow \left(g \le clk[i] \le k_0 - g \right)$

A Technical Lemma

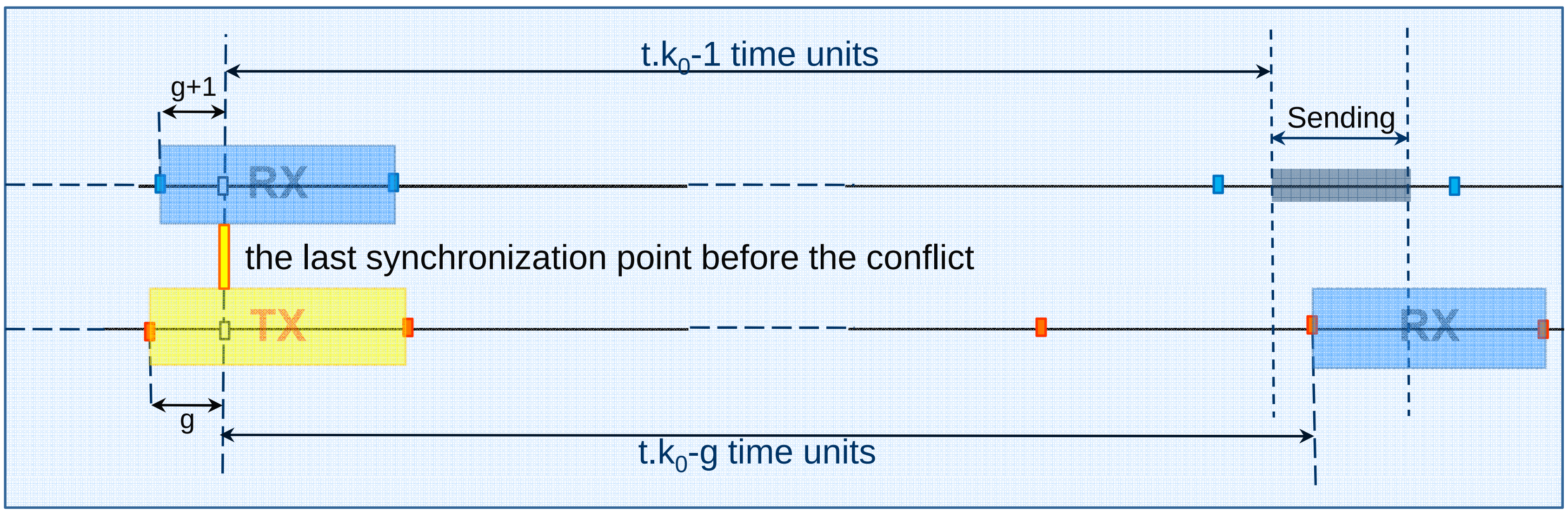
$\frac{(C-d).k_0-g}{(C-d).k_0-1} < \frac{\min}{\max}$ is equivalent to $\left(\forall t; 1 \le t \le (C-d)\right) \frac{t.k_0-g}{t.k_0-1} < \frac{\min}{\max}$

System Invariants (cont.)

☺ If $(\forall t; 1 \le t \le (C-d)), \frac{t.k_0 - g}{t.k_0 - 1} < \frac{\min}{\max}$

then $(\forall i, j; i \ne j)(WSN[i].Wait\_For\_End\_Of\_Slot \wedge clk[i] = 0 \rightarrow \neg WSN[j].Sending)$

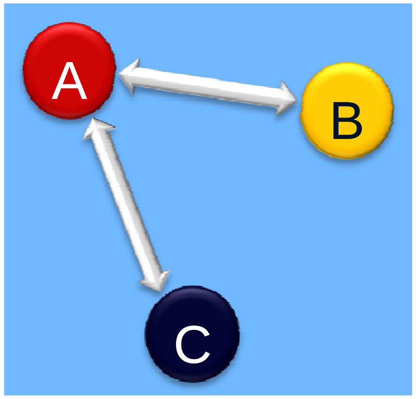
Is an invariant:

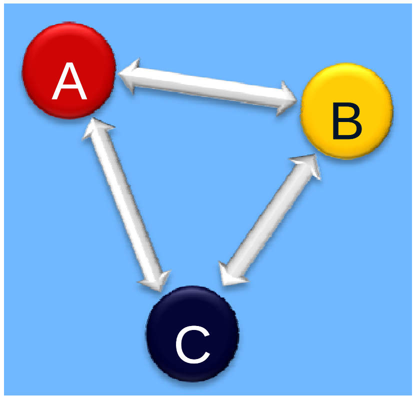


Proving Sufficiency

- We proved the sufficiency part of the theorem by induction on the length of the system execution.

A Fully-Connected Network
vs.
A non-Fully-Connected one





C	6	8	10	12
n	4	4	4	4
k_0	10	10	10	10
g	3	3	3	3
min	**58**	**78**	**98**	**118**
max	**59**	**79**	**99**	**119**
ratio	**0.9830**	**0.9873**	**0.9899**	**0.9916**

C	6	8	10	12
n	4	4	4	4
k_0	10	10	10	10
g	3	3	3	3
min	**19**	**29**	**39**	**49**
max	**20**	**30**	**40**	**50**
ratio	**0.95**	**0.9667**	**0.975**	**0.98**

Conclusion & Future Work

- We studied the fully-connected wireless sensor network and obtained a formula showing what relation between network parameters should be to have a collision-free network.

- Our formula doesn't describe the behavior of a non-fully-connected network.

- Our current challenge is discovering the conditions of collision-freeness in a general wireless sensor network.

Thank You

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