

# Decentralized Frame Synchronization of a TDMA-based Wireless Sensor Network

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**Abstract**—Synchronization is a crucial component of infrastructure for *Wireless Sensor Networks* (WSNs). Most applications of WSNs make extensive use of time synchronization mechanisms like *Time Division Multiple Access* (TDMA) scheduling, accurate timestamping of events, coordinate activities of the network or data fusion. The unique requirements of WSNs, compared to centralized networks, in terms of precision, lifetime, energy and scope of the synchronization achieved, make the synchronization methods developed for centralized systems unsuitable for WSNs. This motivates the research of synchronization methods which are aligned to the specific properties of WSN. Median algorithm is implemented currently on the wireless sensor nodes in MyriaNed<sup>1</sup> project. In this research, the flaw in the Median algorithm is explored and three algorithms have been proposed to achieve a decentralized, stable, and energy-efficient synchronization of a WSN. The algorithms achieve synchronization by using the phase error of a node's wake-up time with that of the neighboring node's, without actually exchanging the information about the clock time of the sender. So, the method avoids time keeping on the messages (time stamping) which reduces the message overload. The algorithms can be integrated with the slot allocation algorithm to form the *Media Access Protocol* (MAC) layer protocol for a better throughput. The research is concluded with the comparison of three algorithms in terms of energy consumption and performance with the Median. A low energy-consumption or a better convergence as well as the length of the guard time can be used as a metric for selecting the algorithm.

**Index Terms**—ad-hoc networks, frame synchronization, decentralized synchronization, wireless sensor networks.

## I. INTRODUCTION

### A. Synchronization in Wireless Sensor Networks

WSNs are distributed networks of low cost sensors, dedicated to closely observing real-world phenomena. Various applications are realized using WSNs[8]. One of the design issues in WSN technology is synchronization, which is a critical piece of infrastructure in any distributed system. There are many reasons why a synchronized time is needed in a WSN, one of them being adjusting the slots in a TDMA-based communication.

In order to have a seamless communication between the nodes, the synchronization of the frames is a necessary part of the MAC protocol. Due to physical factors, the frequency

of the oscillator in the node has a drift. When no provisions are taken, it causes the nodes to run out of synchronization. Given that there is a certain error, the node will adjust its wake-up time with the offset calculated with a synchronization algorithm.

### B. Existing Work on WSN frame synchronization

Several algorithms have been proposed and researched for frame synchronization in WSNs. The *Reference Broadcast Synchronization* (RBS) stated in [1] is an important scheme in the area of WSN synchronization. It achieves a Receiver-Receiver pairwise synchronization to remove sender nondeterminism and results in a good precision of a few microseconds. A decentralized slot synchronization algorithm for TDMA-based networks is shown in [2]. This method uses the topology of the nodes as a means to weigh the phase error of the sender with the receiver.

Another approach stated in [11] establishes a table to correspond the clock of the sender with that of the receiver clock so that a good estimation of the neighbors clock is achieved using different estimation techniques. A different approach to weight-based synchronization for interference elimination for a TDMA based ad-hoc networks is presented in [7]. The algorithm achieves synchronization in a decentralized manner using the node's phase error with its neighbors' to eliminate the impact of interference. Correlation method is used in [12] in order to decode the message and learn about the status of the sender node which is used for synchronization.

### C. Objective and overview of the research

The primary objective of this research is to develop an algorithm to achieve a long-term decentralized frame synchronization of a WSN in an energy-efficient method which employs no timestamping on the messages. The method uses the phase errors between the receiver and its neighbors, without actually estimating the neighbor's clock as a way of achieving long-term frame synchronization. In this research, three algorithms are presented and compared with the Median algorithm in terms of performance and energy consumption.

The remainder of the paper is organized as follows: Section 2 discusses the synchronization error and presents the set back of the Median algorithm. Section 3 presents the mathematical

<sup>1</sup>MyriaNed is a project to create a large functional network of 10,000 nodes with wireless communication for research on protocols, power management, programming models, and security.

models of the proposed algorithms for the synchronization of the network. In Section 4, simulation results are presented and analysis of the results in addition to the comparison of the methods with respect to energy consumption is discussed. Finally, Section 5 draws the conclusions from the research and suggests future work.

## II. PROBLEM FORMULATION

### A. Wireless Sensor Networks: Why different ?

Are synchronization methods developed for centralized systems applicable in the case of WSNs? Many assumptions in the centralized schemes do not hold in the case of WSNs. Some of these factors can be described as follows:

- **Energy Limitation:** Due to the nodes' small size and nature of applications which they are designed for, energy consumption is a major concern in a WSN.
- **Dynamic Nature of the Network:** In a WSN, network dynamics results from various factors which prevents simple static configurations.
- **Diverse Applications:** A variety of applications have totally different needs as far as synchronization is concerned. Some applications might need a global timescale while others can work with a relative timescale.
- **Cost of the Nodes:** Sensor nodes are very small in size and must be cheap cost wise.

### B. Sources of synchronization error

The different factors which give rise to errors between two nodes can be identified as:

**Oscillator Characteristics:** The following two oscillator characteristics are the main sources of errors between the clocks of two different nodes.

- **Accuracy:** is a measure of the difference between oscillators expected (ideal) frequency and actual frequency.
- **Stability:** is oscillator's tendency to stay at the same frequency over time.

**Hardware and Environmental factors:** The non-determinism in the message delivery latency can be categorized in four type of delays:

- **Send Time:** The time spent at the Sender to build the message.
- **Access Time:** Delay occurred while waiting for access to the transmit channel.
- **Propagation Time:** Time required for the message to travel from sender to receiver.
- **Receive Time:** Time needed for processing at the receivers.

All the above factors result in the following errors between two nodes:

- **Phase error:** The oscillators of any two nodes can be out of phase at any given time, resulting into different time on both clocks.
- **Frequency error:** Frequency error, in contrast, measures the difference in the clock rates.
- **Clock drift:** It is not just that the clocks are running at different rates, but the frequency of each clock does not stay constant over a period of time.

### C. Clock Time

From the definition of frequency:

$$f = d\phi/dt, \quad (1)$$

and integrating both sides over time,

$$\phi = \int f(t)dt, \quad (2)$$

where  $f$  is frequency,  $\phi$  is phase, and  $t$  is time.

Thus, the clock time is described as

$$C(t) = \frac{1}{f_o} \int_{t_o}^t f(\tau)d\tau + C(t_o), \quad (3)$$

where  $f(\tau)$  is the frequency of the clock,  $f_o$  is the nominal frequency of the crystal oscillator and  $t_o$  is the start time of the node.

The exact clock drift is hard to predict because it depends on environmental influences but can usually assumed that it clock doesn't exceed a maximum value  $\rho$ . This means that it can be assumed

$$1 - \rho \leq \frac{dC(t)}{dt} \leq 1 + \rho, \quad (4)$$

where  $\rho$  represents the maximum clock drift. Note that different clocks have different maximum clock drift values  $\rho$ .

The frequency of the clock is dependent on different factors and can be given as

$$f_i(t) = f_o + \Delta f + a(t - t_o) + \Delta f_e(t - t_o) + f_r(t) \quad (5)$$

where

$t_o$  = the start time of the clock,

$a$  = aging factor,

$f_o$  = nominal frequency,

$\Delta f$  = calibration error,

$f_r(t)$  = frequency instability (noise) term,

$\Delta f_e$  = frequency error which occurs due to outside factors like temperature, voltage instability.

From (3) and (5),

$$C_i(t) - C_i(t_o) = \frac{1}{f_o} \int_{t_o}^t f_i(\tau)d\tau, \quad (6)$$

$$C_i(t) - C_i(t_o) = \frac{1}{f_o} \int_{t_o}^t [f_o + \Delta f + a(\tau - t_o) + \Delta f_e(\tau) + f_r(\tau)]d\tau,$$

$$C_i(t) = C_i(t_o) + (t - t_o) + \frac{\Delta f}{f_o}(t - t_o) + \frac{a}{2f_o}(t - t_o)^2 + \frac{1}{f_o} \int_{t_o}^t f_e(\tau)d\tau + \frac{1}{f_o} \int_{t_o}^t f_r(\tau)d\tau.$$

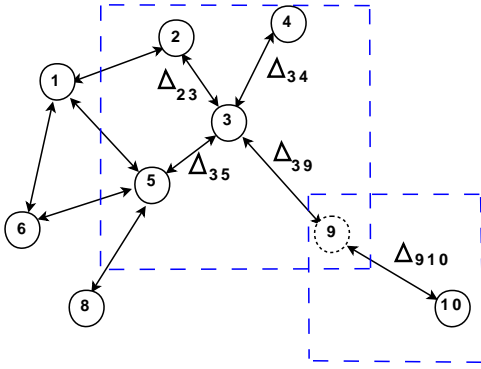


Fig. 1. A WSN scenario

#### D. Median algorithm and the setback

The Median algorithm is currently implemented in the MyriaNode. With the simplicity concerning the computational need, it is the best fit for WSN requirements discussed in the previous section. The algorithm is described as follows:

- 1) Nodes broadcast packets.
- 2) Each receiver records the time that the packet is received according to the local clock.
- 3) Each receiver  $i$  computes its phase error to any other node  $j$  in the neighborhood as

$$\Delta t_{ij}^{(n)} = t_i^{(n)} - t_j^{(n)}, \quad (7)$$

where  $t_i$  is the wake-up time of node  $i$  and  $t_j$  is the wake-up time of node  $j$  at the  $n^{th}$  period.

- 4) Receivers compute the median of the offsets

$$\xi_i^{(n)} = \text{median}(\Delta t_{ij}^{(n)}), \forall j \quad (8)$$

- 5) Receivers adjust their wake-up time by the computed offset value,

$$t_i^{(n+1)} = t_i^{(n)} + T_i^{(n)} - k\xi_i^{(n)}, \quad (9)$$

where  $k$  is the gain factor.

By multiplying the median with a gain factor, i.e.  $k\xi_i^{(n)}$ , the output can be adjusted for better performance.

As per the application of Median, there are setbacks on its implementation. In some test-cases, the algorithm fails to converge or stays unsynchronized for a certain period of time.

A typical scenario is presented where the Median algorithm takes a longer time to achieve synchronization. In Figure 1, a distribution of nodes is shown. Node 10 joins the stable network communication through Node 9. Assume Node 9 belongs to two broadcast domains, Node 3's and Node 10's. Thus, upon the application of the median algorithm, the node tends to adjust its wake-up time with the median of the phase errors from its neighbors, Node 3 and Node 10. Adjusting the wake-up time, the offset is

$$\xi_9 = \frac{\Delta t_{910} + \Delta t_{39}}{2}. \quad (10)$$

With Node 10 being isolated from the well established network, it has a major drift from the other nodes. Being in sync with the other nodes, Node 9 will drift away from the network after its adjustment with the Node 10. It will take

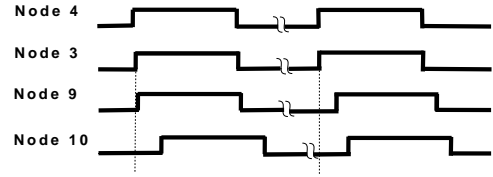


Fig. 2. Using median for phase error correction

more time to synchronize with the network again. Figure 2 shows the state of the network after the synchronization using the Median.

So, in order to address this problem, three algorithms are explored in the next section to realize an energy-efficient, more precise and simple frame synchronization.

### III. SYNCHRONICITY PROTOCOL

#### A. Synchronization Frequency

**Precise synchronization:** Precise synchronization is implemented when a node has already joined in the network, which repeatedly adjusts the diversion of its time slot reference caused by propagation delay and clock drift.

Decreasing the timing of the synchronization and do periodic execution of the synchronization of the algorithm greatly reduces the energy consumption of the WSN. Thus, a synchronization period,  $T_{sync}$ , is defined as the period in which the network can stay synchronized without the application of the synchronization algorithm.

A time slot can be defined as being expressed in a number of clock cycles as

$$t_{slot} = kT, \quad (11)$$

where  $T$  is the period of the time frame in clock cycles and  $t_{slot}$  is the time duration of a TDMA slot and  $k$  is coefficient.

With a synchronization period  $T_{sync}$  and the maximum clock drift of a clock  $\rho$ , the maximum time difference between a sender and a receiver is

$$t_{diff} = 2\frac{T_{sync}}{T}\rho, \quad (12)$$

where the factor of 2 reflects the worst case scenario where each node's clock drifts in the opposite direction.

In achieving frame synchronization in a TDMA scheduling, a guard time is given for a fault tolerance which is used to accommodate the phase errors, as shown in Figure 3. Since

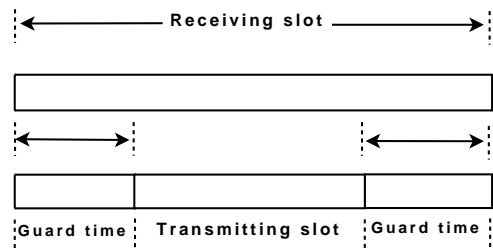


Fig. 3. Guard time of the nodes

the relative time difference between two nodes can be in two direction, the guard time needs to be twice  $t_{diff}$ ,

$$t_{guard} = 2t_{diff} = 4\frac{T_{sync}}{T}\rho. \quad (13)$$

At this end, the minimum duration for a time slot  $t_{slot}$  is

$$t_{slot} \geq t_{guard} + T_{tx}, \quad (14)$$

where  $T_{tx}$  is the time required to send a packet from one node to other.

Two nodes have a good communication link when they synchronize their clocks at least once every  $T$  that gives the following relation,

$$\frac{T_{sync}}{T} \geq 1. \quad (15)$$

To obtain the value of  $T_{sync}$  for a particular system with a given duty cycle, the following equations are to be seen:

$$t_{guard} \geq 4\rho\frac{T_{sync}}{T} \quad (16)$$

$$t_{slot} \geq t_{guard} + T_{tx} \quad (17)$$

$$\frac{T_{sync}}{T} \geq 1. \quad (18)$$

In general, there is a trade-off in determining  $T_{sync}$ : increasing  $T_{sync}$  reduces the energy costs of synchronization, whereas decreasing the network performance.

### B. Mathematical Model

As part of the message, nodes are transmitting the slot number which they are transmitting. This information is used in the determination of the time that the message is sent. Each message that is sent within a time slot is exactly received by a neighbor at a known clock tick number

$$tick_{rx} = T_{tx} + \frac{t_{guard}}{2} \quad (19)$$

Whenever this number is not equal with the desired one, a phase error is observed.

The difference between the wake up times of node  $i$  and node  $j$  is

$$\Delta t_{ij}^{(n)} = t_i^{(n)} - t_j^{(n)}, \quad (20)$$

where  $t_i$  and  $t_j$  are the wake-up times of node  $i$  and node  $j$  at the  $n^{th}$  period.

The wake-up time of a node at a random time after  $n$  periods of firings after it is turned on is

$$t_i^{(n)} = \sum_n T_i^{(n)} + t_{io}, \quad (21)$$

where  $T_i^{(n)}$  is the period of the crystal clock at the  $n^{th}$  period which changes with time according to (5) and  $t_{io}$  is the initial start-up time of the node. The frequency of the node varies due to the different factors mentioned in (5).

The difference in the wake-up time of the nodes is given by

$$\Delta t_{ij} = \sum_n T_i^{(n)} + t_{io} - \left( \sum_n T_j^{(n)} + t_{jo} \right) \quad (22)$$

$$= \left( \sum_n T_i^{(n)} - \sum_n T_j^{(n)} \right) + (t_{io} - t_{jo}). \quad (23)$$

This phase error can be adjusted using two approaches, which are not exclusive. The first one is adjusting the clock frequency to come up with a better wake-up time. The second one is to adjust the next wake-up time depending on the difference between the current wake-up time of the node and its neighbors. In this research, the second option is explored due to:

- The cost of adjusting the frequency of the clock, and
- The complexity of the implementation.

Hence, the next wake-up time of the node is dependent on the current wake-up time of its neighbors in relation to its previous wake-up time,

$$t_i^{(n+1)} = t_i^{(n)} + T_i^{(n)} - \xi_i^{(n)}, \quad (24)$$

where  $T_i$  is the period of the node's clock and  $\xi_i$  is given by

$$\xi_i = f(\Delta t_{ij}). \quad (25)$$

The function  $f$  is based on an algorithm which takes the wake-up time differences between the node and its neighbors and determines the optimal offset to be added to the next wake-up time of the node.

Different algorithms are presented here and discussed with the simulation results presented in the next section of the report.

**1) Weighted Measurements - Interference-phobic approach:** One form of approach to tackle the dynamic behavior of a WSN, due to channel conditions as well as collisions, is a Weighted Measurement (WM) approach. Using this approach, different weights are given to the different measurements. A weight is added to increase the influence of the close by neighbors and ensure faster synchronization. In addition to that, a new joining neighbor can get synchronized with out disturbing the existing neighbors, adjusting its time to the big swarm of nodes.

The offset is then

$$\xi_i^{(n)} = \sum w_{ij}^{(n)} \Delta t_{ij}^{(n)}, \quad (26)$$

where  $\sum w_{ij}^{(n)} = 1$ .

The weighted adjustment is used to modify the next wake-up time of the node,

$$\begin{aligned} t_i^{(n+1)} &= t_i^{(n)} + T_i^{(n)} - \xi_i^{(n)} \\ &= t_i^{(n)} + T_i^{(n)} - \sum_{j=0}^N w_{ij}^{(n)} \Delta t_{ij}^{(n)} \\ &= t_i^{(n)} + T_i^{(n)} - \sum_{j=0}^N w_{ij}^{(n)} (t_i^{(n)} - t_j^{(n)}) \\ &= T_i^{(n)} + \sum_{j=0}^N w_{ij}^{(n)} t_j^{(n)}. \end{aligned}$$

The main task in this algorithm is how to choose the weight factors so that the stability of the network (timewise) is achieved in a faster time. The weight is selected in such a way that the a node joining a network should adjust its time with the network that it is joining. After each measurement, a phase error is associated with the tabled values to recognize

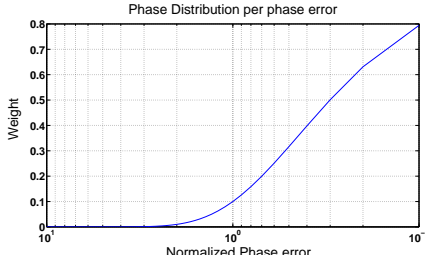


Fig. 4. Weight factors for the phase error distribution

how far is the sending node concerning the time that it drifted away from the receiving node. The values are calculated as:

$$\delta_{ij} = ae^{-b\Delta t_{ij}}. \quad (27)$$

The parameters  $a$  and  $b$  are selected using the initial conditions,  $\delta_{ij} = 0.1$  for  $\Delta t_{ij} = t_{guard}$  and  $\delta_{ij} = 1$  for  $\Delta t_{ij} = 0$ .

As shown in Figure 4, the closer the phase error is to zero, the higher value it is given. The assignment of high values to the small phase errors will decrease the time it takes to synchronize this node with the network.

In the assignment of the weight factor, there is another issue to be considered. If all the phase errors are in the lower region, this corresponds to the fact that the node is a newly joining node to a well-established and synchronized network. In order to incorporate the effect of the distribution of the phase errors, another step is introduced in the calculation of the weight factors so that the node will adjust its wake-up time towards the more stabilized network.

The weight factor is

$$w_{ij} = \begin{cases} 1 - \delta_{ij}, & \text{if } \text{mean}(\delta_{ij}) < 0.5 \\ \delta_{ij}, & \text{if } \text{mean}(\delta_{ij}) > 0.5 \end{cases}$$

where  $m$  is the mean function. Hence, the weight moves towards the large phase errors if the node is a new-comer that wants to join the "already established" network.

As explained earlier, WM is a two step process in which the first one describes how the node has drifted away from its neighbors whereas the second one describes how the node is positioned in the network topology which surrounds it.

Using the WM approach, a series of measurements will be used to estimate the next wake-up time of the node, giving less value/emphasis to the nodes which are out of reach from the neighboring nodes.

## 2) Non Linear Least Squares: Non-Linear Least Squares

Non Linear Least Squares (NLLS) is a method to fit a set of  $n$  observations with a model that is non-linear in  $m$  unknown parameters by minimizing the sum of the squares of the errors ("the residuals").

Using a set of  $n$  data points which are  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , and a curve (model function)  $y = f(x, \beta)$ , that in addition to the variable  $x$  also depends on  $m$  parameters  $(\beta_1, \beta_2, \dots, \beta_m)$ . It is desired to find the vector  $\beta$  of parameters such that the curve best fits the given data in the least squares, that is, the sum of squares

$$S = \sum_{i=1}^n r_i^2, \quad (28)$$

is minimized, where the residuals  $r_i$  are given by

$$r_i = y_i - f(x_i, \beta), \quad (29)$$

for  $i=1, 2, \dots, n$ .

The minimum value of  $S$  occurs when the gradient is zero. Hence, there are  $m$  gradient equations to be solved:

$$\frac{\partial S}{\partial \beta_j} = 2 \sum_i r_i \frac{\partial r_i}{\partial \beta_j} = 0 \quad (j = 1, 2, \dots, m). \quad (30)$$

In a non-linear system, the derivatives  $\frac{\partial r_i}{\partial \beta_j}$  are functions of both the independent variable and the parameters. These gradient equations do not have a closed solution. Instead, initial values must be chosen for the parameters. Then, the parameters are refined iteratively, that is, the values are obtained by successive approximation,

$$\beta_j^{k+1} = \beta_j^k + \Delta \beta_j. \quad (31)$$

for  $j = 1, 2, \dots, m$ . Here,  $k$  is an iteration number and the vector of increments,  $\Delta \beta_j$ , is known as the shift vector. At each iteration, the model is linearized by approximation to a first-order Taylor series expansion about  $\beta^k$ .

$$f(x_i, \beta) \approx f(x_i, \beta^k) + \sum_j \frac{\partial f(x_i, \beta^k)}{\partial \beta_j} (\beta_j^k - \beta_j) \quad (32)$$

$$f(x_i, \beta) = f(x_i, \beta^k) + \sum_j J_{ij} \Delta \beta_j. \quad (33)$$

The Jacobian,  $J$ , is a function of constants, the independent variable and the parameters, so it changes from one iteration to the next. Thus, in terms of the linearized model,

$$\frac{\partial r_i}{\partial \beta_j} = -J_{ij} \quad (34)$$

and the residuals are given by

$$r_i = \Delta y_i - \sum_{j=1}^m J_{ij} \Delta \beta_j, \quad (35)$$

where

$$\Delta y_i = y_i - f(x_i, \beta^k). \quad (36)$$

Substituting these expressions into the gradient equations and equating to 0, a matrix notation can be found

$$(J^T J) \Delta \beta = J^T \Delta y \quad (37)$$

## /Model Design

As the algorithm is decentralized, the next wake-up time of the node depends on the current offset that it has with the other nodes. The distribution of the phase error is the crucial factor in deciding the type of curve to fit in. The phase errors shows how many time units that the neighbor node is out of touch with the node in focus. The larger the phase error is, the more out-of-sync the node is.

The set of data are the measured phase errors of the node with its neighbors. In order to meet the demand of adjusting the time offset and stabilize the network, the logarithmic curve with the model

$$f(x_i, \beta) = \beta_1 + \beta_2 \log(x_i), \quad (38)$$

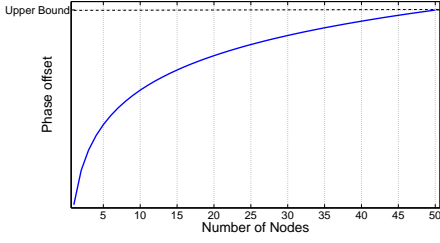


Fig. 5. Curve fitting using logarithmic function

is chosen.

A logarithmic function can be used to represent the distribution of the offsets in the neighborhood, as shown in Figure 5. Normalized phase error values tend to be small and follow a curve in such a way that the maximum value of the phase error should not be greater than the threshold value that was intended to be, the synchronization error.

The initial values of the parameters  $\beta_1$  and  $\beta_2$  is estimated taking into account the state of the network to be.

As each measurement arrives from the neighbors, the parameters are calculated in such a way that the measurement error from a pre-determined offset value is reduced. This ensures that the offset to be added in the next wake-up time doesn't diverge in a large amount from the predicted value.

#### Model Design

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The set of data are the measured phase errors of the node and the parameters  $\beta_1$  and  $\beta_2$  are estimated using a least squares approximation for the function

$$f(x_i, \beta) = \beta_1 + \beta_2 \log(x_i), \quad (39)$$

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#### 3) Discrete time Kalman Filter for synchronization:

##### Discrete time Kalman Filter

The Kalman Filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step.

The Kalman filter addresses the general problem of trying to estimate the state  $x$  of a discrete-time controlled process

that is governed by the linear stochastic difference equation

$$x_k = Hx_{k-1} + Bu_k + w_{k-1}, \quad (40)$$

with a measurement  $z$  that is

$$z_k = Hx_k + v_k. \quad (41)$$

The random variables  $w_k$  and  $v_k$  represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions

$$p(w) \approx N(0, Q), \quad (42)$$

$$p(v) \approx N(0, R). \quad (43)$$

With the initial estimates of  $x_{k-1}$  and  $P_{k-1}$

$$x_k = Hx_{k-1} + Bu_k \quad (44)$$

$$P_k = HP_{k-1}H^T + Q. \quad (45)$$

The measurement update equations are responsible for the feedbacks i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

$$K_k = P_k H^T (HP_k H^T + R)^{-1} \quad (46)$$

$$x_k = x_k + K_k(z_k - Hx_k) \quad (47)$$

$$P_k = (I - K_k H)P_k \quad (48)$$

The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations.

Hence, a priori and a posteriori estimate errors are defined as

$$e_k^- = x_k - \tilde{x}_k^-, \quad (49)$$

$$e_k = x_k - \tilde{x}_k. \quad (50)$$

The a priori estimate error covariance is then

$$P_k^- = E[e_k^- e_k^{-T}] \quad (51)$$

and the a posteriori estimate error covariance is

$$P_k = E[e_k e_k^T] \quad (52)$$

The update equation is

$$\tilde{x}_k = \tilde{x}_k + K(z_k - H\tilde{x}_k). \quad (53)$$

The difference  $z_k - H\tilde{x}_k$  in (53) is called the residual. The residual reflects the discrepancy between the predicted measurement  $H\tilde{x}_k$  and the actual measurement  $z_k$ . A residual of zero means that the two are in complete agreement.

The matrix  $K$  in (46) is chosen to be the gain or blending factor that minimizes the posteriori error covariance. Taking the derivative of the trace of the result with respect to  $K$ , setting that result equal to zero, and then solving for  $K$ ,

$$K_k = P_k H^T (HP_k H^T + R)^{-1} \quad (54)$$

$$K_k = \frac{P_k H^T}{HP_k H^T + R}. \quad (55)$$



Looking at (55), it is seen that as the measurement error covariance  $R$  approaches zero, the gain  $K$  weights the residual more heavily. Specifically,

$$\lim_{R_k \rightarrow 0} K_k = H^{-1}. \quad (56)$$

On the other hand, as the a priori estimate error covariance  $P_k$  approaches zero, the gain  $K$  weights the residual less heavily. Specifically,

$$\lim_{P_k \rightarrow 0} K_k = 0. \quad (57)$$

### Filter design

The transition matrix  $H$  plays an important role in achieving the proper synchronization. It indicates how fast/slow is the next wake up time should be compared to it's previous value and/or neighbors. With the stable network where the nodes remain intact, the wake-up time of the node is expected to be the same despite the clock drift.

The initial estimates are selected from the previous firing time of the node. Hence, the initial values of  $x$  and  $P$  are estimated from the previous values, one period earlier. This ensures that the nodes are on the same track as the previous time, since the neighbors remain the same with some exception of mobility and interference. Hence, to tackle the dynamic nature of the WSN, the new node joining the network should be synchronized with the already established "status quo" of the network. In the filter design, the covariance matrices  $R$  and  $Q$  have a significant role in the overall implementation of the algorithm.  $R$  represents how the measurements are valued to affect the result of the outcome and  $Q$  sends a signal as to how the estimated value is weighed.

## IV. RESULTS AND DISCUSSION

### A. Simulation setup

In the simulation, a Discrete Event Simulator (DES) is used. A DES will break down a simulation into discrete chunks. Every event will occur at some countable time moment and will be given in chronological order. The advantage of this distinction is two-fold. First, simulations will not be dependant on some real-time clock. Second, events can be isolated to perform certain measurements.

One such DES is MiXiM<sup>2</sup>. It provides a component architecture for models. Components are programmed in C++, then assembled into larger components and models using a high-level language. Octave<sup>3</sup> is used for interpretation of the data from the network simulator.

The simulation is conducted 1000 times to counter the effect of randomness introduced in the simulation.

### B. Simulation Results

The nodes are deployed uniformly across the field. The neighborhood is limited to 10 nodes. The start up time of the nodes is random, Gaussian distributed variable,  $t_{io}$ . The

<sup>2</sup>MiXiM (Mixed Simulator) is a simulation framework for wireless and mobile networks using the OMNeT++ [16] simulation engine. It is a collaborative project between TU Berlin, TU Delft and Universitaet Paderborn.

<sup>3</sup>Octave is a free program for performing numerical computations which is mostly compatible with MATLAB. It is part of the GNU project.

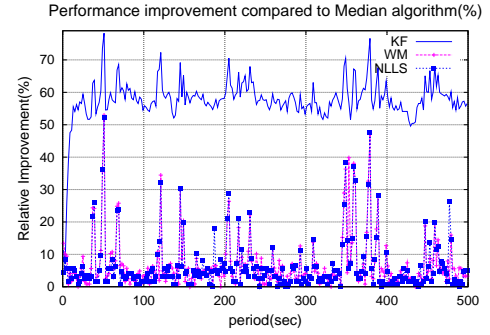


Fig. 8. Relative error of the algorithms w.r.t. Median

synchronization error is the difference between the wake-up time of the nodes in the neighborhood.

1) **Case I:** In the first set of simulation, the synchronization error is simulated for the nodes which are static, hence no effect of mobility. The number of nodes is taken to be 20 and 50 in the set of simulations.

Figure 6a shows the synchronization error for 20 nodes operating in a static environment. KF has the lowest synchronization error, at an average of  $4clk$ . NLLS and WM perform similarly with Median at average of  $12clk$ .

Figure 6b is the result of a simulation for 50 nodes. The synchronization error in general increases as the number of nodes increases the number of neighbors, resulting in more data for the synchronization layer. Comparing the individual algorithms, KF has over  $7clk$  performance improvement than the Median whereas NLLS and WM have a better performance ( $1clk$ ) than the Median.

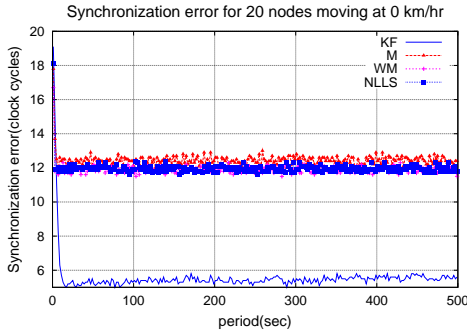
2) **Case II:** In second set of simulations, the mobility of the nodes is taken into account. Here, the number of the nodes is taken to be 20. The simulations are conducted for different speeds, at  $6km/hr$  and  $20km/hr$ . Chosen speeds emulate the speed of a walking man and an average speed of a slowly moving vehicle.

Figure 7a shows 20 nodes with a constant speed of  $6km/hr$  random mobility. As the results shows, WM and NLLS perform better the synchronization of the frame,  $1clk$  each. KF outperforms all the best, with  $8clk$  from the Median algorithm.

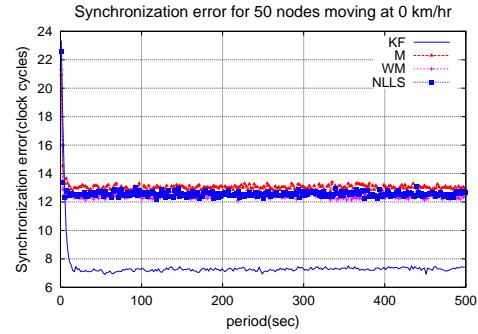
The simulation is also conducted with a speed of  $20km/hr$ , having similar results(Figure 7b). With the increase in the speed of the nodes, the precision of the algorithms improves. KF, with an average of  $10clk$  performs the best, whereas WM and NLLS perform well too compared with the median,  $1clk$  and  $1clk$  respectively.

The relative comparison of the algorithms performance improvement with the Median is shown in Figure 8. WM and NLLS perform, on average, 8 – 10% better than Median. The best performer, KF, has on average 60% better performance than the Median one.

3) **Case III:** In this set of simulations, the number of the nodes is increased to 50. With slowly moving nodes ( $6km/hr$ ), the results are shown in Figure 9a. Large disruptions occur due to nodes moving slowly in the surrounding (leave the network and join again after some time), resulting in a larger drift with neighbors before getting back to the

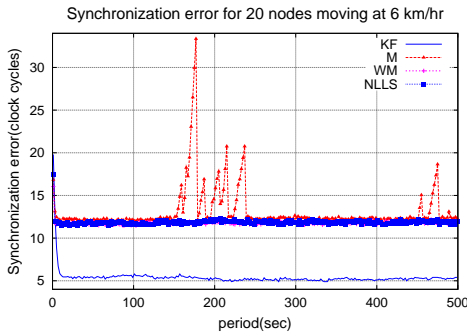


(a) Number of Nodes - 20

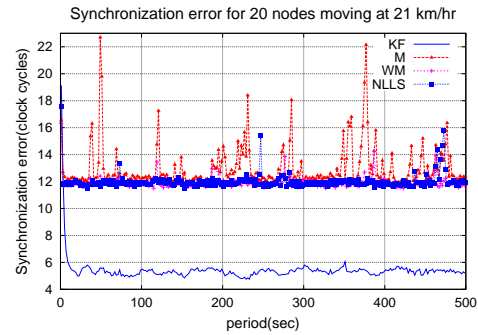


(b) Number of Nodes - 50

Fig. 6. Simulation results for Static Nodes

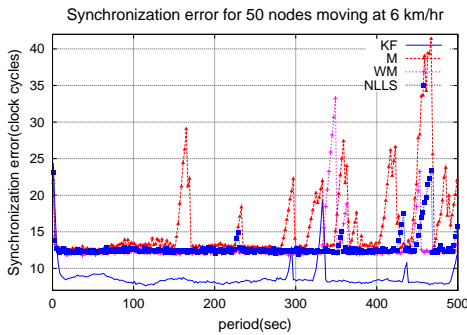


(a) Speed - 6km/hr

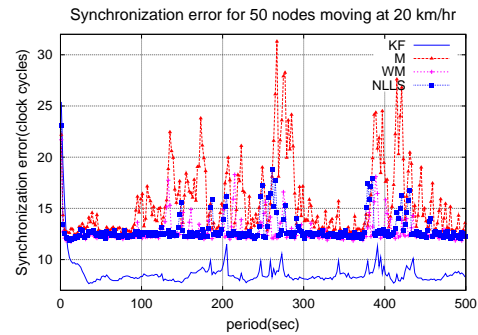


(b) Speed - 20km/hr

Fig. 7. Simulation results for 20 Nodes



(a) Speed - 6km/hr



(b) Speed - 20km/hr

Fig. 9. Simulation results for 50 Nodes

network.

With a speed of  $20\text{ km/hr}$ , the simulation result is presented in Figure 9b. With this speed, the performance is better due to the faster moving nodes, joining the networks at a faster rate. This helps in getting synchronized with the nodes in a faster time, without drifting away for a longer period of time (which is the case in the first set of nodes with speed of  $6\text{ km/hr}$ ). Again, for the set of nodes with a higher speed, a relative comparison is made to see the performance of the nodes with the median algorithm.

The Median algorithm is shown to perform the worst in this case. There are a lot of disruptions in the network, making it more unstable whereas the other algorithms adapt to the

changes faster.

Figure 10 shows that KF performs the best against Median algorithm, 45%. WM and NLLS perform well against Median too, 5% each. It has been shown that the median is prone to error in case of high dynamics in the network. In general, the Kalman filter has a better precision and convergence speed than the rest whereas WM and NLLS has a better tolerance to a dynamic network.

### C. Measurements concerning the energy consumption

As the results in the previous section shows, KF algorithm performs well in all conditions, so do WM and NLLS. The downside in implementing these algorithms, despite the



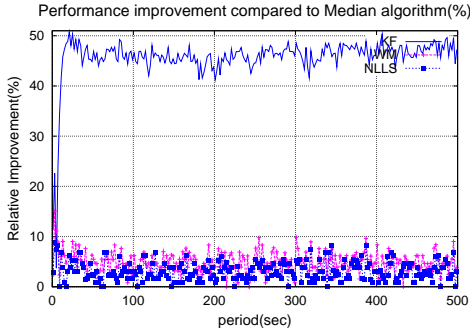


Fig. 10. Relative error of the algorithms w.r.t. Median

performance gain that all have against the Median is the energy consumption. As the algorithms are going to be implemented on the sensor nodes, the energy consumption is a priority in the study of embedded system algorithm development. The algorithms are written in C and implemented on the test nodes to see the effect that they are going to have on the energy consumption of the nodes.

Algorithm	Average Execution Time
M	65 $\mu$ s
NLLS	82.5 $\mu$ s
WM	90 $\mu$ s
KF	150 $\mu$ s

The table shows the average execution time of the algorithms implemented on the MyriaNode. This doesn't show the exact energy consumption of the algorithms but it can give us an approximation on the relative comparison of the algorithms about the energy consumption on the MyriaNode. As the table shows, the consumption of WM is 5% higher than the median algorithms. This is also true for NLLS and KF which consume 9% and 4 % higher than the median algorithm respectively.

In order to reduce the energy that is going to be spent on the algorithm with a better performance, the guard time of the slot can be reduced, due to a better performance showing by KF, WM and NLLS. The energy consumption of the guard

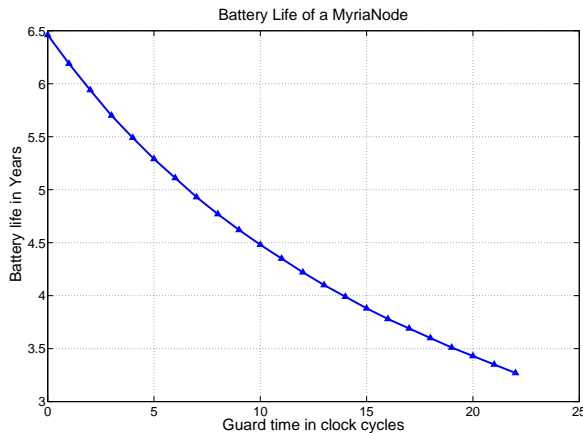


Fig. 11. The energy consumption of Guard time per RX slot

time is shown in Figure 11.

With the performance gain obtained, the guard time of the slot can be decreased, hereby decreasing the energy consumption of the node in general. Hence, by decreasing the guard time in proportion with the performance enhancement, energy for the demand of the algorithm can be saved.

## V. CONCLUSION AND RECOMMENDATION

A decentralized frame synchronization of a TDMA-based WSN is achieved using Weighted Measurements (WM), Non Linear Least Squares(NLLS) and Kalman Filter (KF) methods. A simulation is conducted with different scenarios, especially taking the effect of mobility. A comparison of the algorithms with the currently implemented Median algorithm is conducted and the results are presented.

In a static environment, the KF performs the best whereas the WM and NLLS have shown a similar performance as the Median. In terms of convergence time, all have shown a similar performance.

A 60% improvement in the performance of the synchronization can be achieved on average using the Kalman Filter for dynamic WSN. A lower improvement, 8% and 6% is obtained using WM and NLLS methods for the synchronization of TDMA frames. Both WM and NLLS show a better tolerance against Median in a dynamic network. Having a faster adaptation to the changes in the network made the algorithms preferred ones to a network characterized with dynamic behaviour. KF estimation of the next wake up time is the best with its iterative capability and adaptive nature. KF has also better stability, better resistant to topology changes occurring in the network.

Using these algorithms for a TDMA-based WSN synchronization( WM, NLLS and KF), the precision of the network as well as the stability of the algorithm increases. This in turn has a positive impact on the energy consumption of the nodes because the synchronization period,  $T_{sync}$  can be increased or the guard time of the node's frame,  $t_{guard}$  can be reduced to achieve the same performance as the Median algorithm. Results are presented and discussed.

But in the downside, the energy consumption of the algorithms is greater than the Median algorithm's energy consumption as the algorithms are more complex computationally. Analysis is made and presented about the energy saving made by decreasing the guard time of the slot. Communication is in general around 5 times costlier than computation on MyriaNodes. A net gain of battery life can thus be achieved by reducing the guard time. Median is shown to be still the best choice in a static environment with the current implementation, with less energy consumption and simplicity into consideration. But, for dynamic networks, the other algorithms perform better, in terms of performance. KF has the edge due to its adaptive nature.

Different software power minimization techniques can be applied to further reduce the power consumption of the proposed algorithms implementation, making them more energy efficient for implementation. Upon the completion of the ongoing project on developing a Software Defined Radio (SDR) for

the inspection of the nodes' wake-up times, a proper evaluation and enhancements on the algorithms, being implemented on the MyriaNodes, is achievable. Real time feedbacks can be used to improve the algorithms towards perfection.

## VI. ACKNOWLEDGMENT

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<sup>4</sup>MyriaNed is a project to create a large functional network of 10,000 nodes with wireless communication for research on protocols, power management, programming models, and security.