

The diagram illustrates the derivation of the semantic equivalence (T) from the syntactic equivalence (C). It is structured into two main columns, each representing a sequence of logical steps.

Left Column (Derivation from C to T):

- Top-level expression: $\langle \{\}, \{\}, id, \{[a']\langle f_n^C \langle (a\ c).X, (b\ c)(a\ c).Y \rangle, (c\ d)(b\ d)(a\ d).X \rangle \approx? [b']\langle f_n^C \langle X, Y \rangle, X \rangle\} \rangle$
- Rule: $(\approx? \mathbf{ab})$
- Expression: $\langle \{\}, \{\}, id, \{\langle f_n^C \langle (a\ c).X, (b\ c)(a\ c).Y \rangle, (c\ d)(b\ d)(a\ d).X \rangle \approx? \langle f_n^C \langle (a' \ b').X, (a' \ b').Y \rangle, (a' \ b').X \rangle, a' \#? \langle f_n^C \langle X, Y \rangle, X \rangle\} \rangle$
- Rule: $(\approx? \mathbf{pair})$
- Expression: $\langle \{\}, \{\}, id, \{f_n^C \langle (a\ c).X, (b\ c)(a\ c).Y \rangle \approx? f_n^C \langle (a' \ b').X, (a' \ b').Y \rangle, (c\ d)(b\ d)(a\ d).X \approx? (a' \ b').X, a' \#? \langle f_n^C \langle X, Y \rangle, X \rangle\} \rangle$
- Rule: $(\approx? \mathbf{C})$
- Expression: $\langle \{\}, \{\}, id, \{(a\ c).X \approx? (a' \ b').X, (b\ c)(a\ c).Y \approx? (a' \ b').Y, (c\ d)(b\ d)(a\ d).X \approx? (a' \ b').X, a' \#? \langle f_n^C \langle X, Y \rangle, X \rangle\} \rangle$
- Rule: $(\approx? \mathbf{inv})$
- Expression: $\langle \{\}, \{\}, id, \{(b\ c)(a\ c).Y \approx? (a' \ b').Y, (c\ d)(b\ d)(a\ d).X \approx? (a' \ b').X, a' \#? \langle f_n^C \langle X, Y \rangle, X \rangle, (a\ c)(a' \ b').X \approx? X\} \rangle$
- Rule: $(\approx? \mathbf{inv})$
- Expression: $\langle \{\}, \{\}, id, \{(c\ d)(b\ d)(a\ d).X \approx? (a' \ b').X, a' \#? \langle f_n^C \langle X, Y \rangle, X \rangle, (a\ c)(a' \ b').X \approx? X, (b\ c)(a\ c)(a' \ b').Y \approx? Y\} \rangle$
- Rule: $(\approx? \mathbf{inv})$
- Expression: $\langle \{\}, \{\}, id, \{a' \#? \langle f_n^C \langle X, Y \rangle, X \rangle, (a\ c)(a' \ b').X \approx? X, (b\ c)(a\ c)(a' \ b').Y \approx? Y, (c\ d)(b\ d)(a\ d)(a' \ b').X \approx? X\} \rangle$
- Rule: $(\#? \mathbf{pair})$
- Expression: $\langle \{\}, \{\}, id, \{a' \#? f_n^C \langle X, Y \rangle, a' \#? X, (a\ c)(a' \ b').X \approx? X, (b\ c)(a\ c)(a' \ b').Y \approx? Y, (c\ d)(b\ d)(a\ d)(a' \ b').X \approx? X\} \rangle$
- Rule: $(\#? \mathbf{app})$
- Expression: $\langle \{\}, \{\}, id, \{a' \#? \langle X, Y \rangle, a' \#? X, (a\ c)(a' \ b').X \approx? X, (b\ c)(a\ c)(a' \ b').Y \approx? Y, (c\ d)(b\ d)(a\ d)(a' \ b').X \approx? X\} \rangle$
- Rule: $(\#? \mathbf{pair})$
- Expression: $\langle \{\}, \{\}, id, \{a' \#? Y, a' \#? X, (a\ c)(a' \ b').X \approx? X, (b\ c)(a\ c)(a' \ b').Y \approx? Y, (c\ d)(b\ d)(a\ d)(a' \ b').X \approx? X\} \rangle$
- Rule: $(\#? \mathbf{var})$
- Expression: $\langle \{a' \# Y\}, \{\}, id, \{a' \#? X, (a\ c)(a' \ b').X \approx? X, (b\ c)(a\ c)(a' \ b').Y \approx? Y, (c\ d)(b\ d)(a\ d)(a' \ b').X \approx? X\} \rangle$
- Rule: $(\#? \mathbf{var})$
- Expression: $\langle \{a' \# X, a' \# Y\}, \{\}, id, \{(a\ c)(a' \ b').X \approx? X, (b\ c)(a\ c)(a' \ b').Y \approx? Y, (c\ d)(b\ d)(a\ d)(a' \ b').X \approx? X\} \rangle$
- Rule: (\top)

Right Column (Derivation from C to T):

- Top-level expression: $\langle \{\}, \{\}, id, \{(a\ c).X \approx? (a' \ b').Y, (b\ c)(a\ c).Y \approx? (a' \ b').X, (c\ d)(b\ d)(a\ d).X \approx? (a' \ b').X, a' \#? \langle f_n^C \langle X, Y \rangle, X \rangle\} \rangle$
- Rule: $(\approx? \mathbf{inst})$
- Expression: $\langle \{\}, \{\}, X/(a' \ b')(a\ c).Y, \{(b\ c)(a\ c).Y \approx? (a' \ b')(a\ c)(a' \ b').Y, (a' \ b')(a\ c)(c\ d)(b\ d)(a\ d).Y \approx? (a' \ b')(a\ c)(a' \ b').Y, a' \#? \langle f_n^C \langle (a' \ b')(a\ c).Y, Y \rangle, (a' \ b')(a\ c).Y \rangle\} \rangle$
- Rule: $(\approx? \mathbf{inv})$
- Expression: $\langle \{\}, \{\}, X/(a' \ b')(a\ c).Y, \{(a' \ b')(a\ c)(c\ d)(b\ d)(a\ d).Y \approx? (a' \ b')(a\ c)(a' \ b').Y, a' \#? \langle f_n^C \langle (a' \ b')(a\ c).Y, Y \rangle, (a' \ b')(a\ c).Y \rangle, (b\ c)(a\ c)(a' \ b')(a\ c)(a' \ b').Y \approx? Y\} \rangle$
- Rule: $(\approx? \mathbf{inv})$
- Expression: $\langle \{\}, \{\}, X/(a' \ b')(a\ c).Y, \{a' \#? \langle f_n^C \langle (a' \ b')(a\ c).Y, Y \rangle, (a' \ b')(a\ c).Y \rangle, (b\ c)(a\ c)(a' \ b')(a\ c)(a' \ b').Y \approx? Y, (a' \ b')(a\ c)(c\ d)(b\ d)(a\ d)(a' \ b')(a\ c)(a' \ b').Y \approx? Y\} \rangle$
- Rule: $(\#? \mathbf{pair})$
- Expression: $\langle \{\}, \{\}, X/(a' \ b')(a\ c).Y, \{a' \#? f_n^C \langle (a' \ b')(a\ c).Y, Y \rangle, a' \#? (a' \ b')(a\ c).Y, (b\ c)(a\ c)(a' \ b')(a\ c)(a' \ b').Y \approx? Y, (a' \ b')(a\ c)(c\ d)(b\ d)(a\ d)(a' \ b')(a\ c)(a' \ b').Y \approx? Y\} \rangle$
- Rule: $(\#? \mathbf{app})$
- Expression: $\langle \{\}, \{\}, X/(a' \ b')(a\ c).Y, \{a' \#? \langle (a' \ b')(a\ c).Y, Y \rangle, a' \#? (a' \ b')(a\ c).Y, (b\ c)(a\ c)(a' \ b')(a\ c)(a' \ b').Y \approx? Y, (a' \ b')(a\ c)(c\ d)(b\ d)(a\ d)(a' \ b')(a\ c)(a' \ b').Y \approx? Y\} \rangle$
- Rule: $(\#? \mathbf{pair})$
- Expression: $\langle \{\}, \{\}, X/(a' \ b')(a\ c).Y, \{a' \#? Y, a' \#? (a' \ b')(a\ c).Y, (b\ c)(a\ c)(a' \ b')(a\ c)(a' \ b').Y \approx? Y, (a' \ b')(a\ c)(c\ d)(b\ d)(a\ d)(a' \ b')(a\ c)(a' \ b').Y \approx? Y\} \rangle$
- Rule: $(\#? \mathbf{var})$
- Expression: $\langle \{a' \# Y\}, \{\}, X/(a' \ b')(a\ c).Y, \{a' \#? (a' \ b')(a\ c).Y, (b\ c)(a\ c)(a' \ b')(a\ c)(a' \ b').Y \approx? Y, (a' \ b')(a\ c)(c\ d)(b\ d)(a\ d)(a' \ b')(a\ c)(a' \ b').Y \approx? Y\} \rangle$
- Rule: $(\#? \mathbf{var})$
- Expression: $\langle \{b' \# Y, a' \# Y\}, \{\}, X/(a' \ b')(a\ c).Y, \{(b\ c)(a\ c)(a' \ b')(a\ c)(a' \ b').Y \approx? Y, (a' \ b')(a\ c)(c\ d)(b\ d)(a\ d)(a' \ b')(a\ c)(a' \ b').Y \approx? Y\} \rangle$
- Rule: (\top)