13) a) de la práctica de números complejos

$$\begin{aligned}
z^2-(2+i)\,z-7i &= 0\\
a &= 1, b=-(2+i)\,, c=-7i\\
\Delta &= b^2-4ac\\
z_{1,2} &= \frac{-b+\sqrt{\Delta}}{2a}
\end{aligned}$$

$$w=4+4i-1+28i=3+32i$$

$$\begin{aligned}
|w| &= \sqrt{9+1024}=\sqrt{1033}\\
\arg w &= \arctan\frac{32}{3}=\theta
\end{aligned}$$

$$\sqrt{w}=\left\{\sqrt[4]{1033}_{\frac{\arctan\frac{32}{3}+2k\pi}{2}}:k=0,1\right\}$$

$$\begin{aligned}
k &= 0\to\sqrt[4]{1033}_{\frac{\arctan\frac{32}{3}}{2}}=\sqrt[4]{1033}\left(\cos\frac{\arctan\frac{32}{3}}{2}+i\sin\frac{\arctan\frac{32}{3}}{2}\right)=...\\
k &= 1\to\sqrt[4]{1033}_{\frac{\arctan\frac{32}{3}+2\pi}{2}}=\sqrt[4]{1033}\left(\cos\frac{\arctan\frac{32}{3}+2\pi}{2}+i\sin\frac{\arctan\frac{32}{3}+2\pi}{2}\right)
\end{aligned}$$

Cuenta auxiliar:

$$\begin{aligned}
\tan\theta &= \tan\left(\frac{\theta}{2}+\frac{\theta}{2}\right)=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\\
\tan\theta-\tan^2\left(\frac{\theta}{2}\right)\tan\theta-2\tan\frac{\theta}{2} &= 0\\
x &= \tan\frac{\theta}{2}\to-\tan\theta.x^2-2x+\tan\theta=0\\
-\frac{32}{3}x^2-2x+\frac{32}{3} &= 0\quad\text{resolver}
\end{aligned}$$

$$\begin{aligned}
\cos\frac{\theta}{2} &= \sqrt{\frac{1}{1+\tan^2\frac{\theta}{2}}}\ (\theta\text{ y }\frac{\theta}{2}\text{ son ángulos del primer cuadrante})\\
\sin\frac{\theta}{2} &= ...
\end{aligned}$$

<u>Otra forma</u>:

$$\begin{aligned}
w &= \underbrace{x}_{\mathrm{Re}(w)}+i\underbrace{y}_{\mathrm{Im}(w)}\\
w^2 &= 3+32i\\
x^2-y^2+2xyi &= 3+32i
\end{aligned}$$

$$\left\{ \begin{array}{c} x^2 - y^2 = 3 \\ 2xy = 32 \end{array} \right.$$

$$\begin{aligned}
y &= \frac{16}{x} \\
x^2 - \frac{256}{x^2} &= 3 \\
\frac{x^4 - 256}{x^2} &= 3 \\
x^4 - 256 - 3x^2 &= 0 \\
s &= x^2 \to s^2 - 3s - 256 = 0 \text{ seguir...}
\end{aligned}$$

Ejercicio 5 del TP1:

$$\begin{aligned}
p(x) &= 3x^3 + (3 - 3i)x^2 - (6 + 3i)x + 6i \\
p(x) &= 3(x - i)(x^2 + x - 2) = 3(x - i)(x - 1)(x + 2)
\end{aligned}$$

Ejercicio 15) de la práctica de lógica

$$\begin{aligned}
p &\leftrightarrow q \\
(p \to q) \wedge q &\to p \\
& \quad (\sim p \vee q) \wedge (\sim q \vee p) \\
& \quad (\sim p \wedge \sim q) \vee \underbrace{(\sim p \wedge p)}_{F_0} \vee \underbrace{(q \wedge \sim q)}_{F_0} \vee (q \wedge p) \\
& \quad (\sim p \wedge \sim q) \vee (q \wedge p) \\
&\sim (p \vee q) \vee (q \wedge p) \\
p \vee q &\to q \wedge p
\end{aligned}$$

$$\begin{aligned}
&\sim (\sim (p \vee q) \vee (q \wedge p)) \\
(p \vee q) \wedge &\sim (q \wedge p) \\
& \quad (p \vee q) \wedge (\sim q \vee \sim p) \\
& \quad (p \wedge \sim q) \vee (p \wedge \sim p) \vee (q \wedge \sim q) \vee (q \wedge \sim p) \\
& \quad (p \wedge \sim q) \vee (q \wedge \sim p) \\
p &\veebar q
\end{aligned}$$

Ejercicio teórico para demostrar que la suma de polinomios es cerrada:

$$\begin{aligned}
gr(p) &= n; \quad p(x) = \sum_{i=0}^{n} a_i x^i = \sum_{i=0}^{n} a_i x^i + \sum_{i=n+1}^{m} 0.x^i \\
gr(q) &= m > n; \quad q(x) = \sum_{i=0}^{n} b_i x^i + \sum_{i=n+1}^{m} b_i x^i \\
(p+q)(x) &= \sum_{i=0}^{n} \underbrace{(a_i + b_i)}_{\in\mathbb{C}} x^i + \sum_{i=n+1}^{m} \underbrace{(0 + b_i)}_{\in\mathbb{C}} x^i
\end{aligned}$$