

DIFFUSION IN LANDSCAPE DEVELOPMENT MODELS: ON THE NATURE OF BASIC TRANSPORT RELATIONS

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ABSTRACT

In constructing large-scale landscape development models, processes must be appropriately represented. The diffusion equation is rewritten with separate coefficients for slow, continuous mass movements and rapid, episodic mass movements. Using available transport data, transport/gradient relations are assessed and estimates of diffusivities are given. Diffusivities estimated in this study are compared with values derived in scarp studies and values adopted in landscape development models. The relation between transport and gradient may be non-linear, which would require modification of the simple diffusion equation normally accepted in modelling. New approaches are required to place these essential relations on a firmer foundation. © 1997 by John Wiley & Sons, Ltd.

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INTRODUCTION

The construction of a landscape development model requires the representation of processes over extended scales of space and time. Mechanistic parameterizations of processes that are appropriate locally in the short term are not practical at much larger scales. Suitable methods must be found for the generalization of processes that capture the essential character of landscapes as they evolve. It has been several decades since the diffusion equation was introduced into geomorphological reasoning for this purpose (see Nash (1980a,b) for early references). The basic equation has been used subsequently to model the consequent development of scarps (Nash, 1980a,b; Colman and Watson, 1984; Hanks *et al.*, 1984). In recent years it has been employed in many large-scale landscape development models to simulate slope evolution over long periods (Anderson and Humphrey, 1989; Flemings and Jordan, 1989; Koons, 1989; Willgoose *et al.*, 1991; Anderson, 1994; Kooi and Beaumont, 1994, 1996; Rosenbloom and Anderson, 1994; Tucker and Slingerland, 1994; Rinaldo *et al.*, 1995).

The diffusion equation is attractive for modelling slope evolution in large-scale landscape development models because it eliminates mechanistic details that are resolvable only at smaller scales. It is flexible because the diffusion coefficient can be modified to reflect changes in space and/or time of the controlling variables. However, it remains to be determined whether critical features of landscapes are maintained when using this equation.

The basic fixed coefficient equation assumes a linear dependence of transport on slope tangent. To assess the appropriateness of this assumption, and to calibrate such an equation, if it indeed appears to be reasonable, requires transport estimates of the order of a century or more. But long-term estimates of total sediment transport on hillslopes are not generally available. In this paper, we consider these questions by writing the diffusion equation in a form that allows us to use such transport data as there are.

THE DIFFUSION EQUATION

The basic equation is derived from a statement of sediment continuity:

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$$\frac{\partial h}{\partial t} = - \left[\frac{\partial S}{\partial x} + \frac{\partial S}{\partial y} \right]$$

and a sediment transport relation:

$$S = -k \left[\frac{\partial h}{\partial x} + \frac{\partial h}{\partial y} \right]$$

wherein h is height, t is time, S is volumetric transport rate ($L^3L^{-1}T^{-1}$), k is a diffusion coefficient (L^2T^{-1}) and x and y are space dimensions. These two equations are combined to form the diffusion equation:

$$\frac{\partial h}{\partial t} = k \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right]$$

The equation assumes transport-limited removal of material from the slope (which is a standard assumption in long-term models). Further insight is gained by rewriting it in the following form:

$$\rho_b \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[m \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[m \frac{\partial h}{\partial y} \right]$$

wherein m is a standard mass transfer rate ($ML^{-1}T^{-1}$) at unit gradient, and ρ_b is sediment bulk density. The diffusion coefficient is now represented as a standard mass transfer rate divided by sediment bulk density. This reveals that we can assess the transport/gradient relation using either volumetric or mass transport data.

In order to incorporate both slow and rapid mass-wasting processes, the equation is rewritten with two diffusion terms:

$$\frac{\partial h}{\partial t} = [\alpha + \Omega] \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right]$$

where α is the diffusivity for slow, continuous mass movements (e.g. creep processes) and Ω is the diffusivity for rapid, episodic mass movements (e.g. shallow landslides). (This approach is analogous with the treatment of molecular and eddy viscosity in fluid mechanics.) The incorporation of Ω entails the assumption that, over the long term, rapid mass movement affects the landscape everywhere that is above some threshold gradient.

DIFFUSIVITY FOR SLOW, CONTINUOUS MASS MOVEMENTS

To appraise diffusion coefficients for slow, continuous mass movements, we examined the volumetric creep data presented in Young (1974), Saunders and Young (1983) and several more recent studies. Some results (Owens, 1969; Finlayson, 1981) exhibit a weak, but statistically non-significant, relation between gradient and transport. Others (Williams, 1973) exhibit no correlation at all. Most of the studies are short term and it is possible that such a relation is detectable only at longer time-scales when short-term variability of controlling factors, such as soil moisture, may average out.

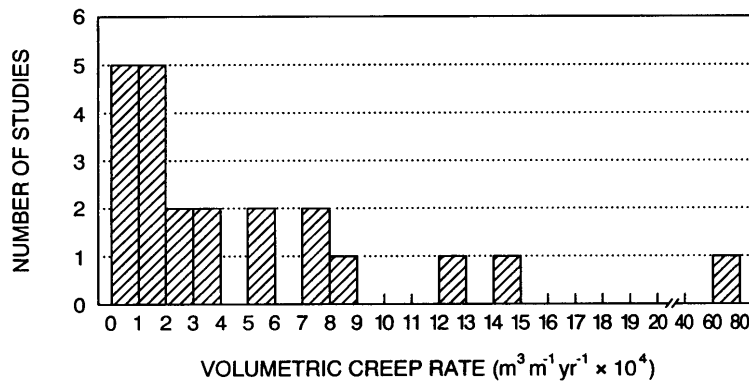


Figure 1. Volumetric creep rates based on the field data given in Saunders and Young (1983) and Young (1974). Other data are from Finalyson (1981) and McKean *et al.* (1993). Each datum represents the average creep rate for a particular study. Note the outlier to the right of scale break: this creep estimate was obtained by McKean *et al.* (1993) using an isotope mass-balance model whereas other studies are direct measurements

The diffusion coefficient is numerically equivalent to the volumetric transport rate at unit gradient. All of the observations were taken on considerably lower gradients but, since functional dependence is observed to be weak, and in the absence of more definitive evidence, we have assumed that the median value from several studies, of $0.0002 \text{ m}^2 \text{ a}^{-1}$ (Figure 1), represents the correct *order of magnitude* value for the creep diffusion coefficient. In a stochastic approach, the variation of diffusivity would be represented by an extremely distributed variate. But there remains a need for additional study as it seems incredible that there is no gradient relation. Results of a field study by Schumm (1964) and a controlled experiment by Van Asch *et al.* (1989) suggest that there may be a non-linear relation between creep rate and gradient. This finding is supported by Andrews and Bucknam (1987), who propose a non-linear relation between transport and gradient on scarps. Confirmation of non-linearity would be an important result because it would necessitate the formulation of a more suitable creep transport equation.

DIFFUSIVITY FOR RAPID EPISODIC MASS MOVEMENTS

In order to approach rapid, episodic sediment transfer at landscape scale, we require data sets that incorporate many events. Here we illustrate our approach by using a data set from the Queen Charlotte Islands, British Columbia (Rood, 1984, 1990). Landsliding rates are high owing to the high annual rainfall (ranging from about 1500 to 5000 mm a^{-1}), rapidly weathered volcanic and sedimentary rocks, and glacially oversteepened slopes. The landslide inventory was completed by identifying landslides on aerial photographs. The data set is believed to cover approximately a 40 year period as landslides older than this were not clearly visible on the photographs. In relation to landscape development, this period is very short. Nevertheless, the data constitute perhaps the most comprehensive, extended record of landsliding activity available. Forested portions of 22 watersheds (clearcut logging occurred in many basins) were analysed in order to obtain natural landsliding rates. The mean area of forested terrain in each basin is 12 km^2 , with a basin average over this area of about 30 landslides.

The areas of each watershed that fall under particular slope gradient classes are calculated. The following calculation is performed for each gradient class for each basin:

$$S_{\text{gradient class}} = \sum_{i=1}^n \left[\frac{v_i}{A_{\text{gradient class}}} \cdot d_i \right]$$

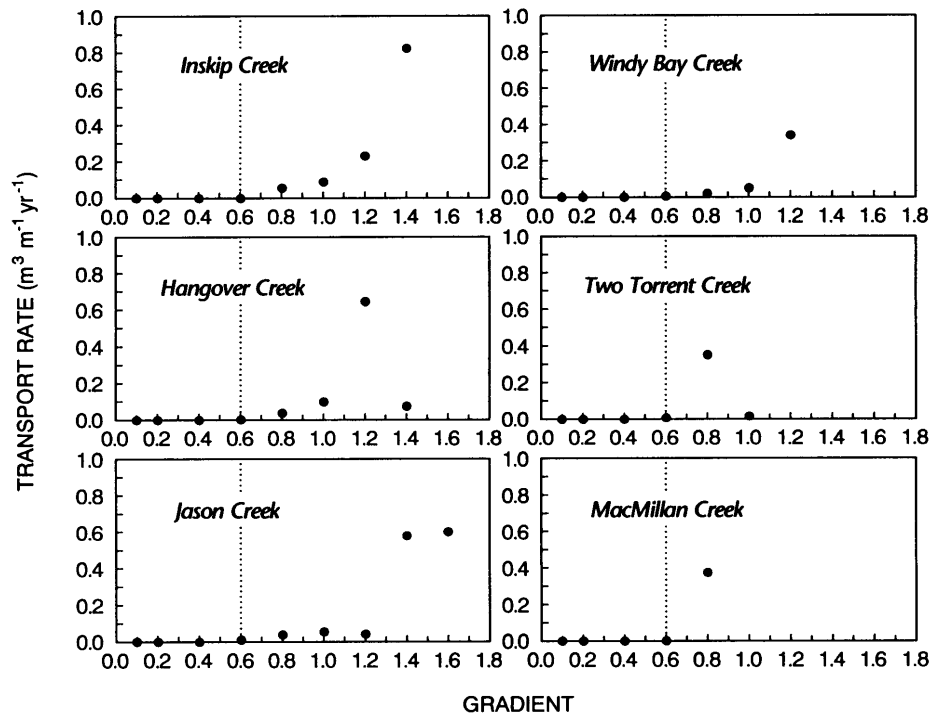


Figure 2. Landslide transport rate versus gradient for drainage basins in the Queen Charlotte Islands, British Columbia. Gradient is represented as slope tangent. The dashed lines represent the threshold for landsliding activity of about 30°

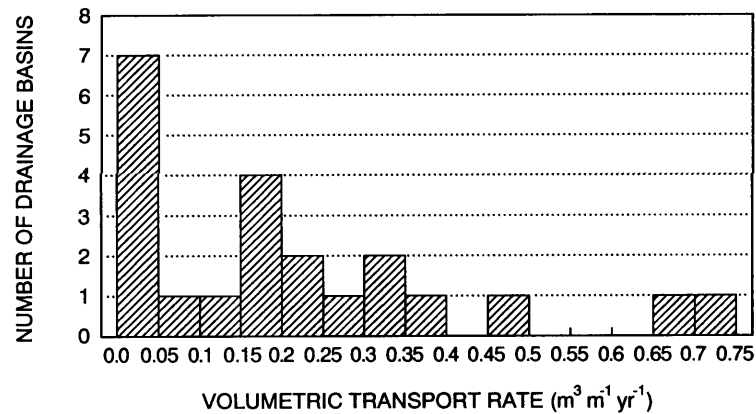


Figure 3. Landslide transport rates at unit gradient for drainage basins in the Queen Charlotte Islands, British Columbia

where S is volumetric transport rate ($\text{m}^3 \text{m}^{-1}$ per 40 years), v is landslide volume, d is transport distance (typically the full length of the slope), A is area and n is the number of landslides. This result is divided by the 40 years that the landslide inventory covers.

Transport rate versus gradient is plotted for several watersheds in Figure 2. Although non-linearities appear in our results, best-fit linear relations were determined in order to obtain the transport rate at unit gradient for each basin. This approach is adopted in order to determine a representative coefficient for the basic form of the

diffusion equation. The median value of $0.2 \text{ m}^2 \text{ a}^{-1}$ derived over all of our study basins (Figure 3) estimates a diffusivity for rapid, episodic mass movements.

A threshold of about 30° is found for landslide activity. At higher gradients, two interpretations are possible. Some basins appear to exhibit a distinctly non-linear relation between transport and slope angle, whilst others show a step increase to a finite transport for slopes in the range of $30\text{--}45^\circ$. On slope angles greater than about 40° results are variable; this is not surprising since some cohesion mechanisms must come into play. A step-change is consistent with the belief that slopes in different gradient classes are dominated by distinct processes. Theory suggests that mass transport rates may be related to the sine of the slope angle rather than the tangent. However, this adjustment does not remove the non-linearities from our data.

Again, a non-linear relation, strictly observed, requires a modification of the governing equation. Non-linearity may appear in the transport/gradient relation for several reasons. One is that transport distance itself depends on slope angle. Kirkby (1992) has elaborated such a model from his general formulation of hillslope evolution. In the long run, this seems to represent an unlikely constraint for rapid mass-wasting processes in steep terrain. However, a second possibility is that starting angle and slope length (which enter our transport estimates) are themselves confounded in our data. In fact they are independent of each other. It finally must be acknowledged that our gradient/transport relations are based on slope angle in the detachment zone. But slopes in the Queen Charlotte Islands are typically relatively short (median length about 50 m) and nearly rectilinear, so this factor appears not to be a source of serious bias. It remains to explore more complex models for these data.

COMPARISON WITH OTHER STUDIES

Previous attempts to estimate sediment diffusivities have been based on scarp erosion studies (Table I). These are local studies in which a diffusion coefficient is fitted to slope profile development. The primary process in operation on such slopes is slow mass movement such as creep. Our diffusivity for creep is at the lower end of the range of scarp values and our landslide diffusivity is at least an order of magnitude greater than these values. From the perspective of landscape modelling, our data – derived from an ensemble of possibly representative hillslopes – are apt to be more informative than results from individual scarp slopes. However, these slopes are relatively steep and possibly indicative of higher values for creep transport near unit gradient.

Diffusivity estimates for the present study are compared with diffusivities adopted in landscape development models (Table II). The diffusivities employed by Anderson (1994) and Rosenbloom and Anderson (1994) represent values for slow mass movements but are much greater than the creep diffusivity found in the present study. Anderson (1994) includes an option of incorporating a non-linear increase in transport due to landsliding activity on steeper slopes.

Table I. Comparison with diffusion coefficients derived from scarp studies.

Study	Diffusion Coefficient ($\text{m}^2 \text{ a}^{-1}$)
Present study	Landslides: 2×10^{-1} Creep: 2×10^{-4}
Nash (1980a)	1.2×10^{-2}
Nash (1980b)	4.4×10^{-4}
Colman and Watson (1984)	9×10^{-4}
Hanks <i>et al.</i> (1984)	1.1×10^{-3} 1.1×10^{-2} 1.6×10^{-2}

Table II. Comparison with diffusion coefficients implemented in landscape development models.

Study	Diffusion Coefficient ($\text{m}^2 \text{a}^{-1}$)
Present study	Landslides: 2×10^{-1} Creep: 2×10^{-4}
Anderson and Humphrey (1989)	10^1
Flemings and Jordan (1989)	1×10^2 to 5×10^3
Koons (1989)	1.5×10^{-1} to 1.5×10^1
Anderson (1994), Rosenbloom and Anderson (1994)	Order of 10^{-2}
Kooi and Beaumont (1994, 1996)	2×10^{-2} to 1×10^2

Note: Willgoose *et al.* (1991), Tucker and Slingerland (1994) and Rinaldo *et al.* (1995) do not quote diffusivity values adopted in their landscape development models.

The diffusivities implemented by Koons (1989) and Kooi and Beaumont (1994, 1996) are assumed to represent all slope processes including landsliding. The values at the higher end of the ranges are used when modelling humid regions. These values are higher than the landslide diffusivity estimated in the present study (and higher even than an upper extreme). The Queen Charlotte Islands are known to have high landsliding rates; only in exceptional circumstances would the landslide rate be expected to exceed the value found in the present study.

The diffusivities used by Anderson and Humphrey (1989) and by Flemings and Jordan (1989) also exceed the values found in the present study. In the former case the diffusion coefficient is derived from an estimate of debris flow delivery rates to fans. The diffusivity values of Flemings and Jordan (1989) are based on estimates of mean regional gradients in mountain belts and basin fill rates; both hillslope and fluvial processes may be reflected in these high values.

CONCLUSIONS

- (i) On balance, relations between transport rate and gradient appear to be non-linear. If this is the case, then the slope evolution equation used in landscape development models must be selected to incorporate these non-linearities. This has not, heretofore, been the rule.
- (ii) The order of magnitude comparison between slow and rapid mass-wasting processes suggests that in susceptible terrain only the latter need be considered in long-term landscape development models.
- (iii) To confirm current appearances, field data must cover longer time periods and must be controlled to identify sources of variability. There are severe limitations to the time-scales of transport rates that are available by direct measurement. Techniques based on absolute dating methods should be subject to much further investigation.
- (iv) Although laboratory experiments do not provide transport estimates that are representative of those found in nature, further experimentation (particularly relevant for creep) might help us to elucidate the nature of the transport/gradient relation.

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