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A Physical Explanation of an Observed Link Area-Slope Relationship

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An observed log-log linear relationship between channel slope and contributing area is explained by the erosional physics that lead to catchment form. It is postulated that tectonic uplift is in balance with the fluvial erosion downwasting that dominates catchment erosion, and it is shown that this relationship results in the observed log-log linear relationship at dynamic equilibrium. In addition, it has been observed that there are deviations from this log-log linear relationship near the catchment divide, with observed slopes being lower than those predicted from the relationship. This is explained by noting that for small areas, fluvial erosion effects are dominated by soil creep and rain splash, modeled by diffusive physics. The area at which this deviation from log-log linearity occurs is that point on the hillslope at which diffusive physics, like soil creep and rain splash, begin to dominate fluvial erosion. These predictions are confirmed by numerical simulations using a catchment evolution model.

INTRODUCTION

Many investigators [*Hack*, 1957; *Flint*, 1974; *Gupta and Waymire*, 1989; *Tarboton et al.*, 1989*a*] have discussed or reported a relationship between area and mean slope in the channels of the form

$$A^{\alpha} S = \text{constant} \tag{1}$$

where A is the contributing area to the point of interest (generally the downstream end of a link), S is the mean slope of the segment above the point of interest, and α is a scaling coefficient between 0.4 and 0.7 (*Tarboton et al.* [1989*a*] and other unpublished data by Tarboton). *Hack* [1957] used regression to relate slope to distance downstream, geology, and bed material. Using the catchment evolution model discussed in the accompanying papers [*Willgoose et al.* this issue (*a*, *b*)], it is possible to derive this relationship for a catchment in dynamic equilibrium. The governing equations used in the model are

$$\frac{\partial z}{\partial t} = c_0(x, y) + \frac{1}{\rho_s(1-n)}\left[\frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y}\right] + D_z\left[\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right] \tag{2}$$

$$q_s = f(Y) q^{m_1} S^{n_1} \tag{3}$$

$$f(Y) = \beta_1 O_t \qquad Y = 0 \quad \text{(hillslope)}$$

$$f(Y) = \beta_1 \qquad Y = 1 \quad \text{(channel)}$$

where z is elevation, t is time, x, y are horizontal directions, $c_0(x, y)$ is the tectonic uplift rate, q_{sx}, q_{sy} are sediment transport per unit width in the x and y directions, respectively, D_z is the diffusivity of elevation, q is the mean peak discharge per unit width, or $q = Q/w$ for channels, Q is mean peak discharge in channels, w is the width of channels, S is slope, O_t is the ratio of hillslope erosion rate to channel erosion rate, β_1 is a sediment transport coefficient, ρ_s is sediment density, and n is sediment porosity.

The first term in (2) is the tectonic uplift, the second term is a continuity term for sediment transport, and the third term is a Fickian diffusion of elevation modeling such processes as rain splash and soil creep. These processes are described in detail elsewhere [*Willgoose et al.*, 1989].

DYNAMIC EQUILIBRIUM AND THE AREA-SLOPE RELATIONSHIP

For a catchment in dynamic equilibrium the tectonic uplift and the erosional loss are in approximate balance everywhere in the basin. This is a generally accepted definition of dynamic equilibrium, at least for short time scales [*Hack*, 1960]. Initially, consider the case where fluvial erosion dominates the diffusion effects of soil creep and rain splash so that the second term in (2) dominates the third term. Constant tectonic uplift is assumed throughout the basin. By continuity in the channel at the outlet of the catchment,

$$c_0 A = \frac{\beta_1 q^{m_1} S^{n_1}}{\rho_s(1-n)} w \tag{4}$$

where A is the catchment area.

A commonly observed relationship between mean peak discharge and area [e.g., *Strahler*, 1964] is

$$Q = \beta_3 A^{m_3} \tag{5}$$

Substituting this relationship into (4) and rearranging yields

$$A^{(m_3 m_1 - 1)/n_1} S = \left[\frac{1}{\beta_3}\right]^{m_1/n_1} \left[\frac{c_0 \rho_s(1-n)}{\beta_1} w^{m_1 - 1}\right]^{1/n_1} \tag{6}$$

which is the form of (1) where the predicted scaling coefficient α_p is

$$\alpha_p = \frac{m_3 m_1 - 1}{n_1} \tag{7}$$

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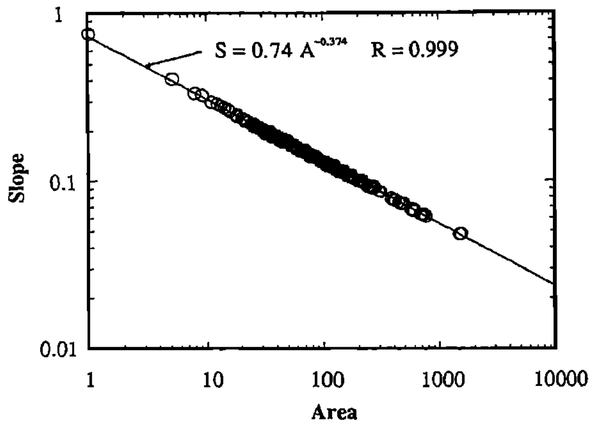


Fig. 1. Relationship between slope and areas for channels, at dynamic equilibrium, for a simulation dominated by fluvial sediment transport ($m_1 = 1.8$, $n_1 = 2.1$, $\rho_s = 1$, $n = 0$, $m_3 = 1.0$, $\beta_3 = 1$, $c_0 = 0.00031$, $\beta_1 = 0.01$, $D_z = 0$).

The form of the right-hand side of (6) is consistent with *Hack* [1957], who found for the Appalachian Mountains in Virginia that for a given area, slopes were highest in the catchments with hardest and thus less erosive materials.

Equation (1) was fitted to output from a catchment simulation which had been allowed to proceed to dynamic equilibrium. The simulation scheme is described by *Willgoose et al.* [this issue (*a*)] and tries to mimic the processes believed to be important in channel and hillslope development and interaction. In this particular exercise only fluvial sediment transport (no diffusion) was considered. A regression between contributing areas and channel slopes at dynamic equilibrium is shown in Figure 1. The fit is very satisfactory ($r > 0.99$). The fitted coefficient in (1) is $\alpha = 0.374$. Using the parameters of the computer simulation with values that are considered to be realistic ($m_3 = 1.0$, $m_1 = 1.8$, $n_1 = 2.1$), the predicted coefficient is $\alpha_p = 0.381$, a very good agreement with the fitted value. This behavior is not unexpected given the above analysis.

Tarboton et al. [1989*a*], using digital maps of continental United States catchments, found a value of the coefficient of about $\alpha = 0.47$. They used mean link slopes and the total contributing area to the bottom of the link. Further data on the sediment transport equation and flood frequencies for their catchments are required to verify that their results correspond to the mechanism postulated above. Nevertheless, the observed scaling coefficient α is within the bounds of the predicted one (α_p) (equation (7)). If, for instance, the parameters of the sediment transport equation are set to $m_1 = 2.1$ and $n_1 = 2.45$ (i.e., 17% higher), then with $m_3 = 1$, the predicted coefficient is $\alpha_p = 0.45$. Thus the scaling coefficient α is fairly dependent on the governing sediment transport equations for the catchment. If the flood frequency distribution with area for the catchment is known, so that m_3 may be estimated, then α, derived from the elevations of the catchment, is potentially an estimator of the exponents in the sediment transport equation.

In addition to the scaling relationship with area (equation (1)) *Tarboton et al.* [1989*a*] fitted relationships of the form

$$(2M - 1)^{\alpha'} S = \text{constant} \tag{8}$$

$$M^{\alpha''} S = \text{constant} \tag{9}$$

where M is the channel link magnitude [*Shreve*, 1966] and α' and α'' are scaling coefficients to be used with $(2M - 1)$ and M, respectively. It is important to determine which relationship (1, 8, or 9) is most appropriate and whether the suggested model is compatible with available observations.

Flint [1974] fitted (8) to 11 catchments and found α' to be in the range 0.37–0.83, with correlation coefficients of around 0.9. Equations (1), (8), and (9) were fitted to the same catchment simulation results discussed above, using definitions of area and slope consistent with those of *Tarboton et al.* (1989*a*) to yield the coefficients $\alpha = 0.33$, $\alpha' = 0.42$, and $\alpha'' = 0.53$, with correlation coefficients of $r = 0.99$, 0.86, and 0.85, respectively. Thus different scaling coefficients are obtained depending on the variable used to represent the size of the catchment. Following is an explanation for those differences and an argument for (1) as the most appropriate relationship.

A commonly accepted relationship for area and magnitude is

$$E[A] = (2M - 1)E[A_l] \tag{10}$$

where $E[\ ]$ is the expectation operator; A_l is the area draining laterally to a link, where the area is assumed to have the same distribution for both exterior and interior links. It has been shown that there is a significant difference between the area draining to an exterior link and the area draining to an interior link [*Shreve*, 1974; *Montgomery and Dietrich*, 1989; *Willgoose et al.*, 1989]. More correctly, then, the mean area should be expressed as

$$E[A] = ME[A_e] + (M - 1)E[A_i] \tag{11}$$

where A_e, A_i are the areas draining laterally to exterior and interior links, respectively.

Equation (8) is equivalent to (1) only if (10) is true. As noted, however, (11) is more satisfactory. The effect of the difference in exterior areas is clear in Figure 2, where M, $(2M - 1)$, and area are plotted versus link slope. For the regressions of $(2M - 1)$ versus slope and M versus slope, the difference in the fitted coefficients is solely in the magnitude 1 links (i.e., exterior links), where slopes are anomalously high, consistent with (10) being only approximately correct. If a regression is fitted to $(2M - 1)$ and slope (and M and slope), ignoring the exterior links, then the fitted coefficient is closer to that based on area. This observation is consistent with (11).

This extra area draining to exterior links, called the source area, is believed to be one possible reason why *Tarboton et al.* [1989*a*] observed a trend $\alpha < \alpha' < \alpha''$ in field data. This behavior is consistent with simulated results.

The differences between α, α', and α'' should be smaller in higher-magnitude channel networks. This is because of the diminishing influence of the exterior links in the regression; asymptotically, for infinite magnitude networks, the three coefficients will be the same. The networks of *Tarboton et al.* [1989*a*] were much larger than those considered here, and consequently, the differences between the scaling coefficients were not so large. For example, *Tarboton et al.* [1989*a*] examined a nested catchment: Big Creek, Idaho (magnitude = 139) nested inside St. Joe River (magnitude = 621). For the smaller Big Creek the coefficients α, α', and α'' were -0.47, -0.50, and -0.58, respectively, while for St. Joe River they were -0.47, -0.48, and -0.56. These num-

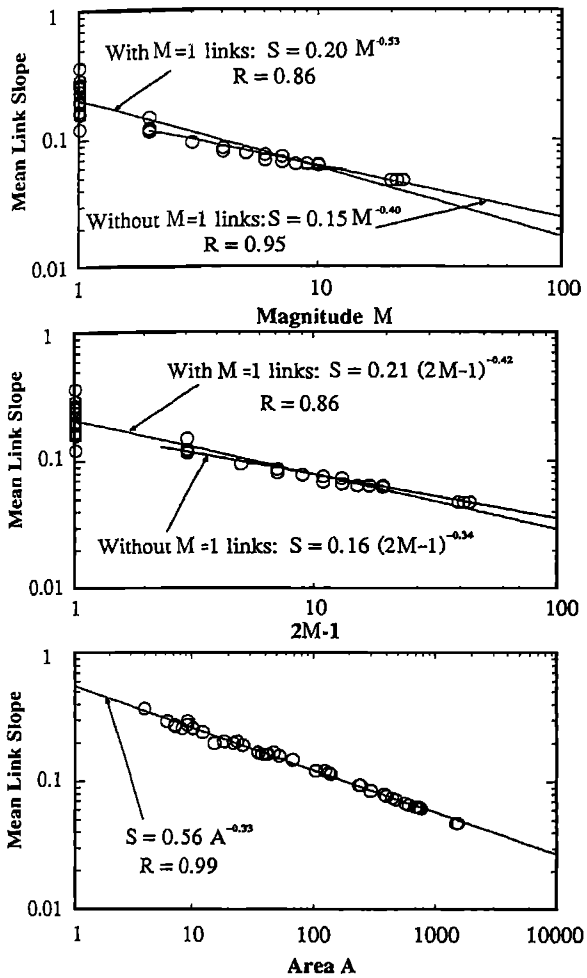


Fig. 2. Link magnitude and area versus mean link slopes, at dynamic equilibrium, for the simulation dominated by fluvial sediment transport in Figure 1.

bers hint at the magnitude dependence of the magnitude-based scaling coefficients and that the area-based scaling coefficients appear to be independent of magnitude, as expected.

Hillslope Behavior

Equation (4) is satisfied in the channel flowing out of the catchment, irrespective of the erosion processes within the catchment. It can, however, be applied with equal validity at the scales less than the hillslope scale. The sediment outflow is that corresponding to transport on the hillslope, so that

$$c_0 A = \frac{\beta_1 O_t q^{m_1} S^{n_1} w}{\rho_s(1-n)} \tag{12}$$

where A is the contributing area of the hillslope, and w is the flow width at the end of the hillslope element. The slope area relationship in hillslopes becomes

$$A^{(m_3 m_1 - 1)/n_1} S = \left[\frac{1}{\beta_3}\right]^{m_1/n_1} \left[\frac{c_0 \rho_s (1-n) w^{m_1 - 1}}{\beta_1 O_t}\right]^{1/n_1} \tag{13}$$

If hillslope fluvial erosion depends on powers of slope and discharge equal to those in channels, then the scaling coefficient for the hillslopes and the channels should be the same.

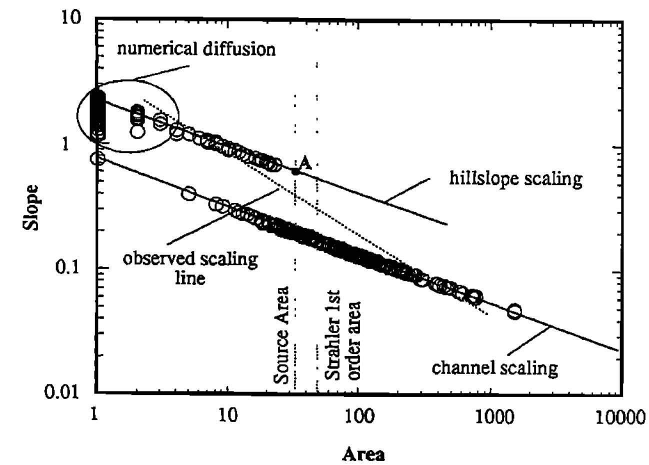


Fig. 3. Relationship between slopes and areas, at dynamic equilibrium, for both channels and hillslopes for the simulation dominated by fluvial sediment transport in Figure 1.

The only difference will be the value of the constant on the right-hand side of (6) and (13).

Figure 3 shows the slopes and areas for the whole catchment, when at dynamic equilibrium, from the same simulation run used in the study of channel behavior. This simulation uses the same fluvial erosion dependence on slope and discharge for channels and hillslopes. For comparison, two common measures of hillslope scale are indicated in this plot. The first is the mean Strahler first-order area. The second is the mean source area. As predicted by (6) and (13), the hillslope line falls higher than the channel line. That is, for the same area the slopes are higher for hillslopes than channels. This difference between channel and hillslope slopes is required for sediment transport continuity at the boundary between the two units. The predicted ratio between the slopes, on the hillslopes, and in the channel for the same contributing area is

$$\frac{S_c}{S_h} = O_t^{1/n_1} \tag{14}$$

where S_h and S_c are the slopes on the hillslopes, and in the channel, respectively, at the channel head, noting that at the channel head the width of the hillslope flow is equal to the width of the channel head. For the simulation studied (O_t = 0.1, n_1 = 2.1, w = 1) the expression yields S_c/S_h = 0.33, which is corroborated by Figure 3.

This sharp differentiation between the slopes of the hillslopes and channels is not as pronounced when the catchment is far from dynamic equilibrium. Figure 4 shows the catchment of Figure 3 at earlier times, when the network is still growing. For pixels or points with contributing areas greater than 10 the sediment transport capacity is large and hence elevations proceed to equilibrium quickly; the differentiation of slopes between the two regimes is clear at early times. For pixels or points with smaller contributing areas, the sediment transport capacity is lower, and elevations take longer to attain equilibrium. In Figure 4 there has been insufficient time to develop differentiation of slopes between the two regimes.

Patton and Schumm [1975] examined the slopes and area contributing to a point in the hillslopes and channels. They

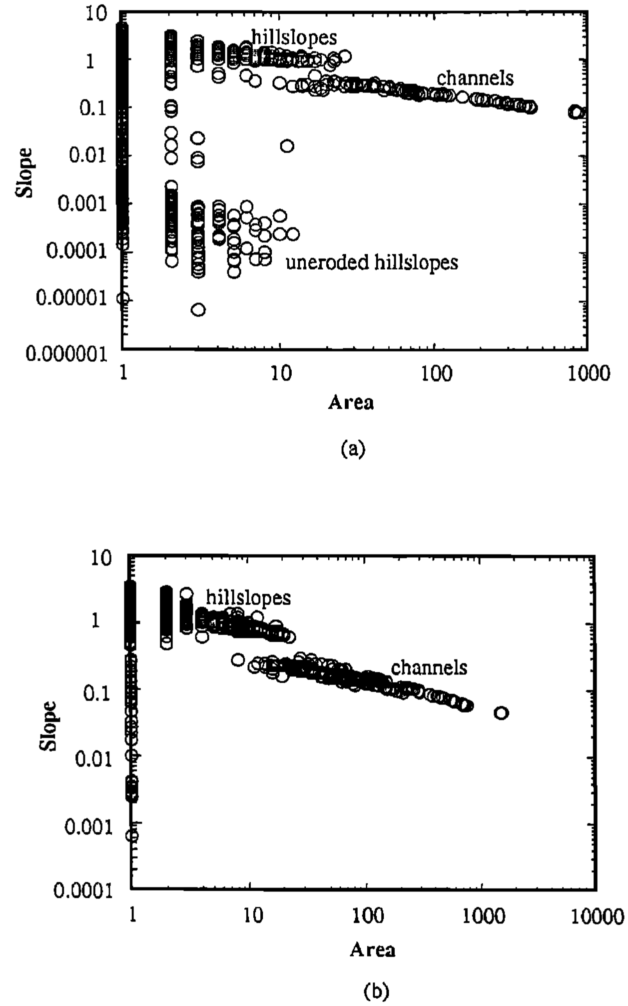


Fig. 4. Relationship between slopes and areas for early stages of catchment development. Nondimensional times of (*a*) 6000, and (*b*) 12,000, with dynamic equilibrium (Figure 3) occurring at 53,000.

found that the data for channels and hillslopes were significantly different at the 1% level, supporting the differentiation between the slopes of the hillslopes and the channels that results from the different sediment transport physics on the hillslopes and in the channels, as parameterized here by O_t and demonstrated in Figure 3.

The sharp difference in the slope-area relationship (i.e., a shift in level) of hillslopes and channels at dynamic equilibrium (Figure 3) was not observed by *Tarboton et al.* [1989*a*, *b*]. Figure 3, however, represents the ideal homogeneous catchment, and there are many reasons why the differentiation in slopes may not be observed. First, both *Tarboton et al.* [1989*a*, *b*] and *Flint* [1974] observed scatter in the plotted slopes about the trend of up to half an order of magnitude. In addition, before dynamic equilibrium the differentiation in slopes is not sharp (Figure 4). Both of these effects would tend to hide any differences between the slopes of the hillslopes and the channels. Furthermore, the model is perfectly deterministic so that the distinction between the hillslopes and channel is perfect. In reality the catchment properties (e.g., runoff, erodability) exhibit considerable variability in space so that the distinction between hillslope to channel will rarely be so clear-cut. Moreover, the sediment transport equations in the model represent mean temporal equations and do not account for stochastic advance and retreat of channel heads [*Calver*, 1978], which will tend to average the sediment transport process near the channel head. All of the above argue for hillslope and channel lines like in Figure 3 but with blurred demarcation between hillslope and channel. If the slopes are averaged for a given area, then the mean line will fall through the channel line at high areas and the hillslope line at small areas. A resulting average relationship is drawn in Figure 3 and is labeled the observed scaling relationship line. The slope of this line, −0.5, to −0.6, is consistent with the slopes calculated by *Tarboton et al.* [1989*a*].

Tarboton et al. [1989*b*] also noted a deviation from the power law trend of slope for small areas. If their studied networks had a high drainage density, they found that with decreasing area, mean slopes reached a maximum value and then slopes leveled off or decreased for further decreases in area. This deviation is not totally unexpected. Equation (1) predicts infinite slopes at the watershed where the contributing area is zero. This is clearly unrealistic and indicates that other physical mechanisms must dominate hillslope form for small areas. It has been previously proposed that near the watershed, where areas are small, the diffusion term of (2) dominates the fluvial term [*Gilbert*, 1909]. Using the full governing equation (2), (4) can be generalized to

$$c_0 A = \frac{w\beta_1 O_t q^{m_1} S^{n_1}}{\rho_s(1-n)} + D_z S w \tag{15}$$

and substituting the discharge-area relationship,

$$c_0 A = \frac{\beta_1 w^{1-m_1} O_t \beta_3^{m_1} A^{m_1 m_3} S^{n_1}}{\rho_s(1-n)} + D_z S w \tag{16}$$

As area diminishes, the second, diffusive term becomes more dominant, and the equation can be approximated by

$$c_0 A = D_z S w \tag{17}$$

or

$$A^{-1} S = \frac{c_0}{D_z w} \tag{18}$$

so that the estimated scaling coefficient α_p for diffusion-dominated hillslopes is (Figure 5) $\alpha_p = -1$. The slopes for areas intermediate between the two regimes are indicated in Figure 5 and are calculated by solving (16) (for $w = 1$). Further catchment simulations were performed to confirm this prediction. The simulations were diffusion dominated in the hillslopes and fluvial transport dominated in the channels. The areas and slopes near dynamic equilibrium and the calculated renormalization line are shown in Figure 6. Note that all slopes fall below the theoretical lines because of interactions between the fluvial sediment transport and the diffusive transport terms.

The problem statement of (15) can be generalized with two (or more) different sediment transport mechanisms so that

$$c_0 A = \beta_1 Q^{m_1} S^{n_1} + \beta_1' Q^{m_1'} S^{n_1'} \tag{19}$$

The main practical problem is to determine the relative magnitudes of the two processes β_1 and β_1'. A useful variable parameterizing the relative importance of diffusion and fluvial sediment transport is the area A_t at which the theoretical scaling relationships of diffusive and fluvial transport intersect. The value of this area is

$$A_t = \left\{ \left[\frac{1}{\beta_3} \right]^{(m_1 n_1' - m_1' n_1)} c_0^{(n_1' - n_1)} \cdot \left[\frac{(\beta_1')^{n_1}}{\beta_1^{n_1'}} \right] \right\}^{[m_3(m_1 n_1' - m_1' n_1) - (n_1' - n_1)]^{-1}} \tag{20}$$

The scale parameter that controls the transition area A_t is

$$\overline{\beta}_1 = \left[\frac{\beta_1'^{n_1}}{\beta_1^{n_1'}} \right]^{m_3(m_1 n_1' - m_1' n_1) - (n_1' - n_1)]^{-1}} \tag{21}$$

For large $\overline{\beta}_1$ the contributing area at which the transition from diffusion to fluvial regimes occurs is large. For small $\overline{\beta}_1$ the area where transition occurs is small. Thus the break in slope in the scaling relationship potentially identifies the scale at which one transport process dominates another. *Tarboton et al.* [1989*b*] argue that this type of behavior can be used to define the scale of hillslopes.

An interesting point is that fluvial processes with identical values for the predicted scaling coefficient α_p but different values of m_1 and n_1 (equation (6)) will yield slope-area relationships on a log-log plot that are indistinguishable, other than possible differences in the constant on the right-hand side. However, in combination with a second process, as exemplified in (19), the two fluvial mechanisms will result in different transitions, from the channel slope-area relationship to that of the diffusion-dominated hillslope. This is also shown in Figure 5. If the scatter observed by *Flint* [1974] and *Tarboton et al.* [1989*a*, *b*] is typical, however, any differences in the transition, due to differences in m_1 and n_1, are unlikely to be distinguishable in the field data.

The slope-area relationship may also be important in the interpretation of the slope and area correlations observed at channel heads [*Montgomery and Dietrich*, 1989]. Their relationship between area and slope at the channel head is fitted very well by a relationship of the form of equation (1) [*Willgoose et al.*, 1990]. *Willgoose et al.* [this issue (*a*)] believe that their data supports the concept of a channel initiation threshold. However, Montgomery and Dietrich may have simply measured the slope-area relationship resulting from the sediment transport equilibrium discussed above. Only a digital elevation analysis of the region examined by Montgomery and Dietrich, using the techniques of *Tarboton et al.* [1989*b*], will categorically answer this question.

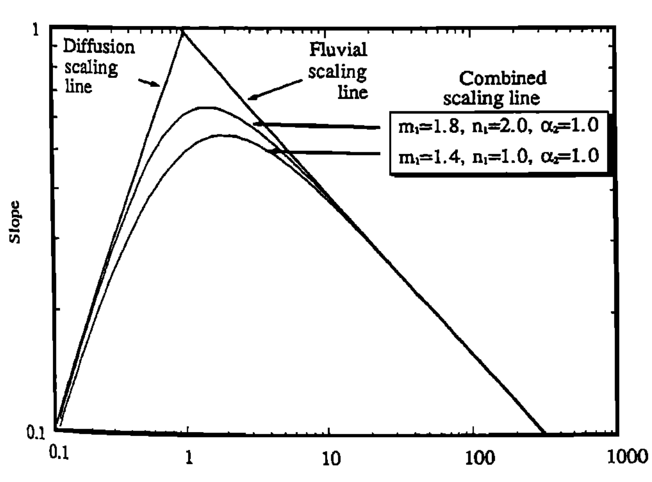
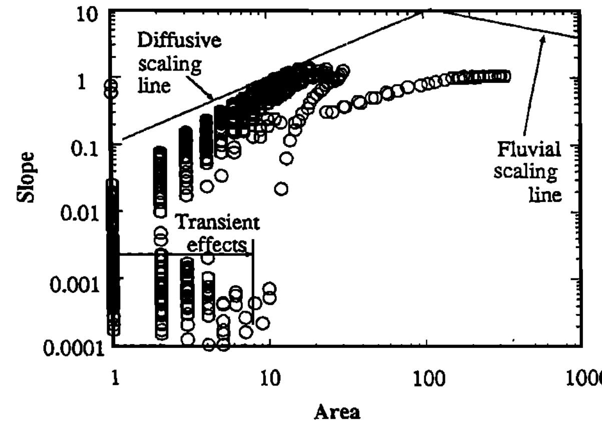
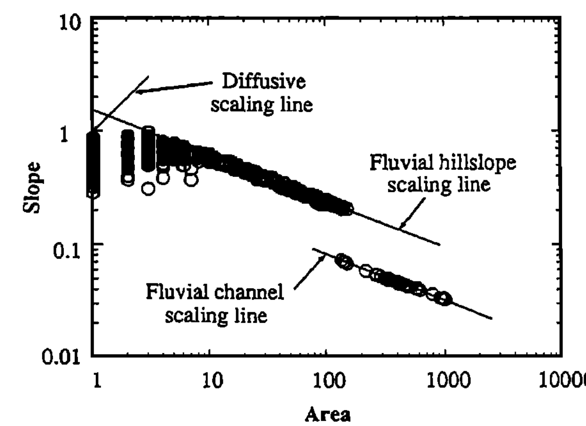


Fig. 5. Diffusive sediment transport processes and their influence on the area-slope relationship near the catchment watershed.



(a)



(b)

Fig. 6. Slope-area relationships for two simulations using different rates of diffusive transport showing the interaction between fluvial sediment transport and diffusive transport. (*a*) Higher diffusion dominance and (*b*) lower diffusion dominance.

CONCLUSIONS

An explanation is given for many of the features of the observed relationship between catchment area and channel and hillslope slope. The explanation is based on the assumption of dynamic equilibrium between tectonic uplift and erosion processes. Since a number of assumptions need to be made regarding the relationship between discharge and area and about the form of the sediment transport function, this explanation cannot be totally verified at this stage. The comparison with field data in this paper was incomplete since flood frequency data and sediment transport relations were not available to check the proposed theoretical relation between slope and area. To completely verify the theory we need simultaneous measurement of areas, slopes, sediment transport, and runoff relations in the same catchment. This type of comprehensive measurements of catchment properties is not presently available.

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