№ 13

$A = A^T > 0$

$\tau > 0$

$$\frac{x^{k+1} - x^k}{\tau} + A\,\frac{x^{k+1} + x^k}{2} = f$$

$$\frac{1}{\tau} \cdot x^{k+1} - \frac{1}{\tau} x^k + \frac{1}{2} \cdot A x^{k+1} + \frac{1}{2} \cdot A x^k = f$$

$$\frac{1}{\tau} \cdot x^{k+1} + \frac{1}{2} \cdot A x^{k+1} + \frac{1}{2} A \cdot x^k - \frac{1}{\tau} x^k = f$$

$$\frac{1}{\tau} \cdot E \cdot x^{k+1} + \frac{1}{2} A x^{k+1} + \frac{1}{2} A \cdot x^k - \frac{1}{\tau} E x^k = f$$

$$\left(\frac{1}{\tau} \cdot E + \frac{1}{2} A\right) x^{k+1} + \left(\frac{1}{2} A - \frac{1}{\tau} E\right) x^k = f$$

$$\left(\frac{1}{\tau}\cdot E+\frac{1}{2}A\right)x^{k+1}-\left(\frac{1}{\tau}E-\frac{1}{2}A\right)x^k=f$$

$$\left(\frac{1}{\tau}\cdot E+\frac{1}{2}A\right)x^{k+1}-\left(\left(\frac{1}{\tau}\cdot E+\frac{1}{2}A\right)-\frac{2}{2}A\right)x^k=f$$

$$\left(\frac{1}{\tau}\cdot E+\frac{1}{2}A\right)\left(x^{k+1}-x^k\right)+Ax^k=f$$

$$\tau\cdot\left(\frac{1}{\tau}\cdot E+\frac{1}{2}A\right)\cdot\frac{\left(x^{k+1}-x^k\right)}{\tau}+Ax^k=f$$

$$\boxed{\left(E+\frac{\tau}{2}A\right)\cdot\frac{x^{k+1}-x^k}{\tau}+Ax^k=f}$$

$\widetilde{B}=E+\frac{\tau}{2}A$

$\widetilde{A}=A$

по теор. Самарского:

$\tau>0$ V (по усл)

$A=A^T>0$ V (по усл)

$A>0$ V

$B>\frac{1}{2}\tau A$?

$B>\frac{1}{2}\tau A \Leftrightarrow B-\frac{1}{2}\tau A>0$

$E+\frac{\tau}{2}A-\frac{\tau}{2}A>0$

$E>0$

$E>0$ по кр. Сильвестра $\Rightarrow B>\frac{1}{2}\tau A \Rightarrow$

$\Rightarrow$ сх-ть $\forall\tau>0$