Лемма: $f(x_1 \dots x_n) = \sum\limits_{i=1}^{n} f_i \prod\limits_{\substack{j=1 \\ i\neq j}}^{n} \frac{1}{x_i - x_j}$

▷ Пусть $f(x_1 \dots x_n) = \sum\limits_{i=1}^{n} f_i \prod\limits_{\substack{j=1 \\ i\neq j}}^{n} \frac{1}{x_i - x_j}$

Д-ть: $f(x_1 \dots x_{n+1}) = \sum\limits_{i=1}^{n+1} f_i \prod\limits_{\substack{j=1 \\ i\neq j}}^{n+1} \frac{1}{x_i - x_j}$

По индукции!

База: $n=2$ - доказали на семинаре

$n \Rightarrow n+1$: $f(x_1 \dots x_{n+1}) \overset{\text{опр}}{=} \frac{f(x_2 \dots x_{n+1}) - f(x_1 \dots x_n)}{x_{n+1} - x_1} \overset{\text{И.П.}}{=}$

$$= \frac{\left[\sum\limits_{i=2}^{n+1} f_i \prod\limits_{i\neq j;\, j=2}^{n+1} \frac{1}{x_i - x_j}\right] - \left[\sum\limits_{i=1}^{n} f_i \prod\limits_{j=1;\, i\neq j}^{n} \frac{1}{x_i - x_j}\right]}{x_{n+1} - x_1} =$$

$$= \frac{f_{n+1} \cdot \prod\limits_{j=2}^{n} \frac{1}{x_{n+1} - x_j} + \sum\limits_{i=2}^{n} f_i \prod\limits_{\substack{i\neq j \\ j=2}}^{n+1} \frac{1}{x_i - x_j} - \sum\limits_{i=2}^{n} f_i \prod\limits_{j=1;\, i\neq j}^{n} \frac{1}{x_i - x_j} - f_1 \prod\limits_{j=2}^{n} \frac{1}{x_1 - x_j}}{x_{n+1} - x_1} =$$

$$= f_{n+1} \cdot \prod\limits_{j=1}^{n} \frac{1}{x_{n+1} - x_j} + f_1 \cdot \prod\limits_{j=2}^{n+1} \frac{1}{x_1 - x_j} + \frac{1}{x_{n+1} - x_1} \cdot \left[\sum\limits_{i=2}^{n} f_i \left(\prod\limits_{\substack{i\neq j \\ j=2}}^{n} \frac{1}{x_i - x_j}\right) \cdot\right.$$

$$\cdot \frac{1}{x_i - x_{n+1}} - \sum_{i=2}^{n} f_i \left(\prod_{\substack{i \neq j \\ j=2}}^{n} \frac{1}{x_i - x_j} \right) \cdot \frac{1}{x_i - x_1} \Bigg] =$$

$$= f_{n+1} -//- + f_1 -//- + \frac{1}{x_{n+1} - x_1} \Bigg[\sum_{i=2}^{n} f_i \prod_{\substack{i \neq j \\ j=2}}^{n} \frac{1}{x_i - x_j} \cdot \frac{1}{x_i - x_{n+1}} -$$

$$+ \sum_{i=2}^{n} f_i \prod_{\substack{i \neq j \\ j=2}}^{n} \frac{1}{x_i - x_j} \cdot \frac{1}{x_1 - x_i} \Bigg] =$$

$$= -//- + \frac{1}{x_{n+1} - x_1} \Bigg[\sum_{i=2}^{n} f_i \Bigg[\prod_{\substack{i \neq j \\ j=2}}^{n} \frac{1}{x_i - x_j} \Bigg] \cdot \left(\frac{1}{x_i - x_{n+1}} + \frac{1}{x_1 - x_i} \right) \Bigg] =$$

$$= -//- + \frac{1}{x_{n+1} - x_1} \Bigg[\sum -//- \cdot \left(\frac{x_1 - x_i + x_i - x_{n+1}}{(x_i - x_{n+1})(x_1 - x_i)} \right) \Bigg] =$$

$$= -//- + \frac{1}{x_{n+1} - x_1} \Bigg[\sum -//- \cdot \left(\frac{x_1 - x_{n+1}}{(x_i - x_{n+1})(x_1 - x_i)} \right) \Bigg] =$$

$$= -//- + \frac{1}{x_{n+1} - x_1} \Bigg[\sum -//- \cdot \left(\frac{x_{n+1} - x_1}{(x_i - x_{n+1})(x_i - x_1)} \right) \Bigg] =$$

$$= -//- + \Bigg[\sum -//- \cdot \frac{1}{(x_i - x_{n+1})(x_i - x_1)} \Bigg] =$$

$$= -//- + \sum_{i=2}^{n} f_i \prod_{\substack{i \neq j \\ j=2}}^{n} \frac{1}{x_i - x_j} \cdot \frac{1}{x_i - x_{n+1}} \cdot \frac{1}{x_i - x_1} =$$

$$= -//- + \sum_{i=2}^{n} f_i \prod_{\substack{j=1 \\ i \neq j}}^{n+1} \frac{1}{x_i - x_j} \Rightarrow$$

$$= f_1 \cdot \prod_{j=2}^{n+1} \frac{1}{x_1 - x_j} + \sum_{i=2}^{n} f_i \prod_{\substack{j=1 \\ j \neq i}}^{n+1} \frac{1}{x_i - x_j} + f_{n+1} \prod_{j=1}^{n} \frac{1}{x_{n+1} - x_j} =$$

$$= \sum_{i=1}^{n+1} f_i \prod_{\substack{j=1 \\ j \neq i}}^{n+1} \frac{1}{x_i - x_j}$$

Ч. Т. Д. (Ура, наконец-то)