

⊕ B) 1) Вычислить $\|A\|_1$ и $\|A\|_\infty$:

$$\begin{pmatrix} 2 & 7 & -1 & 5 \\ 0 & 3 & 2 & 1 \\ -1 & -2 & -3 & 7 \\ 16 & 5 & 2 & 24 \end{pmatrix}$$

$$\|A\|_1 = \max_j \sum_i |a_{ij}| = \max(19, 17, 8, 37) = 37$$

$$\|A\|_\infty = \max_i \sum_j |a_{ij}| = \max(15, 6, 13, 47) = 47$$

$$2+0+1+16=19$$

$$2+7+1+5=15$$

$$2+3+2+5=12$$

$$0+3+2+1=6$$

$$1+2+3+2=8$$

$$1+2+3+7=13$$

$$5+1+7+24=37$$

$$16+5+2+24=47$$

$$A > 0 \Leftrightarrow A + A^T > 0$$

Докажем

$$A > 0, \text{ если } \forall x: (Ax, x) > 0$$

$$\Rightarrow \nexists ((A + A^T)x, x):$$

$$\forall x: ((A + A^T)x, x) \stackrel{(A+B)x = Ax+Bx}{=} (Ax + A^T x, x) \stackrel{(x,y,z) \rightarrow (x,z)+(y,z)}{=} (Ax, x) + (A^T x, x)$$

$$= (Ax, x) + (A^T x, x) \stackrel{(x,y)=(y,x)}{=} (Ax, x) + (x, A^T x) =$$

$$\stackrel{(Ax,y)=(x,A^T y)}{=} \underbrace{(Ax, x)}_{>0} + \underbrace{(x, A^T x)}_{>0} > 0 \quad \text{чтд}$$

$$\Leftarrow A + A^T > 0, \text{ значит, не оп!}$$

$$((A + A^T)x, x) > 0$$

$$(Ax + A^T x, x) = (Ax, x) + (A^T x, x) = (Ax, x) + (x, A^T x) =$$

$$= (Ax, x) + (Av, x) = 2 \cdot (Ax, x) > 0 \Rightarrow (Ax, x) > 0 \quad \text{4TA}$$

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 4 \\ 6x_1 + x_2 + 2x_3 + x_4 = 10 \\ 2x_1 + 3x_2 + x_3 + 6x_4 = 12 \\ x_1 + x_2 + 4x_3 + x_4 = 7 \end{cases} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 6 & 1 & 2 & 1 & 10 \\ 2 & 3 & 1 & 6 & 12 \\ 1 & 1 & 4 & 1 & 7 \end{pmatrix} \begin{matrix} \\ \cdot 6 \\ \\ \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & 5 & 4 & 5 & 14 \\ 0 & -1 & 1 & -4 & -4 \\ 0 & 0 & -3 & 0 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4/5 & 1 & 14/5 \\ 0 & -1 & 1 & -4 & -4 \\ 0 & 0 & -3 & 0 & -3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4/5 & 1 & 14/5 \\ 0 & 0 & 9/5 & -5 & -34/5 \\ 0 & 0 & -3 & 0 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4/5 & 1 & 14/5 \\ 0 & 0 & 1 & +25/9 & +34/9 \\ 0 & 0 & -3 & 0 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4/5 & 1 & 14/5 \\ 0 & 0 & 1 & +25/9 & +34/9 \\ 0 & 0 & 0 & -25/3 & -25/3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4/5 & 1 & 14/5 \\ 0 & 0 & 1 & 25/9 & 34/9 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \Rightarrow \begin{cases} x_1 + x_2 + x_3 + x_4 = 4 \\ x_2 + \frac{4}{5}x_3 + x_4 = \frac{14}{5} \\ x_3 + \frac{25}{9}x_4 = \frac{34}{9} \\ x_4 = 1 \end{cases} \quad \begin{cases} x_1 = 1 \\ x_2 = \frac{14}{5} - \frac{4}{5} - 1 = 1 \\ x_3 = \frac{34}{9} - \frac{25}{9} = \frac{8}{9} \\ x_4 = 1 \end{cases}$$

$$x = \begin{pmatrix} 1 \\ 1 \\ \frac{8}{9} \\ 1 \end{pmatrix}$$

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$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 6 & 1 & 2 & 1 & 10 \\ 2 & 3 & 1 & 6 & 12 \\ 1 & 1 & 1 & 1 & 7 \end{array} \right) \sim \left(\begin{array}{cccc|c} 6 & 1 & 2 & 1 & 10 \\ 1 & 1 & 1 & 1 & 4 \\ 2 & 3 & 1 & 6 & 12 \\ 1 & 1 & 1 & 1 & 7 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 1 & 1 & 1 & 1 & 4 \\ 2 & 3 & 1 & 6 & 12 \\ 1 & 1 & 1 & 1 & 7 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 0 & -5/6 & -4/6 & -5/6 & -14/6 \\ 0 & -16/6 & -2/6 & -34/6 & -52/6 \\ 0 & -5/6 & -22/6 & -5/6 & -32/6 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 0 & -16/6 & -2/6 & -34/6 & -52/6 \\ 0 & -5/6 & -4/6 & -5/6 & -14/6 \\ 0 & -5/6 & -22/6 & -5/6 & -32/6 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 0 & 1 & 1/8 & 17/8 & 26/8 \\ 0 & -5 & -4 & -5 & -14 \\ 0 & -5 & -22 & -5 & -32 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 0 & 1 & 1/8 & 17/8 & 26/8 \\ 0 & 0 & 27/8 & -45/8 & -18/8 \\ 0 & 0 & 171/8 & -45/8 & 126/8 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 0 & 1 & 1/8 & 17/8 & 26/8 \\ 0 & 0 & 171/8 & -45/8 & 126/8 \\ 0 & 0 & 27/8 & -45/8 & -18/8 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 0 & 1 & 1/8 & 17/8 & 26/8 \\ 0 & 0 & 1 & -45/171 & 126/171 \\ 0 & 0 & 27 & -45 & -18 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 0 & 1 & 1/8 & 17/8 & 26/8 \\ 0 & 0 & 1 & -45/171 & 126/171 \\ 0 & 0 & 0 & 120/19 & 720/19 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1/6 & 2/6 & 1/6 & 10/6 \\ 0 & 1 & 1/8 & 17/8 & 26/8 \\ 0 & 0 & 1 & -45/171 & 126/171 \\ 0 & 0 & 0 & 1 & 420/19 \end{array} \right)$$

$$\left\{ \begin{array}{l} x_1 + \frac{1}{6}x_2 + \frac{2}{6}x_3 + \frac{1}{6}x_4 = \frac{10}{6} \\ x_2 + \frac{1}{8}x_3 + \frac{17}{8}x_4 = \frac{26}{8} \\ x_3 + \frac{-45}{171}x_4 = \frac{126}{171} \\ x_4 = 1 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} x_1 = \frac{10}{6} - \frac{1}{6} - \frac{2}{6} - \frac{1}{6} = \frac{6}{6} = 1 \\ x_2 = \frac{26}{8} - \frac{1}{8} - \frac{17}{8} = \frac{8}{8} = 1 \\ x_3 = \frac{126}{171} + \frac{45}{171} = \frac{171}{171} = 1 \\ x_4 = 1 \end{array} \right. \Rightarrow x = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$