$$\begin{cases} y_0 = 1 \\ \dfrac{y_{j+1} - y_j}{h} - y_j = 0 ; j = 0,1,2 \\ \dfrac{y_{j+1} - y_j}{h} - \dfrac{y_{j+1} + y_j}{2} = 0 ; j \geq 3 \end{cases}$$

> $y_0 = 1$

> $\dfrac{y_1 - y_0}{h} - y_0 = 0 \Rightarrow y_1 - y_0 - h y_0 = 0$

$y_1 = (1+h) y_0 \Rightarrow \boxed{y_{j+1} = (1+h) y_j ; j = 0,1,2.}$

> $\dfrac{y_{j+1} - y_j}{h} - \dfrac{y_{j+1} + y_j}{2} = 0$

$2 y_{j+1} - 2 y_j - h y_{j+1} - h y_j = 0$

$$y_{j+1}\cdot(2-h)-y_j(2+h)=0$$

$$y_{j+1}\cdot(2-h)=y_j(2+h)$$

$$y_{j+1}=\frac{2+h}{2-h}\,y_j\ ;\ j\geq 3$$

$$y_0=1$$

$$y_1=(1+h)y_0$$

$$y_2=(1+h)y_1=(1+h)^2y_0$$

$$y_3=(1+h)y_2=(1+h)^3y_0$$

$$y_4=\frac{2+h}{2-h}y_3=\frac{2+h}{2-h}\cdot(1+h)^3y_0$$

$$y_5=\frac{2+h}{2-h}y_4=\frac{(2+h)^2}{(2-h)^2}(1+h)^3\cdot y_0$$

$$y_j=\frac{(2+h)^{j-3}}{(2-h)^{j-3}}(1+h)^3y_0=\frac{\left(1+\frac{h}{2}\right)^{j-3}}{\left(1-\frac{h}{2}\right)^{j-3}}(1+h)^3y_0$$

$$y_j=\frac{\left(1+\frac{h}{2}\right)^{j-3}}{\left(1-\frac{h}{2}\right)^{j-3}}(1+h)^3y_0$$

$$y_j=\left(\frac{1+\frac{h}{2}}{1-\frac{h}{2}}\right)^{j-3}(1+h)^3y_0\overset{y_0=1}{=}\left(\frac{1+\frac{h}{2}}{1-\frac{h}{2}}\right)^{j-3}(1+h)^3$$

$$y_j=\left(\frac{1+\frac{h}{2}}{1-\frac{h}{2}}\right)^{\left[\frac{x_j}{h}-3\right]}=(*)\quad\cdot(1+h)^3$$

$$(*)=e^{\left(\frac{x_j}{h}-3\right)\left(\ln\left(1+\frac{h}{2}\right)-\ln\left(1-\frac{h}{2}\right)\right)}=$$

$$= \exp\left(\left(\frac{x}{h} - 3\right) \cdot \left(h + \frac{h^3}{12} + \frac{h^5}{60} + O(h^7)\right)\right) =$$

$$= \exp\left(x + \frac{xh^2}{12} + \frac{xh^4}{60} + O(h^6) - 3h - \frac{3h^3}{12} - \frac{3h^5}{60} + O(h^7)\right) =$$

$$= \exp\left(x - 3h + \frac{xh^2}{12} - \frac{3h^3}{12} + \frac{xh^4}{60} - \frac{3h^5}{60} + O(h^6)\right) =$$

$$= \exp(x) \cdot \exp\left(-3h + \frac{xh^2}{12} - \frac{h^3}{4} + \frac{xh^4}{60} - \frac{h^5}{20} + O(h^6)\right) =$$

$$= \exp(x) \cdot \left(1 - 3h + \frac{xh^2}{12} - \frac{h^3}{4} + \frac{xh^4}{60} - \frac{h^5}{20} + O(h^6) +\right.$$

$$\left. + \frac{9h^2}{2} + \frac{x^2h^4}{24} + \ldots\right) \approx$$

$$\approx \exp(x) \cdot \left(1 - 3h + \frac{xh^2}{12} + O(h^3) + \frac{9h^2}{2} + O(h^4)\right) =$$

$$= \exp(x) \cdot \left(1 - 3h + \frac{h^2}{12}(x + 54) + O(h^3)\right)$$

$$y_j = \exp(x) \cdot \left(1 - 3h + \frac{h^2}{12} \cdot (x + 54) + O(h^3)\right) \cdot (1 + h)^3 =$$

$$= \exp(x) \cdot \left(1 - 3h + \frac{h^2}{12} \cdot (x + 54) + O(h^3)\right) \cdot (1 + 3h + 3h^2 + h^3) =$$

$$= \exp(x) \cdot \left(1 - \frac{18}{12}h^2 + \frac{x}{12}h^2 + \frac{x}{12}h^5 + \frac{54}{12}h^5 + \frac{22}{4}h^3 + \frac{42}{4}h^4 + \frac{x}{4}h^3 + \frac{x}{4}h^4\right.$$

$$\left. + O(h^3)\right) \approx \exp(x) \cdot \left(1 + h^2 \cdot \frac{1}{12}(x - 18) + O(h^3)\right) \approx$$

$$\approx e^x + e^x \cdot \frac{1}{12}(x - 18) \cdot h^2 + O(h^3)$$

$\uparrow$

2-й порядок

1) $y_{j+4} + 2y_{j+3} - 3y_{j+2} - 4y_{j+1} + 4y_j = 0$

$x^4 + 2x^3 - 3x^2 - 4x + 4 = 0$

	1	2	-3	-4	4
1	1	3	0	-4	0

$x_1 = 1$

$x_2 = 1$

$x_3 = -2$

$x_4 = -2$

$x^3 + 3x^2 + 0x - 4 = 0$

$x^3 + 3x^2 - 4 = 0$

	1	3	0	-4
1	1	4	4	0

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$y_j = c_1 \cdot 1^j + c_2 \cdot j \cdot 1^j + c_3 \cdot (-2)^j + c_4 \cdot j \cdot (-2)^j$

$\cancel{3)\ y_{j+4} + 2y_{j+3} - 3y_{j+2} - 4y_j}$

2) $y_{j+4} + 2y_{j+2} + y_j = 0$

$x^4 + 0x^3 + 2x^2 + 0x + 1 = 0$

$x^4 + 2x^2 + 1 = 0 \qquad // \quad i^4 + 2i^2 + 1 = i^2 \cdot i^2 + 2i^2 + 1 =$

$x^4 + x^2 + x^2 + 1 = 0 \qquad // \quad = -1 \cdot (-1) - 2 + 1 = 1 - 2 + 1 = 0 \Rightarrow$

$x^2(x^2+1) + 1 \cdot (x^2+1) = 0 \qquad // \Rightarrow \quad (x-i)(x+i) = x^2 - i^2 = x^2 + 1$

$(x^2+1) \cdot (x^2+1) = 0 \qquad x_1 = -i \quad x_3 = i$

$(x-i)(x+i)(x-i)(x+i) = 0 \qquad x_2 = -i \quad x_4 = i$

$$\mu_1 = 1 \cdot e^{i \cdot \frac{\pi}{2}}$$

$$\mu_2 = 1 \cdot e^{-i \frac{\pi}{2}}$$

$$\hat{y}_j^0 = \frac{e^{i \frac{\pi}{2} j} + e^{-i \frac{\pi}{2} j}}{2} = \cos\left(j \frac{\pi}{2}\right)$$

$$y_j = c_1 \cdot \rho^j \cdot \cos\left(j \frac{\pi}{2}\right) + c_2 \cdot \rho^j \cdot j \cdot \cos\left(j \frac{\pi}{2}\right) + c_3 \cdot \rho^j \cdot \sin\left(\frac{\pi}{2} j\right) + c_4 \cdot \rho^j \cdot j \cdot \sin\left(\frac{\pi}{2} \cdot j\right)$$

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3) $y_{j+4} + 5y_{j+3} + 9y_{j+2} + 7y_{j+1} + 2y_j = 0$

$$x^4 + 5x^3 + 9x^2 + 7x + 2 = 0$$

	1	5	9	7	2
-1	1	4	5	2	0
-1	1	3	2	0	
-1	1	2	0		

$x^2 + 3x + 2 = 0$

$x + 2 = 0$

$x_1 = -1$

$x_2 = -1$

$x_3 = -1$

$x_4 = -2$

$$y_j = c_1 \cdot (-1)^j + c_2 \cdot j \cdot (-1)^j + c_3 \cdot j^2 \cdot (-1)^j + c_4 \cdot (-2)^j$$