Схема Лакса-Вендроффа!

$$u_t + a u_x = 0$$

$$\frac{u_j^{n+1} - u_j^n}{\tau} + a\frac{u_{j+1}^n - u_{j-1}^n}{2h} = \frac{\tau}{2}a^2\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2}$$

$$\frac{1}{\tau}\left[\lambda^1 - 1\right] + \frac{a}{2h}\left[e^{i\alpha} - e^{-i\alpha}\right] = \frac{\tau a^2}{h^2 2}\left[e^{i\alpha} - 2 + e^{-i\alpha}\right]$$

$$\lambda - 1 + \frac{\tau a}{2h}\left[e^{i\alpha} - e^{-i\alpha}\right] = \frac{\tau^2 a^2}{h^2 \cdot 2}\left[e^{i\alpha} - 2 + e^{-i\alpha}\right]$$

$$\lambda - 1 + \frac{1}{2} r\left[e^{i\alpha} - e^{-i\alpha}\right] = r^2 \cdot \frac{1}{2}\left[e^{i\alpha} - 2 + e^{-i\alpha}\right]$$

$$\lambda = 1 - \frac{1}{2} r\overbrace{\left[e^{i\alpha} - e^{-i\alpha}\right]}^{i \cdot 2\sin\alpha} + r^2 \cdot \frac{1}{2}\overbrace{\left[e^{i\alpha} - 2 + e^{-i\alpha}\right]}^{2\cos\alpha - 2}$$

$$\lambda = 1 - \frac{1}{2} r e^{i\alpha} + \frac{1}{2} r e^{-i\alpha} + \frac{1}{2} r^2 e^{i\alpha} - 2 \cdot \frac{1}{2} r^2 + \frac{1}{2} r^2 e^{-i\alpha}$$

$$\lambda = 1 - r^2 - \frac{1}{2} r e^{i\alpha} + \frac{1}{2} r^2 e^{i\alpha} + \frac{1}{2} r e^{-i\alpha} + \frac{1}{2} r^2 e^{-i\alpha}$$

$$\lambda = 1 - r^2 + \frac{1}{2} r e^{i\alpha}(r-1) + \frac{1}{2} r e^{-i\alpha}(1+r)$$

$$\lambda = 1 - r^2 + \frac{1}{2} r \cdot (r-1)(\cos\alpha + i\sin\alpha) + \frac{1}{2} r (1+r)(\cos\alpha - i\sin\alpha)$$

$$\lambda = 1 - r^2 + \frac{1}{2} r \cdot (r-1)\cos\alpha + \frac{1}{2} r(r-1) \cdot i\sin\alpha + \frac{1}{2} r \cdot (r+1)\cos\alpha - \frac{1}{2} r(r+1)\, i\sin\alpha$$

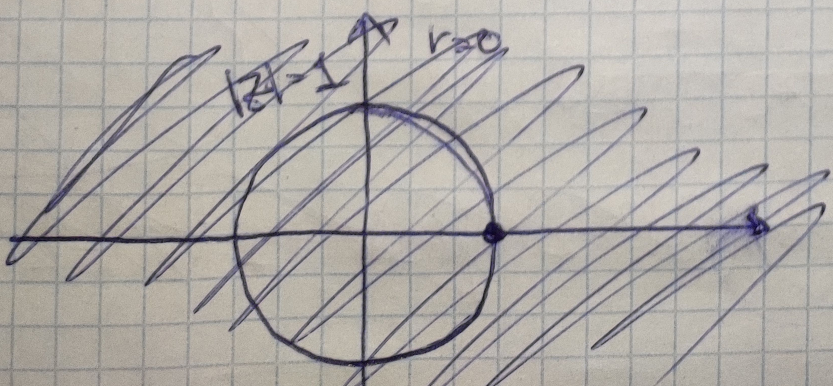
$$\lambda = 1 - r^2 + \frac{1}{2} r^2\cos\alpha - \frac{1}{2} r\cos\alpha + \frac{1}{2} r^2 \cdot i\sin\alpha - \frac{1}{2} r \cdot i \cdot \sin\alpha + \frac{1}{2} r^2 \cdot \cos\alpha + \frac{1}{2} r\cos\alpha - \frac{1}{2} r^2\, i\sin\alpha - \frac{1}{2} r \cdot i\sin\alpha$$

$$\lambda = 1 - r^2 + r^2\cos\alpha - r \cdot i\sin\alpha$$

$$\lambda = 1 + r^2(\cos\alpha - 1) - ir \cdot \sin\alpha$$

$$-1 \le \cos\alpha \le 1$$

$$-2 \le \cos\alpha - 1 \le 0$$



$$|\lambda|^2 = \left(1 + r^2(\cos\alpha - 1)\right)^2 + r^2\sin^2\alpha$$

$$\lambda = 1 + r^2(\cos\alpha - 1) - i \cdot r \cdot \sin\alpha$$

~~если $r \le 1$, то $|r \cdot \sin\alpha|$ ...~~

$-r \le r \cdot \sin\alpha \le r \Rightarrow$ если $|r| > 1$, то <u>неуст</u>

Доделка

$$|\lambda|^2 = [1 - r^2(1-\cos\alpha)]^2 + r^2\sin^2\alpha \le 1$$

$$1 - 2r^2(1-\cos\alpha) + r^4(1-\cos\alpha)^2 + r^2\sin^2\alpha \le 1$$

$$-2r^2(1-\cos\alpha) + r^4(1-\cos\alpha)^2 + r^2\sin^2\alpha \le 0 \quad \Big|: r^2 \ge 0$$

$$-2(1-\cos\alpha) + (1-\cos\alpha)^2\cdot r^2 + \sin^2\alpha \le 0$$

$$-2\left(2\sin^2\frac{\alpha}{2}\right) + \left(2\sin^2\frac{\alpha}{2}\right)^2\cdot r^2 + \sin\alpha\cdot\sin\alpha \le 0$$

$$-4\sin^2\frac{\alpha}{2} + 4\sin^4\frac{\alpha}{2}\, r^2 + \left(2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)^2 \le 0$$

$$-4\sin^2\frac{\alpha}{2} + 4\sin^4\frac{\alpha}{2}\, r^2 + 4\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2} \le 0$$

$$-4\sin^2\frac{\alpha}{2} + 4\sin^4\frac{\alpha}{2}\, r^2 + 4\sin^2\frac{\alpha}{2}\left(1-\sin^2\frac{\alpha}{2}\right) \le 0$$

$$\cancel{-4\sin^2\frac{\alpha}{2}} + 4\sin^4\frac{\alpha}{2}\, r^2 + \cancel{4\sin^2\frac{\alpha}{2}} - 4\sin^4\frac{\alpha}{2} \le 0$$

$$4\sin^4\frac{\alpha}{2}\, r^2 \le 4\sin^4\frac{\alpha}{2}$$

$$r^2 \le 1 \quad \Rightarrow |\lambda|^2 \le 1 \Leftrightarrow r^2 \le 1$$

$$\frac{u_j^{n+1} - u_j^n}{\tau} - a^2 \frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{h^2} = 0$$

$$\frac{1}{\tau}[\lambda - 1] - \frac{a^2}{h^2}[\lambda e^{i\alpha} - 2\lambda + \lambda e^{-i\alpha}] = 0$$

$$\lambda - 1 - \tau \frac{a^2}{h^2}[\lambda e^{i\alpha} - 2\lambda + \lambda e^{-i\alpha}] = 0$$

$$\lambda - 1 - r\lambda e^{i\alpha} - 2\lambda r + r\lambda e^{-i\alpha} = 0$$

$$\lambda - re^{i\alpha} \cdot \lambda - 2r \cdot \lambda + re^{-i\alpha} \cdot \lambda = 1$$

$$\lambda(1 - re^{i\alpha} - 2r + re^{-i\alpha}) = 1$$

$$\lambda = \frac{1}{1 - 2r + r(e^{-i\alpha} - e^{i\alpha})} = \frac{1}{1 - 2r + r(\cos\alpha - i\sin\alpha - \cos\alpha - i\sin\alpha)} =$$

$$= \frac{1}{1 - 2r + r\cdot(-2i\sin\alpha)} = \frac{1}{1 - 2r - 2ri\sin\alpha}$$

$$|\lambda|^2 = \frac{1}{\underbrace{1 - 4r + 4r^2}_{\geq 0} + \underbrace{4r^2}_{\geq 0} \cdot \underset{\in[0;1]}{\sin^2\alpha}} \leq \frac{1}{1 - 4r + 4r^2} = \frac{1}{(1-2r)^2}$$

$$(1-2r)^2 \geq 1 \Rightarrow \text{устойч.}$$

$$1 - 2r \geq 1; \quad 1 - 2r \leq -1$$

$$r \leq 0 \qquad -2r \leq -2$$

$$r \geq 1$$

$$r \in (-\infty; 0] \cup [1; +\infty) - \text{уст}$$

Если $1 - 2r < 1$, то при $\alpha = 0$!

$$\frac{1}{(1-2r)^2 + 4r^2 \cdot \sin^2 0} = \frac{1}{(1-2r)^2} > 1 \Rightarrow$$

$\Rightarrow$ неуст