Симметрия

$M = \{-1, 0, 1\}$

$k = 2$ - чёт

$n = 2$ - чёт

$$\begin{cases} a_0 + 2 \cdot a_1 = 0 \\ a_1 \cdot 1^2 = \frac{2!}{2} = 1 \end{cases}$$

$m = 0$

$n + k - 2 = 2$

$$\begin{cases} a_0 = -2a_1 = -2 \\ a_1 = 1 \end{cases}$$

$$a_{-1} = a_1$$

$$\Downarrow$$

$A_4 = (-1)^4 \cdot 1 + 0 + 1^4 \cdot 1 =$

$= 2$

Ответ:

$$\Lambda_h = \frac{1 \cdot y(x-h) - 2 \cdot y(x) + y(x+h)}{h^2}$$

$$\Delta_h = \frac{y^{(4)}(x) \cdot h^2 \cdot 2}{24} = \frac{y^{(4)}(x) \cdot h^2}{12}$$

$M = \{-3, -1, 0, 1, 3\}$

$k = 2$ - res

$n = 4$ - res

$n + k - 2 = 2 + 4 - 2 = 4$

$n - 2 = 2$

$$\Lambda_h = \frac{\frac{1}{6}y(x-3h) - \frac{3}{2}y(x-h) + \frac{8}{3}y(x) - \frac{3}{2}y(x+h) + \frac{1}{6}y(x+3h)}{h^4}$$

$$\Delta_h = \frac{y^{(6)}(x) \cdot h^2 \cdot 240}{6!} = \frac{y^{(6)}(x) \cdot h^2}{3}$$

$$\begin{cases} a_0 + 2 \cdot a_1 + 2 \cdot a_3 = 0 & m=0 \\ 1^2 a_1 + 3^2 a_3 = 0 & m=2 \\ 1^4 a_1 + 3^4 a_3 = \frac{4!}{2} = 12 \end{cases}$$

$$\begin{cases} a_0 + 2 \cdot a_1 + 2 \cdot a_3 = 0 \\ a_1 + 9 a_3 = 0 \\ a_1 + 81 a_3 = 12 \end{cases}$$

$$\begin{cases} a_0 = \frac{8}{3} \\ a_1 = -\frac{3}{2} \\ a_3 = \frac{1}{6} \end{cases}$$

$a_{-1} = a_1 = -\frac{3}{2}$

$a_{-3} = \frac{1}{6}$

$$A_6 = \frac{1}{6} \cdot (-3)^6 - \frac{3}{2} \cdot (-1)^6 + 0 - \frac{3}{2} (1)^6 + \frac{1}{6} 3^6 =$$

$= 240$

$M = \{-5, -3, -1, 1, 3, 5\}$

$k = 4, \; n = 3$
чет нечет

$n + k - 2 = 4 + 3 - 2 = 5$

$n - 2 = 1$

$$\Lambda_h = \frac{\frac{1}{64} y(x-5h) - \frac{13}{64} y(x-3h) + \frac{17}{32} y(x-h) - \frac{17}{32} y(x+h) + \frac{13}{64} y(x+3h) + \frac{-1}{64} y(x+5h)}{h^3}$$

$$\Delta_h = \frac{y^{(7)} \cdot h^4 \cdot (-1554)}{7!} = \frac{y^{(7)} \cdot h^4 \cdot 37}{120}$$

$$\begin{cases} a_1 \cdot 1^1 + a_3 \cdot 3^1 + a_5 \cdot 5^1 = 0 & m=1 \\ a_1 \cdot 1^3 + a_3 \cdot 3^3 + a_5 \cdot 5^3 = \frac{3!}{2} \\ a_1 \cdot 1^5 + a_3 \cdot 3^5 + a_5 \cdot 5^5 = 0 & m=5 \end{cases}$$

$$\begin{cases} a_1 \cdot 1 + a_3 \cdot 3 + a_5 \cdot 5 = 0 \\ a_3 \cdot 1 + a_3 \cdot 27 + a_5 \cdot 125 = \frac{3 \cdot 2}{2} = 3 \\ a_5 \cdot 1 + a_3 \cdot 243 + a_5 \cdot 3125 = 0 \end{cases}$$

$$\begin{cases} a_1 = -\frac{17}{32} \\ a_3 = \frac{13}{64} \\ a_5 = \frac{-1}{64} \end{cases} \qquad \begin{cases} a_{-1} = \frac{17}{32} \\ a_{-3} = -\frac{13}{64} \\ a_{-5} = \frac{1}{64} \end{cases}$$

$$A_7 = +\frac{1}{64}\cdot(-5)^7 - \frac{13}{64}\cdot(-3)^7 + \frac{17}{32}\cdot(-1)^7 - \frac{17}{32}\cdot 1^7 + \frac{13}{64}\cdot 3^7 - \frac{1}{64}\cdot 5^7 =$$

$$= -1554$$

$y'(x)-g(x)=0$

$n=1 \quad k=2$

$m=\{0,1,2\}$

$$\begin{cases} a_0\cdot 0^0+a_1\cdot 1^0+a_2\cdot 2^0=0 \\ a_0\cdot 0^1+a_1\cdot 1^1+a_2\cdot 2^1=1!=1 \\ a_0\cdot 0^2+a_1\cdot 1^2+a_2\cdot 2^2=0 \end{cases}$$

$$\begin{cases} a_0+a_1+a_2=0 \\ a_1+2a_2=1 \\ a_1+4a_2=0 \end{cases}$$

$$\begin{cases} a_0=-\frac{3}{2} \\ a_1=2 \\ a_3=-\frac{1}{2} \end{cases}$$

$$\Lambda_h=\left[-\frac{3}{2}y(x)+2y(x+h)-\frac{1}{2}y(x+2h)\right]-g(x)=0$$

$$\Delta_h=\frac{y^{(3)}(x)h^2}{6}\cdot\left(2\cdot 1^3+\left(-\frac{1}{2}\right)\cdot 2^3\right)=\frac{y^{(3)}(x)\cdot h^2}{6}\cdot(2-4)=$$

$$=\frac{-2y^{(3)}(x)\cdot h^2}{3}$$

k=2
симметр
разн.
схема

$$y'''(x) + y''(x) + y(x) = 0$$

для $y'''(x)$:

$\boxed{n=3}$ - нечет $\quad n+k-2=3$

$$|M_+|+1 = l+s$$

$$\left.\begin{array}{l} 2l-1=n \Rightarrow l=2 \\ 2s=k \Rightarrow s=1 \end{array}\right| \Rightarrow |M_+|=2$$

Подходит $\{-2;-1;0;1;2\}$, $a_0=0$

$$\begin{cases} a_1\cdot 1 + a_2\cdot 2 = 0 \\ a_1\cdot 1 + a_2\cdot 8 = 6/2 = 3 \end{cases}$$

$$\left.\begin{cases} a_1 = -1 \\ a_2 = 1/2 \end{cases}\right| \Rightarrow \begin{cases} a_{-2} = -1/2 \\ a_{-1} = 1 \\ a_0 = 0 \\ a_1 = -1 \\ a_2 = 1/2 \end{cases}$$

для $y''(x)$:

$\boxed{n=2}$ - чет $\quad n+k-2=2$

$$|M_+|+1 = \frac{n}{2}+\frac{k}{2} = 1+1 = 2 \Rightarrow |M_+|=1$$

Подходит $\{-1;0;1\}$

$$\begin{cases} a_0 + 2\cdot a_1 = 0 \\ a_1 \cancel{1^2} = \frac{2!}{2} = 2/2 = 1 \end{cases} \qquad \begin{cases} a_0 = 0 - 2\cdot a_1 = 0-2\cdot 1 = -2 \\ a_1 = 1 \end{cases}$$

$$\begin{cases} a_0 = -2 \\ a_1 = 1 \\ a_{-1} = 1 \end{cases}$$

$$\boxed{\Lambda_h = \frac{1}{h^3}\left[-\frac{1}{2}y(x-2h) + 1y(x-h) + 0 - 1y(x+h) + \frac{1}{2}y(x-2h)\right] + \frac{1}{h^2}\left[1y(x-h) - 2y(x) + 1y(x+h)\right] + y(x) = 0}$$

$$\Delta_h = \frac{y^{(5)}(x)\cdot h^2}{5!}\cdot\left(-\frac{1}{2}\cdot(-2)^5 + 1\cdot(-1)^5 + 0 - 1\cdot 1^5 + \frac{1}{2}\cdot 2^5\right) +$$

$$+ \frac{y^{(4)}(x)\cdot h^2}{4!}\left((-1)^4 + (-2)\cdot 0^4 + 1\cdot(1)^4\right) =$$

$$= \frac{y^{(5)}(x)\cdot h^2}{5!}\cdot 30 + \frac{y^{(4)}(x)\cdot h^2}{4!}\cdot 2 = \boxed{\frac{y^{(5)}(x)\cdot h^2}{4} + \frac{y^{(4)}(x)\cdot h^2}{12}}$$