$$y''(x) - g(x) = 0$$

$$\Omega_h = \frac{y(x) - 2y(x-h) + y(x-2h)}{h^2} - \frac{g(x-2h) + g(x)}{2} = 0$$

$$r_h = \frac{1}{h^2}\Big[ y - 2\Big(y - hy' + h^2 y'' \cdot \frac{1}{2!} - h^3 y''' \frac{1}{3!} + h^4 y^{IV} \cdot \frac{1}{4!} + O(h^5)\Big)$$

$$+ y - 2hy' + 4h^2 y'' \cdot \frac{1}{2!} - 8h^3 y''' \cdot \frac{1}{3!} + 16h^4 y^{IV} \cdot \frac{1}{4!} + O(h^5)\Big] -$$

$$- \cancel{\frac{g(x-2h)+g(x)}{2}} \; \frac{1}{2}\Big[ g - 2hg' + 4h^2 g'' \cdot \frac{1}{2!} - 8h^3 g''' \cdot \frac{1}{3!} + 16h^4 \cdot$$

$$\cdot g^{IV} \cdot \frac{1}{4!} + O(h^5) + g \Big] =$$

$$= \frac{1}{h^2}\Big[ \cancel{y} - \cancel{2y} + 2hy' - 2h^2 y'' \cdot \frac{1}{2!} + 2h^3 y''' \cdot \frac{1}{3!} - 2h^4 y^{IV} \cdot \frac{1}{4!} + O(h^5) + \cancel{y} -$$

$$- 2hy' + 4h^2 y'' \cdot \frac{1}{2!} - 8h^3 y''' \cdot \frac{1}{3!} + 16h^4 y^{IV} \cdot \frac{1}{4!} + O(h^5)\Big] -$$

$$- \frac{1}{2}\Big[ 2g - 2hg' + 4h^2 g'' \cdot \frac{1}{2!} - 8h^3 g''' \cdot \frac{1}{3!} + 16h^4 \cdot g^{IV} \cdot \frac{1}{4!} + O(h^5)\Big]$$

$$= \frac{1}{h^2}\Big[ 2 \cdot \cancel{\tfrac{1}{2}} h^2 y'' - 6 \cdot \frac{1}{3!} h^3 y''' + 14 h^4 y^{IV} \cdot \frac{1}{4!} + O(h^5)\Big] -$$

$$- g + hg' - 2h^2 g'' \cdot \frac{1}{2} + 4h^3 g''' \cdot \frac{1}{3!} - 8h^4 \cdot g^{IV} \cdot \frac{1}{4!} + O(h^5) =$$

$$= y'' - hy''' + \frac{7}{12} h^2 y^{IV} + O(h^3) - g + hg' - h^2 g'' + \frac{2}{3} h^3 g''' -$$

$$- \frac{1}{3} h^4 g^{IV} + O(h^5) = F[y] - h(y''' - g') + \frac{7}{12} h^2 y^{IV} + O(h^3) -$$

$$-h^2 g''(x) + \frac{2}{3} h^3 g''' - \frac{1}{3} h^4 g^{IV} + O(h^5) =$$

$$= 0 + 0 + \frac{7}{12} h^2 y^{IV} - h^2 g''(x) + O(h^3)$$

$$\Delta_h = h^2 \cdot \left[\frac{7}{12} y^{IV}(x) - g''(x)\right]$$ – 2-й порядок

$$\Lambda_h = \frac{y(x+h) - 2y(x) + y(x-h)}{h^2} - \frac{g(x+h) + g(x-h)}{2} =$$

$\boxed{\Lambda_h = 0}$

$$= \frac{1}{h^2}\Big[ y(x) + \cancel{hy'} + h^2 y''\cdot\frac{1}{2!} + \cancel{h^3 y'''}\cdot\frac{1}{3!} + h^4 y^{IV}\cdot\frac{1}{4!} +$$

$$+ O(h^5) - 2y + y - \cancel{hy'} + h^2 y''\cdot\frac{1}{2!} - \cancel{h^3 y'''}\cdot\frac{1}{3!} + h^4 y^{IV}\cdot\frac{1}{4!} +$$

$$+ O(h^5)\Big] - \frac{1}{2}\Big[ g + hg' + h^2 g''\cdot\frac{1}{2!} + h^3 g'''\cdot\frac{1}{3!} + O(h^4) +$$

$$+ g - \cancel{hg'} + h^2 g''\cdot\frac{1}{2} - h^3 g'''\cdot\frac{1}{3!} + O(h^4)\Big] =$$

$$= \frac{1}{h^2}\Big[2\cdot\frac{1}{2!}h^2 y'' + 2\cdot\frac{1}{4!}\cdot h^4 y^{IV} + O(h^5)\Big] - \frac{1}{2}\Big[2g + 2h^2 g''\cdot\frac{1}{2!}$$

на всякий

$$+ 2h^4 g^{IV}\cdot\frac{1}{4!} + O(h^5)\Big] =$$

$$= y'' + \frac{1}{2}h^2 y^{IV} + O(h^3) - g + \frac{1}{2}h^2 g'' + h^4 g^{IV}\cdot\frac{1}{4!} + O(h^5) =$$

$$= F\{y\} + \frac{1}{2}h^2\left(y^{IV} + g''\right) + O(h^3)$$

$\Delta_h = \frac{1}{2}h^2\left(y^{IV} + g''\right)$ – 2-й порядок $\Rightarrow$ совпадают

$y'(x) - g(x) = 0$

$K = 4$

$\{-1, 0, 1\}$

$$\frac{y(x+h) - y(x-h)}{2h} - \alpha g(x+h) - (1-2\alpha) g(x) - \alpha g(x-h) = 0$$

$$\frac{1}{2h}\Big[ y + hy' + h^2 y'' \cdot \frac{1}{2!} + h^3 y''' \cdot \frac{1}{3!} + h^4 \cdot y^{IV} \cdot \frac{1}{4!} + O(h^5) -$$

$$-\big( y(x) - hy' + h^2 y'' \cdot \frac{1}{2!} - h^3 y''' \cdot \frac{1}{3!} + h^4 y^{IV} \cdot \frac{1}{4!} + O(h^5) \big)$$

$$\Big] - \alpha \Big[ g + hg' + h^2 g'' \cdot \frac{1}{2!} + h^3 g''' \cdot \frac{1}{3!} + h^4 \cdot g^{IV} \cdot \frac{1}{4!} + O(h^5) \Big] -$$

$$-(1-2\alpha) g(x) - \alpha \Big[ g - hg' + h^2 g'' \cdot \frac{1}{2!} - h^3 g''' \cdot \frac{1}{3!} + h^4 \cdot g^{IV} \cdot \frac{1}{4!} + O(h^5) \Big]$$

$$= \frac{1}{2h}\Big[ \cancel{y} + hy' + h^2 \cancel{y''} \cdot \frac{1}{2!} + h^3 y''' \cdot \frac{1}{3!} + \cancel{h^4 \cdot y^{IV}} \cdot \frac{1}{4!} + O(h^5) -$$

$$- \cancel{y} + hy' - h^2 \cancel{y''} \cdot \frac{1}{2!} + h^3 y''' \cdot \frac{1}{3!} - \cancel{h^4 \cdot y^{IV}} \cdot \frac{1}{4!} + O(h^5) \Big] -$$

$$- \cancel{\alpha} g - \cancel{\alpha h} g' - \alpha h^2 g'' \cdot \frac{1}{2!} - \cancel{\alpha h^3} g''' \cdot \frac{1}{3!} - \alpha h^4 \cdot g^{IV} \cdot \frac{1}{4!} - O(h^5) -$$

$$-(g - 2\alpha g) = -g + 2\cancel{\alpha g}$$

$$-(1-2\alpha) g$$

$$- \cancel{\alpha} g + \cancel{\alpha h} g - \alpha h^2 g'' \cdot \frac{1}{2!} + \cancel{\alpha h^3} g''' \cdot \frac{1}{3!} - \alpha h^4 \cdot g^{IV} \cdot \frac{1}{4!} + O(h^5) =$$

$$= \Big[\frac{1}{2h}\Big] \Big[ 2hy' + 2h^3 y''' \cdot \frac{1}{3!} + O(h^5) \Big] - g - \alpha h^2 g'' - \alpha h^4 \cdot g^{IV} \cdot \frac{1}{12} +$$

$$+ O(h^5) = y' + h^2 y''' \cdot \frac{1}{6} + O(h^4) - g - \alpha h^2 g'' - \alpha h^4 \cdot g^{IV} \cdot \frac{1}{12} + O(h^5)$$

$$= F[y] + h^2\left[y''' \cdot \frac{1}{6} - \alpha g''\right] + \frac{1}{2}h^4 \cdot y^{V} \cdot \frac{1}{60} + O(h^5) -$$

$$\left\{ O(h^5) \to h^5 \cdot y^{V} \cdot \frac{1}{5!} + h^6 \cdot y^{VI} \cdot \frac{1}{6!} + O(h^7) + h^5 \cdot y^{V} \cdot \frac{1}{5!} - h^6 \cdot y^{VI} \cdot \frac{1}{6!} + O(h^7) = h^5 \cdot y^{V} \cdot \frac{1}{60} + O(h^7) \to \frac{1}{2h} h^5 y^{V} \cdot \frac{1}{60} + O(h^6) \right.$$

$$- \alpha h^4 \cdot g^{IV} \cdot \frac{1}{12} + O(h^5) = \left[\alpha = \frac{1}{6}\right] =$$

$$= 0 + h^2 \cdot \frac{1}{6}\left[y''' - g''\right] + \frac{1}{2}h^4 \cdot y^{V} \frac{1}{60} - \frac{1}{6}h^4 \cdot g^{IV} \cdot \frac{1}{12} + O(h^5)$$

$$\Rightarrow \Delta_h = \frac{1}{120} h^4 y^{V} - \frac{1}{6} h^4 g^{IV} \cdot \frac{1}{12} = \frac{1}{12} h^4 \left(\frac{1}{10} y^{V} - \frac{1}{6} g^{IV}\right) -$$

– 4-й порядок при $\boxed{\alpha = \frac{1}{6}}$

$M = \{0, 2, 3\}$

$k = 3, \; n = 2$

$y'' - y = 0$

$|M| = 3$

a) $n \le |M| - 1$ ok

б) $k = 2|M| - n - 1$ ok

$$
\begin{array}{l}
m=0 \\[3em] m=1 \\[3em] m=2=n \\[3em] m>n \\[8em]
\end{array}
\left\{
\begin{aligned}
a_0 + a_2 + a_3 &= 0 \\
2a_2 + 3a_3 &= 0 \\
4a_2 + 9a_3 &= 2! = 2 \\
b_0 + b_2 + b_3 &= 1 \\
\frac{a_2 \cdot 2^3 + a_3 \cdot 3^3}{3!} &= \frac{b_2 \cdot 2^1 + b_3 \cdot 3^1}{1!} \\
\frac{a_2 \cdot 2^4 + a_3 \cdot 3^4}{4!} &= \frac{b_2 \cdot 2^2 + b_3 \cdot 3^2}{2!}
\end{aligned}
\right.
$$

$$\begin{cases} a_0 + a_2 + a_3 \ldots = 0 \\ 2a_2 + 3a_3 = 0 \\ 4a_2 + 9a_3 = 2 \\ \qquad\qquad\qquad b_0 + b_2 + b_3 = 1 \\ \frac{2^3}{3!}a_2 + \frac{3^3}{3!}a_3 - \qquad \frac{2}{1}b_2 - \frac{3}{1}b_3 = 0 \\ \frac{2^4}{4!}a_2 + \frac{3^4}{4!}a_3 - \qquad \frac{2^2}{2!}b_2 - \frac{3^2}{2!}b_3 = 0 \end{cases}$$

$$\frac{2\cdot 2\cdot 2}{3\cdot 2} = \frac{4}{3}$$

$$\frac{2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 3\cdot 2} = \frac{2}{3}$$

$$\frac{27}{8} = \frac{3\cdot 3\cdot 3\cdot 3}{2\cdot 2\cdot 3\cdot 2} \qquad \frac{2\cdot 2}{2\cdot 1} = 2$$

$$\frac{3\cdot 3}{2} = \frac{9}{2}$$

$$\begin{cases} a_0 = \frac{1}{3} \\ a_2 = -1 \\ a_3 = \frac{2}{3} \\ b_0 = \frac{5}{36} \\ b_2 = \frac{11}{12} \\ b_3 = -\frac{1}{18} \end{cases}$$

$$\Lambda_h = \frac{1}{h^2}\left[\frac{1}{3}y(x) - y(x+2h) + \frac{2}{3}y(x+3h)\right] -$$

$$-\frac{5}{36}g(x) - \frac{11}{12}g(x+2h) - \frac{1}{18}g(x+3h)$$

$$\Delta_h = h^3\left[\frac{1}{5!}\left[-1\cdot 2^5 + \frac{2}{3}\cdot 3^5\right] -\right.$$

$$\left.-\frac{1}{3!}\left[\frac{11}{12}\cdot 2^3 + \frac{1}{18}\cdot 3^3\right]\right] y^{(n+k)}(x) = h^3 \cdot \frac{-7}{18}\cdot y^{(n+k)}(x)$$

$M = \{0, 2, 3\}$ $\quad y' - g = 0$

$k = 4 \quad |M| = 3$

$n = 1$

$n \leq |M| - 1$ ok

$k = 2 \cdot |M| - n - 1$ ok

$$\frac{2\cdot 2}{2\cdot 1} = 2 \quad \frac{3\cdot 3}{2} = \frac{9}{2}$$

$$\frac{2\cdot 2\cdot 2}{3\cdot 2} = \frac{4}{3} \quad \frac{3\cdot 3\cdot 3}{3\cdot 2} = \frac{9}{2}$$

$$\begin{cases} a_0 + a_2 + a_3 = 0 \\ 2a_2 + 3a_3 = 1! = 1 \\ b_0 + b_2 + b_3 = 1 \\ \dfrac{a_2 \cdot 2^2 + a_3 \cdot 3^2}{2!} = \dfrac{b_2 \cdot 2^1 + b_3 \cdot 3^1}{1!} \\ \dfrac{a_2 \cdot 2^3 + a_3 \cdot 3^3}{3!} = \dfrac{b_2 \cdot 2^2 + b_3 \cdot 3^2}{2!} \\ \dfrac{a_2 \cdot 2^4 + a_3 \cdot 3^4}{4!} = \dfrac{b_2 \cdot 2^3 + b_3 \cdot 3^3}{3!} \end{cases}$$

$$\begin{cases} a_0 + a_2 + a_3 = 0 \\ 2a_2 + 3a_3 = 1 \\ b_0 + b_2 + b_3 = 1 \\ 2a_2 + \frac{9}{2}a_3 - 2b_2 - 3b_3 = 0 \\ \frac{4}{3}a_2 + \frac{9}{2}a_3 - 2b_2 - \frac{9}{2}b_3 = 0 \\ \frac{2}{3}a_2 + \frac{27}{8}a_3 - \frac{4}{3}b_2 - \frac{9}{2}b_3 = 0 \end{cases}$$

$$\frac{2\cdot 2}{2\cdot 1} = 2$$

$$\frac{3\cdot 3}{2\cdot 1} = \frac{9}{2}$$

$$\frac{2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 3\cdot 2} = \frac{2}{3}$$

$$\frac{3\cdot 3\cdot 3\cdot 3}{2\cdot 2\cdot 3\cdot 2} = \frac{27}{8}$$

$$\frac{2\cdot 2\cdot 2}{3\cdot 2} = \frac{4}{3} \quad \frac{3\cdot 3\cdot 3}{3\cdot 2} = \frac{9}{2}$$

$$\begin{cases} a_0 = -\frac{5}{42} \\ a_2 = -9/14 \\ a_3 = \frac{16}{21} \\ b_0 = 1/14 \\ b_2 = 9/14 \\ b_3 = 2/7 \end{cases}$$

$$\Lambda_h = \frac{1}{h}\left[-\frac{5}{42}y(x) - \frac{9}{14}y(x+2h) + \frac{16}{21}y(x+3h)\right] - \frac{1}{14}g(x) - \frac{9}{14}g(x+2h) - \frac{2}{7}g(x+3h)$$

$$\Delta_h = h^4\left\{\frac{1}{5!}\left(-\frac{5}{42}\cdot 2^5 + \frac{9}{14}\cdot 3^5\right) - \frac{1}{4!}\left(\frac{9}{14}\cdot 2^4 + \frac{2}{7}\cdot 3^4\right)\right\} \cdot y^{(n+k)}(x) =$$

$$\Rightarrow \Delta_h = h^4 \cdot \frac{-619}{5040} y^{(n+k)}(x)$$