

④/3

$$x_1 \quad 1$$

$$x_2 \quad -7$$

$$x_3 \quad 4$$

$$x_4 \quad 2$$

$$\begin{array}{c|c} f_1 & 4 \\ \hline f_2 & 1 \\ \hline f_3 & -3 \\ \hline f_4 & 0 \end{array}$$

Лагранж

$$L_n(x) = \sum_{i=1}^n f_i \prod_{\substack{j=1 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j}$$

$$L_4(x) = 4 \cdot \frac{(x+7)(x-4)(x-2)}{(1+7)(1-4)(1-2)} + 1 \cdot \frac{(x-1)(x-4)(x-2)}{(-7-1)(-7-4)(-7-2)} - 3 \cdot$$

$$\cdot \frac{(x-1)(x+7)(x-2)}{(4-1)(4+7)(4-2)} + 0 \cdot \cancel{\dots} =$$

$$= \frac{4}{8 \cdot (-3) \cdot (-1)} \cdot (x+7)(x-4)(x-2) + \frac{1}{(-8) \cdot (-11) \cdot (-9)} (x-1)(x-4)(x-2) -$$

$$-\frac{3 \cdot}{3 \cdot 11 \cdot 2} (x-1)(x+7)(x-2) =$$

$$= \frac{1}{6} \cdot (x^3 + x^2 - 34x + 56) + \frac{-1}{72 \cdot 11} (x^3 - 7x^2 + 14x - 8) -$$

$$- \frac{1}{22} \cdot (x^3 + 4x^2 - 19x + 14) =$$

$$= \frac{95}{792} x^3 - \frac{5}{792} x^2 - \frac{1909}{396} x + \frac{862}{99}$$

Ответ: $\frac{95}{792} x^3 - \frac{5}{792} x^2 - \frac{1909}{396} x + \frac{862}{99}$

Многочлен: $x_1 = 1; x_2 = -7; x_3 = 4; x_4 = 2$

$$L(x) = f_1 + f_{12}(x-x_1) + f_{123}(x-x_1)(x-x_2) + f_{1234}(x-x_1)(x-x_2)(x-x_3)$$

$$\begin{aligned} 4 &= f_1 \\ 1 &= f_2 \\ -3 &= f_3 \\ 0 &= f_4 \end{aligned}$$

$\begin{array}{c} f_{12} \\ f_{123} \\ f_{1234} \end{array}$

$\begin{array}{c} f_{23} \\ f_{234} \end{array}$

$$f_{12} = \frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 4}{-7 - 1} = \frac{-3}{-8} = \frac{3}{8}$$

$$f_{23} = \frac{f_3 - f_2}{x_3 - x_2} = \frac{-3 - 1}{4 + 7} = \frac{-4}{11}$$

$$f_{34} = \frac{f_4 - f_3}{x_4 - x_3} = \frac{0 + 3}{2 - 4} = \frac{3}{-2} = -\frac{3}{2}$$

$$f_{123} = \frac{f_{23} - f_{12}}{x_3 - x_1} = \frac{-\frac{4}{11} - \frac{3}{8}}{4 - 1} = -\frac{65}{264}$$

$$f_{234} = \frac{f_{34} - f_{23}}{x_4 - x_2} = \frac{-\frac{3}{2} + \frac{4}{11}}{2 + 7} = -\frac{25}{198}$$

$$f_{1234} = \frac{f_{234} - f_{123}}{x_4 - x_1} = \frac{-\frac{25}{198} + \frac{65}{264}}{2 - 1} = \frac{95}{792}$$

$$L_4(x) = 4 + \frac{3}{8}(x-1) - \frac{65}{264}(x-1)(x+7) + \frac{95}{792}(x-1)(x+7)(x-4)$$

$$= 4 + \frac{3}{8}x - \frac{3}{8} - \frac{65}{264}(x^2 + 6x - 7) + \frac{95}{792}(x^3 + 2x^2 - 31x + 28) =$$

$$= \frac{862}{96} + \frac{-1909}{396}x - \frac{5}{792}x^2 + \frac{95}{792}x^3 \quad \leftarrow \text{cobnagator}$$

$$f(x) = e^x \quad [3, 4] \quad h = \frac{4-3}{4} = 0,25$$

$L_5(x)$ ~~затем~~ пять разных

$$\left\{ \begin{array}{l} \frac{d}{dx} e^x = e^x \\ \exp(4) \cdot 0,25^5 \end{array} \right.$$

$$|f(x) - L_5(x)| \leq \frac{M_5 \cdot 0,25^5}{5} = \frac{\exp(4) \cdot 0,25^5}{5} =$$

$$= \frac{\exp(4) \cdot \left(\frac{25}{100}\right)^5}{5} = \frac{\exp(4) \cdot}{4^5 \cdot 5} = \frac{\exp(4)}{5120} \approx 0,0107$$

$$|f(x) - L_n(x)| < 10^{-16} ? n:$$

$$< \frac{M_n h^n}{n} = \frac{\exp(4) \cdot \cancel{h^n}}{n(n-1)^n} < \frac{\exp(4)}{(n-1)^{n+1}} < 10^{-16}$$

$$\frac{(n-1)^{n+1}}{\exp(4)} > 10^{16} \quad | \quad \exp(4) > 0$$

$$(n+1) \ln(n-1) > \ln[\exp(4) \cdot 10^{16}] = \ln[\exp(4)] + 16 \ln 10$$

$$(n+1) \ln(n-1) > 4 + 16 \ln 10 \approx 40,84136$$

$$n = 14, 18, \dots X$$

$$n = 15; \quad 42,22492 \quad \checkmark$$

Orber: nmu $n > 15$



Калькулятор

$$\left\{ \begin{array}{l} \frac{95}{792}x^3 - \frac{5}{792}x^2 - \frac{1909}{396}x + \frac{862}{99} = y \\ x = -7 \end{array} \right.$$



$$(x, y) = (-7, 1)$$

Показать решение →



$$\left\{ \begin{array}{l} \frac{95}{792}x^3 - \frac{5}{792}x^2 - \frac{1909}{396}x + \frac{862}{99} = y \\ x = 1 \end{array} \right.$$



$$(x, y) = (1, 4)$$

[Показать решение →](#)

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sin cos
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Калькулятор

$$\left\{ \begin{array}{l} \frac{95}{792}x^3 - \frac{5}{792}x^2 - \frac{1909}{396}x + \frac{862}{99} = y \\ x = 2 \end{array} \right.$$



$$(x, y) = (2, 0)$$

Показать решение →

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$$\left\{ \begin{array}{l} \frac{95}{792}x^3 - \frac{5}{792}x^2 - \frac{1909}{396}x + \frac{862}{99} = y \\ x = 4 \end{array} \right.$$



$$(x, y) = (4, -3)$$

[Показать решение →](#)

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