$M = \{0, 1, 2, 3\}$

$k = 3$

$n = 1$

$$\Lambda_h = \frac{1}{h} \sum_{j=0}^{3} a_j \, y(x + jh)$$

$$\begin{cases} a_0 \cdot \overset{1}{\overset{\shortparallel}{0^0}} + a_1 \cdot 1^0 + a_2 \cdot 2^0 + a_3 \cdot 3^0 = 0 \\ a_0 \cdot 0^1 + a_1 \cdot 1^1 + a_2 \cdot 2^1 + a_3 \cdot 3^1 = 1 \\ a_0 \cdot 0^2 + a_1 \cdot 1^2 + a_2 \cdot 2^2 + a_3 \cdot 3^2 = 0 \\ a_0 \cdot 0^3 + a_1 \cdot 1^3 + a_2 \cdot 2^3 + a_3 \cdot 3^3 = 0 \end{cases}$$

$$\begin{cases} a_0 + a_1 + a_2 + a_3 = 0 \\ a_1 + 2a_2 + 3a_3 = 1 \\ a_1 + 4a_2 + 9a_3 = 0 \\ a_1 + 8a_2 + 27a_3 = 0 \end{cases}$$

Вжух!

$$\begin{cases} a_0 = -\frac{11}{6} \\ a_1 = 3 \\ a_2 = -\frac{3}{2} \\ a_3 = \frac{1}{3} \end{cases}$$

$A_4 = 3 + 16 \cdot \frac{-3}{2} + 81 \cdot \frac{1}{3} = 3 - 24 + 27 \neq 0 = 6$

Ответ: $\Lambda_h = \dfrac{-\frac{11}{6}y(x) + 3y(x+h) + \frac{-3}{2}y(x+2h) + \frac{1}{3}y(x+3h)}{h}$

$$\Delta_h = \frac{y^{(n+k)}(x)\cdot 6\cdot h^3}{24} = \frac{y^{(4)}(x)\cdot h^3}{4}$$

$M = \{-1, 0, 1\}$

$k = 1$

$n = 2$

$$\Lambda_h = \sum_{j=-1}^{1} a_j\, y(x + jh) \cdot \frac{1}{h^2}$$

$$\begin{cases} a_0 \cdot 0^0 + a_1 \cdot 1^0 + a_{-1}(-1)^0 = 0 \\ a_0 \cdot 0^1 + a_1 \cdot 1^1 + a_{-1}(-1)^1 = 0 \\ a_0 \cdot 0^2 + a_1 \cdot 1^2 + a_{-1}(-1)^2 = 2 \end{cases}$$

$$\begin{cases} a_0 + a_1 + a_{-1} = 0 \\ a_1 + a_{-1} \cdot (-1) = 0 \\ a_1 + a_{-1} \cdot 1 = 2 \end{cases}$$

ВХУХ!

$$\begin{cases} a_0 = -2 \\ a_1 = 1 \\ a_2 = 1 \end{cases}$$

$A_3 = 0 + 1 \cdot 1^3 + 1 \cdot (-1)^3 = 0$

$A_4 = 0 + 1 \cdot 1^4 + 1 \cdot (-1)^4 = 1 + 1 = 2$

Ошибка: $\dfrac{y^{(4)}(x) \cdot 2 \cdot h^2}{6 \cdot 4}$

Ответ: $\dfrac{-2 \cdot y(x) + 1\, y(x-h) + 1\, y(x+h)}{h^2}$

M ... $0^0 + a_1 \cdot 1^0 + a_p \cdot p^0 = 0$

$M=\{-1,0,1,\cancel{10}\ p\}$

$k=2$

$n=2$

$$\begin{cases} a_{-1}\cdot(-1)^0 + a_0\cdot 0^0 + a_1\cdot 1^0 + a_p\cdot p^0 = 0 \\ a_{-1}\cdot(-1)^1 + a_0\cdot 0^1 + a_1\cdot 1^1 + a_p\cdot p^1 = 0 \\ a_{-1}\cdot(-1)^2 + a_0\cdot 0^2 + a_1\cdot 1^2 + a_p\cdot p^2 = 2 \\ a_{-1}\cdot(-1)^3 + a_0\cdot 0^3 + a_1\cdot 1^3 + a_p\cdot p^3 = 0 \end{cases}$$

$$\begin{cases} a_{-1} + a_0 + a_1 + a_p = 0 \\ a_{-1}\cdot(-1) + a_1 + a_p\cdot p = 0 \\ a_{-1} + a_1 + a_p\cdot p^2 = 2 \\ a_{-1}\cdot(-1) + a_1 + a_0\cdot p^3 = 0 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ -1 & 0 & 1 & p & 0 \\ 1 & 0 & 1 & p^2 & 2 \\ -1 & 0 & 1 & p^3 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & p+1 & 0 \\ 0 & 1 & 0 & 1-p^2 & -2 \\ 0 & 1 & 2 & 1+p^3 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & p+1 & 0 \\ 0 & 0 & 2 & p+p^2 & 2 \\ 0 & 0 & 0 & p-p^3 & 0 \end{array}\right)$$

$$\begin{cases} a_{-1}+a_0+a_1+a_p=0 \\ a_0+2a_1+(p+1)a_p=0 \\ 2a_1+(p+p^2)a_p=2 \\ (p-p^3)a_p=0 \end{cases} \Longrightarrow \begin{cases} a_{-1}+a_0+a_1+a_p=0 \\ a_0+2a_1+(p+1)a_p=0 \\ 2a_1+(p+p^2)a_p=2 \\ a_p=0 \end{cases} \Rightarrow$$

$$\begin{cases} a_{-1}=0-1+2=1 \\ a_0=0-2\cdot 1=-2 \\ a_1=(2-0)/2=1 \\ a_p=0 \end{cases} \qquad \begin{cases} a_{-1}=1 \\ a_0=-2 \\ a_1=1 \\ a_p=0 \end{cases}$$

$$\Delta_4 = 1\cdot(-1)^4+1\cdot 1^4+0\cdot p^4 = 2$$

$$\Lambda_h = \frac{y(x-h)-2y(x)+y(x+h)}{h^2} \qquad \Delta_h = \frac{y^{(4)}(x)\cdot 2\cdot h^2}{24} = \frac{y^{(4)}(x)h^2}{12}$$

$M = \{0, 1, 2, 3, 4\}$

$k = 2$

$n = 3$

$$\begin{cases} a_0 \cdot 0^0 + a_1 \cdot 1^0 + a_2 \cdot 2^0 + a_3 \cdot 3^0 + a_4 \cdot 4^0 = 0 \\ a_0 \cdot 0^1 + a_1 \cdot 1^1 + a_2 \cdot 2^1 + a_3 \cdot 3^1 + a_4 \cdot 4^1 = 0 \\ a_0 \cdot 0^2 + a_1 \cdot 1^2 + a_2 \cdot 2^2 + a_3 \cdot 3^2 + a_4 \cdot 4^2 = 0 \\ a_0 \cdot 0^3 + a_1 \cdot 1^3 + a_2 \cdot 2^3 + a_3 \cdot 3^3 + a_4 \cdot 4^3 = 3! = 6 \\ a_0 \cdot 0^4 + a_1 \cdot 1^4 + a_2 \cdot 2^4 + a_3 \cdot 3^4 + a_4 \cdot 4^4 = 0 \end{cases}$$

$$\begin{cases} a_0 + a_1 + a_2 + a_3 + a_4 = 0 \\ a_1 + 2a_2 + 3a_3 + 4a_4 = 0 \\ a_1 + 4a_2 + 9a_3 + 16a_4 = 0 \\ a_1 + 8a_2 + 27a_3 + 64a_4 = 6 \\ a_1 + 16a_2 + 81a_3 + 256a_4 = 0 \end{cases}$$

Вжух!

$$\begin{cases} a_0 = -\frac{5}{2} \\ a_1 = 9 \\ a_2 = -12 \\ a_3 = 7 \\ a_4 = -\frac{3}{2} \end{cases}$$

$\cancel{A_5 = 0 + 9 \cdot 1 + (-12) \cdot 243 + 7}$

$A_5 = 0 + 9 \cdot 1 - 12 \cdot 32 + 7 \cdot 243 - \frac{3}{2} \cdot 1024 =$

$= -210$

$$\Lambda_h = \frac{-\frac{5}{2} y(x) + 9 y(x+h) - 12 y(x+2h) + 7 y(x+3h) - \frac{3}{2} y(x+4h)}{h^3}$$

$$\Delta_h = \frac{y^{(5)}(x)(-210) \cdot h^2}{120} = \frac{-y^{(5)}(x) \cdot 7 \cdot h^2}{4}$$

$$\Lambda_h^{\text{3 сим}} = \frac{1}{h^3}\left(y\left(x + \frac{3}{2}h\right) - 3y\left(x + \frac{1}{2}h\right) + 3y\left(x - \frac{1}{2}h\right) - y\left(x - \frac{3}{2}h\right)\right) =$$

$$= \frac{y(x) + \frac{3}{2} h y'(x) + \frac{9}{4} h^2 y''(x) \cdot \frac{1}{2!} + \frac{27}{8} h^3 y'''(x) \cdot \frac{1}{3!} + \frac{81}{16} h^4 y''''(x) \cdot \frac{1}{4!} +}{h^3}$$

$+ y^{(V)}(x) \cdot \frac{1}{5!} \cdot \frac{243}{32} h^5 + O(h^6) - 3y(x) + \frac{-3}{2} h y'(x) + \frac{-3}{4} h^2 y''(x) \cdot \frac{1}{2!} + \frac{-3}{8} h^3 y^{(3)}(x) \cdot \frac{1}{3!}$

$+ \frac{-3}{16} h^4 y^{(4)}(x) \cdot \frac{1}{4!} + y^{(5)}(x) \cdot \frac{1}{5!} \cdot \frac{-3}{32} h^5 + O(h^6) + 3 \cdot y(x) - \frac{3}{2} h y'(x) + \frac{3}{4} h^2 y''(x) \cdot \frac{1}{2} +$

$+ \frac{-3}{8} h^3 y^{(3)}(x) \cdot \frac{1}{3!} + \frac{3}{16} h^4 y^{(4)}(x) \cdot \frac{1}{4!} - \frac{3}{32} h^5 y^{(5)}(x) \cdot \frac{1}{5!} + O(h^6) -$

$- y(x) - \frac{3}{2} h y'(x) - \frac{9}{4} h^2 y''(x) \cdot \frac{1}{2} - \frac{-27}{8} h^3 y^{(3)}(x) \cdot \frac{1}{3!} - \frac{81}{16} h^4 y^{(4)}(x) \cdot \frac{1}{4!} -$

$- \frac{-243}{32} h^5 y^{(5)}(x) \cdot \frac{1}{5!} + O(h^6)$

$=$

$$= \frac{\frac{27-3-3+27}{8} h^3 y^{(3)}(x) \cdot \frac{1}{3!} + \frac{81-3+3-81}{16} h^4 y^{(4)}(x) \cdot \frac{1}{4!} +}{h^3}$$

24=

$$\frac{+ \frac{243-3-3+243}{32} h^5 y^{(5)}(x) \cdot \frac{1}{5!}}{} + O(h^3) =$$

$$= \frac{h^3 y^{(3)}(x) \frac{48}{8} \cdot \frac{1}{6} + h^5 y^{(5)}(x) \cdot \frac{1}{5 \cdot 4 \cdot 3 \cdot 2} \cdot \frac{480}{32}}{h^3} + O(h^3) =$$

$$= y'''(x) + \frac{h^5 y^{(5)}(x) \cdot \frac{1}{8}}{h^3} + O(h^3) = y'''(x) + \frac{y^{(5)}(x) \cdot h^2}{8} + O(h^3)$$

$\boxed{\text{Ответ: } \frac{y^{(5)}(x) \cdot h^2}{8}}$ – 2-й порядок

$$\Lambda_h^3 = \frac{1}{h^3}\left(y(x+h) - 3y(x) + 3y(x-h) - y(x-2h)\right) =$$

$$= \frac{1}{h^3}\Big[\cancel{y(x)} + \cancel{h y'(x)} + \cancel{h^2 y''(x)\cdot\frac{1}{2!}} + h^3 y'''(x)\cdot\frac{1}{3!} + h^4 y^{(IV)}(x)\cdot\frac{1}{4!} +$$

$$+ h^5 y^{(V)}(x)\cdot\frac{1}{5!} + O(h^6) - \cancel{3y(x)} + \cancel{3y(x)} - \cancel{3h y'(x)} + \cancel{3h^2 y''(x)\cdot\frac{1}{2!}} -$$

$$- 3\cdot h^3 y'''(x)\cdot\frac{1}{3!} + 3h^4 y^{(IV)}(x)\cdot\frac{1}{4!} - 3h^5 y^{(V)}(x)\cdot\frac{1}{5!} + O(h^6) -$$

$$- \cancel{y(x)} + \cancel{2h y'(x)} - \cancel{4h^2 y''(x)\cdot\frac{1}{2!}} + 8h^3 y'''(x)\cdot\frac{1}{3!} - 16 h^4 y^{(IV)}(x)\cdot\frac{1}{4!} +$$

$$+ 32 h^5 y^{(V)}(x)\cdot\frac{1}{5!} + O(h^6) \Big) =$$

$$= \frac{1}{h^3} \left[h^3 y'''(x) \cdot \frac{1}{3!} \cdot (8-3+1) + h^4 y^{(IV)}(x) \cdot \frac{1}{4!} \cdot (1+3-16) + \right.$$

$$\left. + h^5 y^{(V)}(x) \cdot \frac{1}{5!} \cdot (1-3+32) + O(h^6) \right] =$$

$$= y'''(x) + h^1 y^{(IV)}(x) \cdot \frac{-1}{2} + h^2 y^{(V)}(x) \cdot \frac{1}{4} + O(h^3) =$$

$$= y'''(x) + h^1 y^{(IV)}(x) \cdot \frac{-1}{2} + O(h^2)$$ – первый порядок аппр.