■ Study Guide: Number Systems in Number Theory

1. Natural Numbers (■)

Definition: Numbers used for counting.
Two conventions: {1,2,3,...} or {0,1,2,3,...}.
Properties: whole, non-negative, infinite.

2. Whole Numbers

Same as natural numbers but always include 0.
{0,1,2,3,...}.

3. Integers (■)

Extend whole numbers to include negatives.
{...,-3,-2,-1,0,1,2,3,...}.

4. Rational Numbers (■)

Numbers that can be expressed as fractions of integers.
Examples: 1/2, -3/4, 5, 0.333...
Decimals either terminate or repeat.

5. Irrational Numbers

Cannot be expressed as a fraction of integers.
Decimals never terminate or repeat.
Examples: π, $\sqrt{2}$, e.

6. Real Numbers (■)

All rational and irrational numbers together.
Represented on the number line.

7. Complex Numbers (■)

Include imaginary unit i where $i^2=-1$.
Form: a+bi.
Examples: 3+2i, -1-4i.

8. Set Hierarchy

■ ⊂ ■ ⊂ ■ ⊂ ■ ⊂ ■

9. Visual Representation

Nested diagram:
■ inside ■, inside ■, inside ■, inside ■.
Examples: 2∈■, -5∈■, 1/2∈■, π∈■, 3+2i∈■.

10. Decision Tree for Classifying Numbers

1. Involves i? → Complex.
2. Can it be a fraction? Yes→Rational, No→Irrational (Real).
3. If Rational: Whole number? Yes→Integer, No→Fractional Rational.
4. If Integer: Positive/0? Yes→Natural, No→Negative Integer.

11. Practice Exercises

Examples classified:
-7 ∈ ■,■,■,■
0 ∈ ■,■,■,■ (maybe ■)
3/4 ∈ ■,■,■
√5 ∈ ■,■
4+i ∈ ■
12 ∈ ■,■,■,■,■
-2/3 ∈ ■,■,■
π ∈ ■,■