

■ Master Study Guide: Number Systems in Number Theory

1. Natural Numbers (■)

Counting numbers. Two conventions: $\{1,2,3,\dots\}$ or $\{0,1,2,3,\dots\}$. Whole, non-negative, infinite.

2. Whole Numbers

Naturals plus 0: $\{0,1,2,3,\dots\}$.

3. Integers (■)

Whole numbers and negatives: $\{\dots,-3,-2,-1,0,1,2,3,\dots\}$.

4. Rational Numbers (■)

Fractions of integers, repeating/terminating decimals. Examples: $1/2$, $-3/4$, $0.333\dots$, 5 .

5. Irrational Numbers

Not fractions, non-repeating decimals. Examples: π , $\sqrt{2}$, e .

6. Real Numbers (■)

All rational and irrational numbers (number line).

7. Complex Numbers (■)

Form $a+bi$, includes imaginary unit i . Examples: $3+2i$, $-1-4i$.

8. Set Hierarchy

■ \subset ■ \subset ■ \subset ■ \subset ■

9. Visual Representation

Nested sets: ■ inside ■ inside ■ inside ■ inside ■.

10. Decision Tree for Classifying Numbers

Step-by-step classification using i, fraction test, whole number test, positivity.

11. Practice Exercises

Examples classified: -7 , 0 , $3/4$, $\sqrt{5}$, $4+i$, 12 , $-2/3$, π .

12. Integer Subfamilies

Prime Numbers: Integers >1 with exactly two divisors (1 and itself). Examples: 2, 3, 5, 7, 11. Infinite supply; key in number theory.

Composite Numbers: Integers >1 that are not prime; can be factored into smaller primes. Examples: 4, 6, 8, 9, 10. Every composite has a unique prime factorization.

Special Cases: 1 is neither prime nor composite. 0 and negatives are not prime/composite by definition.

13. Intervals

Definition: Sets of real numbers between endpoints.

Types:

- Open: $(a,b) \rightarrow a$ Closed: $[a,b] \rightarrow a \leq x \leq b$
- Half-open: $(a,b]$ or $[a,b)$
- Infinite: $(-\infty,b)$, $[a,\infty)$

Notation: Interval vs set-builder (e.g., $(2,5] = \{x \in \mathbb{R} \mid 2 < x \leq 5\}$)

Examples:

- Between -3 and 7 (excluding 7): $(-3,7)$
- Non-negative reals: $[0,\infty)$
- Integers between 1 and 10: $\{2,3,4,5,6,7,8,9\}$

14. Practice Exercises (New Content)

Integer Families:

- 1) Is 19 prime or composite?
- 2) Factorize 84 into primes.
- 3) Why is 1 not considered prime or composite?

Intervals:

- 1) Express all real numbers greater than -2 and ≤ 6 in interval notation.
- 2) Write $(-\infty,3)$ in words.
- 3) Which integers belong to $[4,9)$?