財務工程 - Week7

https://cool.ntu.edu.tw/courses/1195

PecuLab peculiar change the world 蔡芸琤 (Peculiar)

http://www.peculab.org/

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Github 作業繳交列表

https://docs.google.com/spreadsheets/d/1ZGO3ciyAhqZerbG3wUZd9zhHIxgqh8mvHbw0sqpQGN4/edit?usp=sharing

作業二完整度高之作品

1. https://github.com/pjwu1997/FinancialEngineering/tree/master/HW2
2. https://github.com/manamimebom/Financial_Engineering/tree/master/HW2
3. https://github.com/weiooo/Financial_Engineering/tree/master/HW2
4. https://github.com/changchiaruei/Financial-Engineering/tree/master/HW_2
5. https://github.com/KatherineChu/Financial-Engineering/tree/master/HW2
6. https://github.com/fatdanny77/Financial_Engineering/tree/master/HW2
7. https://github.com/mengjelee/Financial_Engineering/tree/master/hw2
8. https://github.com/jenny56402/Financial_Engineering/tree/master/HW2
9. https://github.com/EnChiSu/Financial-Engineering/tree/master/HW2
10. https://github.com/feiyuehchen/Financial_Engineering/tree/master/HW2

編輯

請大家務必要記得提交每次的作業連結到 NTU COOL

4月10日 9:45

蔡芸琤

所有班別

各位同學

雖然大家都有在第一次提供過 github，但還是請大家每次作業上傳繳交截止前，提交連結到 NTU COOL 的作業區。

原因是，同儕互評系統是自動從大家繳交的狀況下去分配評分人與被評分人，如果你沒有提交，就不會被系統選入同儕互評中。

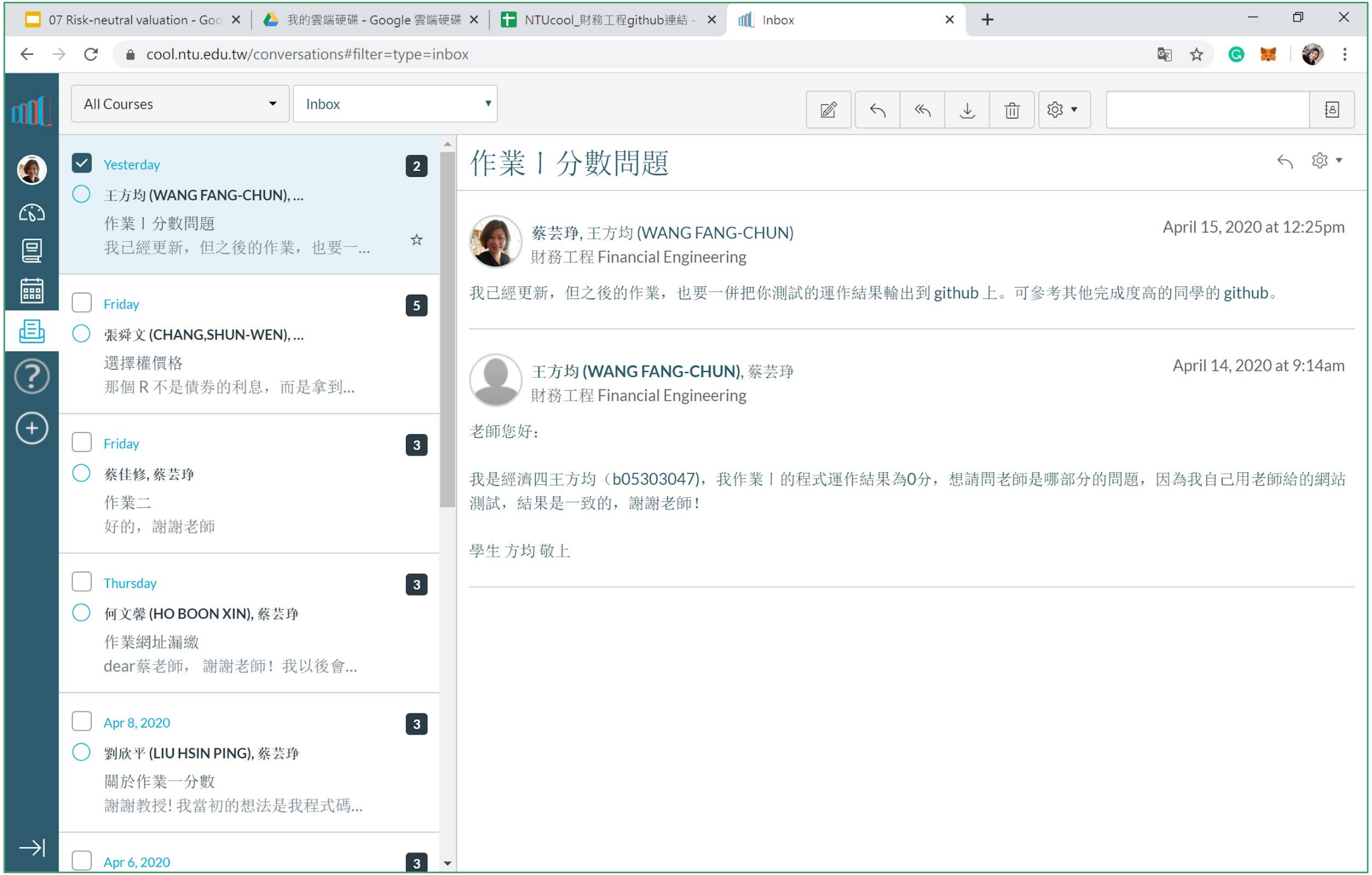
而我在整理繳交列表時，也都是去作業區點繳交的狀況。第二次作業有幾位同學忘記在作業區繳交，我已經都更新至 https://docs.google.com/spreadsheets/d/1ZGO3ciyAhqZerbG3wUZd9zhHIxgqh8mvHbw0sqpQGN4/edit?usp=sharing

沒被登記到的，盡快跟我說。希望第三次作業，大家能改善以上狀況。

Pecu

搜尋條目或作者 | 未讀

回覆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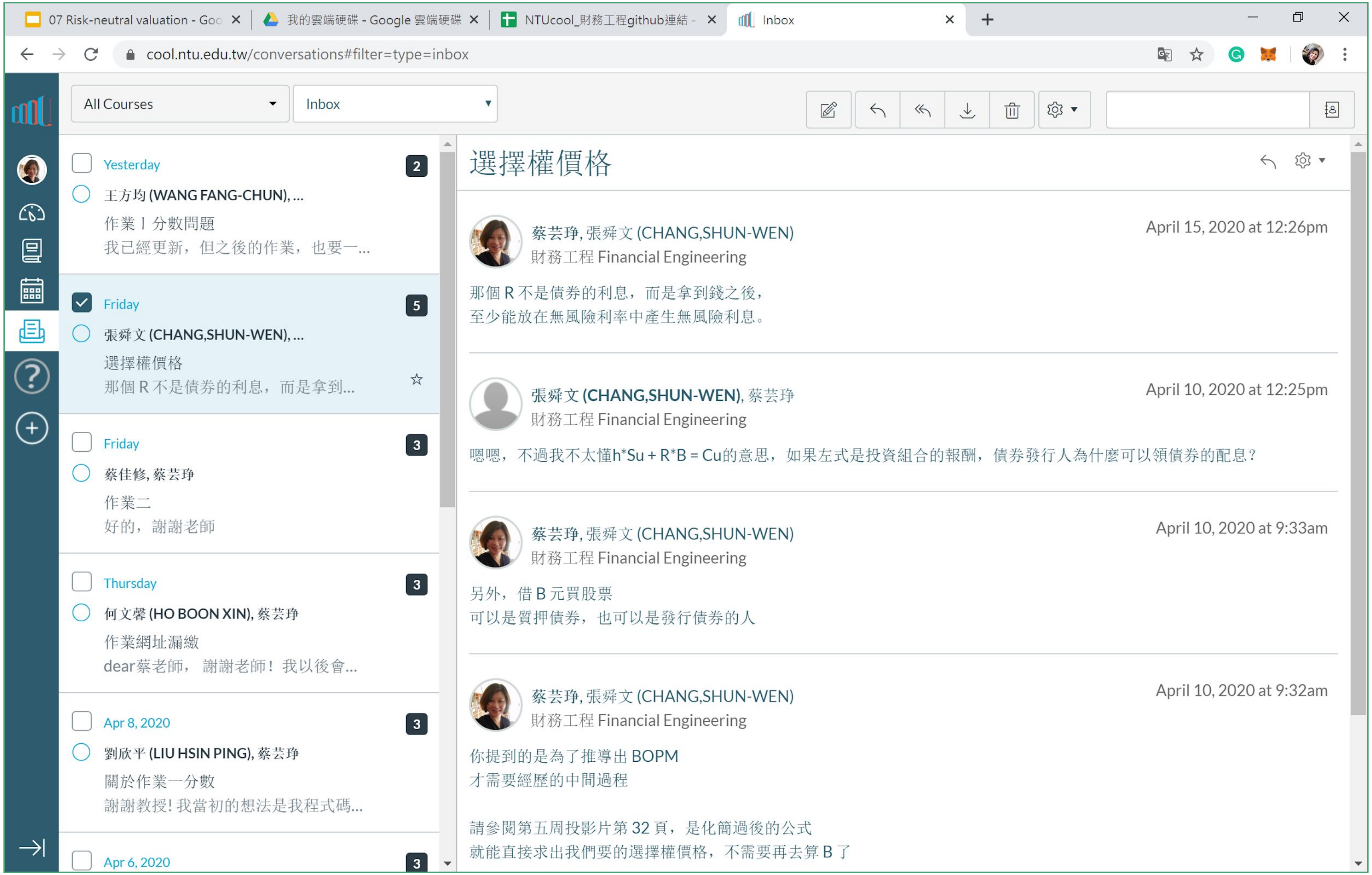
作業Ｉ分數問題
蔡芸琤, 王方均 (WANG FANG-CHUN)
財務工程 Financial Engineering
April 15, 2020 at 12:25pm
我已經更新，但之後的作業，也要一併把你測試的運作結果輸出到 github 上。可參考其他完成度高的同學的 github。
王方均 (WANG FANG-CHUN), 蔡芸琤
財務工程 Financial Engineering
April 14, 2020 at 9:14am
老師您好：
我是經濟四王方均（b05303047），我作業Ｉ的程式運作結果為0分，想請問老師是哪部分的問題，因為我自己用老師給的網站測試，結果是一致的，謝謝老師！
學生 方均 敬上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Apr 8, 2020
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關於作業一分數
謝謝教授! 我當初的想法是我程式碼...
Apr 6, 2020
3
選擇權價格
蔡芸琤, 張舜文 (CHANG,SHUN-WEN)
April 15, 2020 at 12:26pm
財務工程 Financial Engineering
那個 R 不是債券的利息，而是拿到錢之後，
至少能放在無風險利率中產生無風險利息。
張舜文 (CHANG,SHUN-WEN), 蔡芸琤
April 10, 2020 at 12:25pm
財務工程 Financial Engineering
嗯嗯，不過我不太懂h*Su + R*B = Cu的意思，如果左式是投資組合的報酬，債券發行人為什麼可以領債券的配息？
蔡芸琤, 張舜文 (CHANG,SHUN-WEN)
April 10, 2020 at 9:33am
財務工程 Financial Engineering
另外，借 B 元買股票
可以是質押債券，也可以是發行債券的人
蔡芸琤, 張舜文 (CHANG,SHUN-WEN)
April 10, 2020 at 9:32am
財務工程 Financial Engineering
你提到的是為了推導出 BOPM
才需要經歷的中間過程
請參閱第五周投影片第 32 頁，是化簡過後的公式
就能直接求出我們要的選擇權價格，不需要再去算 B 了

風險中性定價理論
(Risk-Neutral Valuation)

風險中性定價理論於資本市場中的意義 (1/2)

1. 市場不存在任何套利可能性的條件下，如果衍生證券的價格依然依賴於可交易的基礎證券，那麼這個**衍生證券的價格是與投資者的風險態度無關的**。
2. 這個結論在**數學上表現為衍生證券定價的微分方程中並不包含有受投資者風險態度的變數**，尤其是期望收益率。

風險中性定價理論於資本市場中的意義 (2/2)

1. 這種定價原理與投資者的風險制度無關，從而推廣到對任何衍生證券都適用，所有投資者都是風險中性的，這個價格的決定，又是適用於任何一種風險志度的投資者。
2. 在風險中性的經濟環境中，投資者並不要求任何的風險補償或風險報酬，所以基礎證券與衍生證券的期望收益率都恰好等於無風險利率。
3. 由於不存在任何的風險補償或風險報酬，市場的貼現率也恰好等於無風險利率，所以基礎證券或衍生證券的任何盈虧經無風險利率的貼現就是它們的現值。

The valuation of options for alt ×

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Outline

Abstract

References

Journal of Financial Economics

Volume 3, Issues 1–2, January–March 1976, Pages 145-166

The valuation of options for alternative stochastic processes ☆

John C. Cox, Stephen A. Ross

Show more

https://doi.org/10.1016/0304-405X(76)90023-4

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Abstract

This paper examines the structure of option valuation problems and develops a new technique for their solution. It also introduces several jump and diffusion processes which have not been used in previous models. The technique is applied to these processes to find explicit option valuation formulas, and solutions to some previously unsolved problems involving the pricing of securities with payouts and potential bankruptcy.

Previous article in issue Next article in issue

Risk-Neutral 風險中立

1. The “risk-neutral” technique is frequently used to value derivative securities. It was developed by John Cox and Stephen Ross in a 1976 article “The Valuation of Options for Alternative Stochastic Processes” Journal of Financial Economics 3, p.145-66.
2. This course considers the intuition for this technique and the technique’s important applications for the case of the risk-free interest rate being constant.

Recall Week5 BOPM

The one-period expected payoff on a call option is

$$\frac{pC_u + (1-p)\,C_d}{R}$$

The expected payoff on a call option

Thus, the equation could be written as

$$C_t = \frac{1}{R_f} \hat{E}[\tilde{C}_{t+1}]$$

where C_t is the call option at time t, and $\hat{E}[\,\tilde{C}_{t+1}\,]$ is the *expected value* of the call at time $t+1$ *in a risk-neutral world.* Therefore, C_t is the "risk-neutral expected value" of $\tilde{C}_{t+1}$ discounted at the risk-free rate, that is,

$$\frac{1}{R_f} = e^{-\mathrm{r}}.$$

where r is defined as the continuously-compounded one-period interest rate. As we saw earlier, this "risk-neutral" valuation result is not just coincidental to options but will hold whenever markets are complete. For example, it can hold for valuing other securities, such as forward contracts.

A forward contract is an agreement

1. A forward contract is an agreement where two parties agree to exchange an asset at some future date for a pre-agreed price.
2. The long (short) party agrees to receive (deliver) the asset in exchange for paying (receiving) this pre-agreed price at a future date, say date T.
3. No payment occurs between the parties at the time of the agreement of the contract, only at the maturity of the contract.

The payoff to the long party at the maturity of the forward contract can be written as

$$\tilde{S}_T - K$$

the previously agreed upon "delivery" price for the forward contract, which is said to equal the "forward price" at the agreement date of the contract.

the underlying asset price at the maturity of the forward contract.

Portfolios

1. Assume that the underlying asset pays no dividends and
2. define f as the current date t value of the forward contract and
3. $\tau \equiv T - t$ as its time until maturity.

At date t:

Portfolio A: A long position in one forward contract written on an asset having current value of S and having a forward price of K

Portfolio B: One share of the underlying asset plus borrowing an amount $e^{-r\tau}K$

At date T:

Portfolio A: $\tilde{S}_T - K$

Portfolio B: $\tilde{S}_T - K$

Since the two portfolios produce exactly the same cash flow at date T, the absence of arbitrage implies that their values at date t must also be the same, that is, $f = S - e^{-r\tau}K$. What can also be shown is that the risk-neutral technique can also be used to derive this no-arbitrage value for f. Applying the above risk-neutral method, the current value of a long position, f, is

$$f = e^{-r\tau}\hat{E}[\tilde{S}_T - K] = e^{-r\tau}\hat{E}[\tilde{S}_T] - e^{-r\tau}K.$$

What is $\hat{E}[\,\tilde{S}_T\,]$, the expected value of $\tilde{S}_T$ in a risk-neutral world?In a risk-neutral world, the expected rate of return on S would be r , and $\hat{E}[\,\tilde{S}_T\,] = Se^{\mathrm{r}\,\tau}$. Thus,

$$f = e^{-\mathrm{r}\,\tau} S e^{\mathrm{r}\,\tau} - e^{-\mathrm{r}\,\tau} K = S - e^{-\mathrm{r}\,\tau} K$$

which is the same as our earlier no-arbitrage derivation. Thus, the "risk-neutral" technique also works for forward contracts. Note when the contract is agreed to initially, $f = 0$ and $K = F$, where F is the initial forward price. Therefore,

$$0 = S - Fe^{-\mathrm{r}\,\tau}, \qquad \text{or}$$

$$F = Se^{\mathrm{r}\,\tau}.$$

To gain more intuition, consider the standard approach to valuing risky cashflows. The present value of a long position, f, equals its *expected cashflows* discounted at a *risk-adjusted rate of return*:

$$f = e^{-\theta\tau} E[\tilde{S}_T] - K e^{-\mathrm{r}\tau}$$

where θ is the *true* expected rate of return on the risky asset and $E[\tilde{S}_T]$ is the *true* expected value of $\tilde{S}_T$. K is discounted by the risk-free rate, r, since it is a certain, not risky, cashflow. θ depends on risk aversion, and might be the result of the CAPM, for example, $\theta = \mathrm{r} + \beta\,(E[\tilde{r}_m] - \mathrm{r})$.

Capital Asset Pricing Model, CAPM

If θ is the true expected rate of return on the asset, then $E[\tilde{S}_T] = Se^{\theta\tau}$. Substituting in, we have

$$f = e^{-\theta\tau} S e^{\theta\tau} - e^{-\mathrm{r}\tau} K = S - K e^{-\mathrm{r}\tau}$$

which is the same result as before. Note how the risk-neutral technique "works." By replacing $e^{-\theta\tau} E[\tilde{S}_T]$ with $e^{-\mathrm{r}\tau}\hat{E}[\tilde{S}_T]$, we "pretend" the expected rate of return on the asset is r (rather than θ) and discount by r (rather than θ). However, these two "mistakes" cancel out, giving us the correct answer:

$$e^{-\mathrm{r}\tau}\hat{E}[\tilde{S}_T] = e^{-\theta\tau} E[\tilde{S}_T] = S.$$

資本資產定價模型 (CAPM) 的假設 (1/2)

CAPM是建立在馬科威茨模型基礎上的，馬科威茨模型的假設自然包含在其中：

1. 投資者希望財富越多愈好，效用是財富的函數，財富又是投資收益率的函數，因此可以認為效用為收益率的函數。
2. 投資者能事先知道投資收益率的概率分佈為正態分佈。
3. 投資風險用投資收益率的方差或標準差標識。
4. 影響投資決策的主要因素為期望收益率和風險兩項。
5. 投資者都遵守主宰原則(Dominance rule)，即同一風險水平下，選擇收益率較高的證券；同一收益率水平下，選擇風險較低的證券。以在無風險折現率R的水平下無限制地借入或貸出資金。

資本資產定價模型 (CAPM) 的假設 (2/2)

CAPM的附加假設條件:

1. 可以在無風險折現率R的水平下無限制地借入或貸出資金。
2. 所有投資者對證券收益率概率分佈的看法一致。
3. 所有投資者具有相同的投資期限, 而且只有一期。
4. 所有的證券投資可以無限制的細分, 在任一投資組合里可以含有非整數股份。
5. 買賣證券時沒有稅負及交易成本。
6. 所有投資者可以及時免費獲得充分的市場信息。
7. 不存在通貨膨脹, 且折現率不變。
8. 投資者具有相同預期, 即他們對預期收益率、標準差和證券之間的協方差具有相同的預期值。

CAPM

$$\theta = \mathrm{r} + \beta\left(E[\tilde{r}_m] - \mathrm{r}\right).$$

1. CAPM 公式中的右邊第一個是無風險收益率，比較典型的無風險回報率是 10 年期的美國政府債券。
2. 如果股票投資者需要承受額外的風險，那麼他將需要在無風險回報率的基礎上多獲得相應的溢價。
3. 股票市場溢價（equity market premium）就等於市場期望回報率減去無風險回報率。證券風險溢價就是股票市場溢價和一個 β 繫數的乘積。

Risk-Neutral Derivation of the Black-Scholes Formula

We now apply the risk-neutral technique to valuing a European call option on a non-dividend paying stock:

$$c = e^{-\mathrm{r}\,\tau}\,\hat{E}[\,max(0,\,\tilde{S}_T - X)\,].$$

To evaluate the "risk-neutral" expectation, we need to make an assumption regarding the probability distribution of $\tilde{S}_T$. If we assume that the stock price follows the binomial "tree" process considered earlier, we obtain the Cox, Ross, Rubinstein binomial option formula.

Let us now assume a different distribution for $\tilde{S}_T$, namely, the lognormal distribution. Define

$$\begin{aligned}\mu &= \text{annual expected rate of return on the stock,}\\ \sigma^2 &= \text{annualized variance of the rate of return on the stock.}\end{aligned}$$

If $\tilde{S}_T$ is a lognormally distributed random variable, then $\ln(\tilde{S}_T)$ is a normally distributed random variable:

$$\ln(\tilde{S}_T) \sim n\left(\ln(S) + (\mu - \sigma^2/2)\tau,\, \sigma^2\tau\right)$$

where $n(\,\cdot\,)$ is the normal probability density function. The lognormal distribution is attractive because it allows $\tilde{S}_T$ to take any possible value over the range 0 to ∞. Continuously compounded returns on the stock over unit time intervals, $\ln(\tilde{S}_{t+1}) - \ln(\tilde{S}_t) = \ln(\tilde{S}_{t+1}/\tilde{S}_t)$, are therefore normally distributed.

Let's now calculate $\hat{E}[\,max(0,\, \tilde{S}_T - X)\,]$, the call's expected payoff in a risk-neutral world. If investors were risk-neutral, then $\mu = \mathrm{r}$, so that

$$\ln(\tilde{S}_T) \;\sim\; n\left(\ln(S) \,+\, (\mathrm{r} - \sigma^2/2)\tau,\, \sigma^2\tau\right).$$

Using this distributional assumption, we have

$$c \,=\, e^{-\mathrm{r}\,\tau}\hat{E}[\,max(0,\, \tilde{S}_T - X)\,] \,=\, e^{-\mathrm{r}\,\tau}\int_X^\infty (S_T - X)\, g(S_T)\, dS_T$$

where $g(S_T)$ is the lognormal probability density function assuming $\mu = \mathrm{r}$. Evaluating the integral in (14), one obtains the Black-Scholes formula

$$c = SN\left(d_1\right) - Xe^{-\mathrm{r}\,\tau}N\left(d_2\right)$$

where $N\left(\cdot\right)$ is the normal distribution function and

$$\begin{aligned} d_1 &= \frac{\ln\left(S/X\right) + \left(\mathrm{r} + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= d_1 - \sigma\sqrt{\tau} \end{aligned}$$

In probability theory, a **normal** (or **Gaussian** or **Gauss** or **Laplace–Gauss**) **distribution** is a type of continuous probability distribution for a real-valued random variable. The general form of its probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The parameter μ is the mean or expectation of the distribution (and also its median and mode); and σ is its standard deviation. The variance of the distribution is σ^2. A random variable with a Gaussian distribution is said to be **normally distributed** and is called a **normal deviate**.

Normal distributions are important in statistics and are often used in the natural and social sciences to represent real-valued random variables whose distributions are not known.[1][2] Their importance is partly due to the central limit theorem. It states that, under some conditions, the average of many samples (observations) of a random variable with finite mean and variance is itself a random variable whose distribution converges to a normal distribution as the number of samples increases. Therefore, physical quantities that are expected to be the sum of many independent processes (such as measurement errors) often have distributions that are nearly normal.[3]

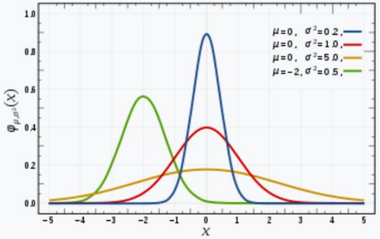
Moreover, Gaussian distributions have some unique properties that are valuable in analytic studies. For instance, any linear combination of a fixed collection of normal deviates is a normal deviate. Many results and methods (such as propagation of uncertainty and least squares parameter fitting) can be derived analytically in explicit form when the relevant variables are normally distributed.

A normal distribution is sometimes informally called a **bell curve**. However, many other distributions are bell-shaped (such as the Cauchy, Student's *t*, and logistic distributions).

Contents [hide]

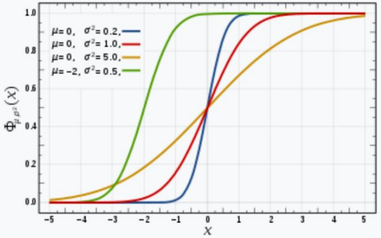
Normal distribution

Probability density function



The red curve is the *standard normal distribution*

Cumulative distribution function



Notation	$\mathcal{N}(\mu, \sigma^2)$
Parameters	$\mu \in \mathbb{R}$ = mean (location) $\sigma^2 > 0$ = variance (squared scale)
Support	$x \in \mathbb{R}$
PDF	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
CDF	$\boxed{\frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]}$
Quantile	$\mu + \sigma\sqrt{2}\operatorname{erf}^{-1}(2p-1)$
Mean	μ
Median	μ
Mode	μ
Variance	σ^2
MAD	$\sigma\sqrt{2/\pi}$
Skewness	0
Ex.	0

Using put-call parity, the price of a European put option is

$$
\begin{aligned}
p &= c + Xe^{-\mathrm{r}\,\tau} - S \\
&= Xe^{-\mathrm{r}\,\tau}\, N(-d_2) - S\, N(-d_1).
\end{aligned}
$$

Some Properties of the Black-Scholes Formula

Consider the value of the European calls and puts when S becomes large. As $S \to \infty$, $d_1 \to \infty$ and $d_2 \to \infty$. Therefore $N(d_1) \to 1$ and $N(d_2) \to 1$, but $N(-d_1) \to 0$ and $N(-d_2) \to 0$. Hence, as $S \to \infty$, for European options:

$$\begin{aligned}
&\text{Call:} \quad && \lim_{S\to\infty} \left[S\,N(d_1) - Xe^{-r\tau} N(d_2) \right] && = \infty, \\
&\text{Put:} \quad && \lim_{S\to\infty} \left[Xe^{-r\tau} N(-d_2) - S\,N(-d_1) \right] && = 0
\end{aligned}$$

as we would expect.

Conversely, as $S \to 0$, $d_1 \to -\infty$ and $d_2 \to -\infty$. Therefore $N(d_1) \to 0$ and $N(d_2) \to 0$, but $N(-d_1) \to 1$ and $N(-d_2) \to 1$. Hence, as $S \to 0$, for European options:

$$\begin{aligned} \text{Call:} \quad & \lim_{S \to 0} \left[S\, N(d_1) \, - \, X e^{-\mathrm{r}\,\tau} N(d_2) \right] && = 0, \\ \text{Put:} \quad & \lim_{S \to 0} \left[X e^{-\mathrm{r}\,\tau}\, N(-d_2) - S\, N(-d_1) \right] && = X e^{-\mathrm{r}\,\tau} \end{aligned}$$

Consider what happens when the stock's volatility, σ, becomes small. If $\ln(S/X) + r\tau > 0$ (equivalent to $S > Xe^{-\mathrm{r}\,\tau}$), then as $\sigma \to 0$, $d_1 \to \infty$ and $d_2 \to \infty$ so that

$$\begin{aligned} &\text{Call:} && \lim_{\sigma \to 0} c \Big|_{S > Xe^{-\mathrm{r}\,\tau}} && = S - Xe^{-\mathrm{r}\,\tau}, \\ &\text{Put:} && \lim_{\sigma \to 0} p \Big|_{S > Xe^{-\mathrm{r}\,\tau}} && = 0. \end{aligned}$$

Instead, if $S < Xe^{-\mathrm{r}\,\tau}$, as $\sigma \to 0$, $d_1 \to -\infty$ and $d_2 \to -\infty$ so that

$$\begin{aligned} &\text{Call:} && \lim_{\sigma \to 0} c \Big|_{S < Xe^{-\mathrm{r}\,\tau}} && = 0, \\ &\text{Put:} && \lim_{\sigma \to 0} p \Big|_{S < Xe^{-\mathrm{r}\,\tau}} && = Xe^{-\mathrm{r}\,\tau} - S. \end{aligned}$$

Estimating the Volatility Parameter

1. The Black-Scholes formula depends on S, X, r, τ, and σ.
2. r is usually taken as the risk-free interest rate on an investment maturing in τ periods, at the expiration of the option contract.
3. The only parameter that is not directly observable is the stock's volatility, σ.
4. This can be estimated using historical data on stock prices or stock returns.

Recall that if μ is assumed to be constant, then $\ln(\tilde{S}_T) \sim n\big(\ln(S) + (\mu - \sigma^2/2)\tau,\, \sigma^2\tau\big)$, so that $\ln(\tilde{S}_T/S) \sim n\big((\mu - \sigma^2/2)\tau,\, \sigma^2\tau\big)$. Now suppose we observe $n+1$ stock prices (or n stock returns) at intervals of length τ. Define $u_i \equiv \ln(S_i/S_{i-1})$ as the continuously compounded stock return over the i^{th} interval. Then $u_i \sim n\big((\mu - \sigma^2/2)\tau,\, \sigma^2\tau\big)$ and we can use the usual estimates for a sample mean and sample variance to calculate σ

$$\text{Sample mean: } \overline{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$$

$$\text{Sample variance: } \widehat{s}^2 = \frac{1}{n-1}\sum_{i=1}^{n} (u_i - \overline{u})^2$$

Since, $\hat{s}^2$ is the estimate of $\sigma^2\tau$, the estimate for σ to be used in the Black-Scholes formula is

$$\sigma = \sqrt{\hat{s}^2/\tau} = \frac{\hat{s}}{\sqrt{\tau}}$$

where $\hat{s}$ = sample standard deviation of the u_i's.

If daily data is used to calculate σ, rather than using $\tau = \frac{1}{365}$, the practice is to use $\tau = \frac{1}{250}$, where 250 is approximately the number of trading days (days stock exchanges are open) per year. Reason: empirical evidence finds stock volatility is much higher on trading days. We can ignore days when exchanges are closed.

Modifying the Formula for Dividends

Consider a European option on a dividend paying stock. Assume that the stock's dividends are known over the life of the option. The current value of the stock, S, equals the present value of (assumed) riskless dividends paid during the life of the option, D, and a risky component, $\hat{S}$, equal to the expected present value of both capital gains and other risky dividends.

$$S = D + \hat{S}$$

Since the price of the stock will decline when dividends are paid, D will not be received by the holder of a call, nor paid by the holder of a put. Therefore, the option is essentially written on $\hat{S}$, not on $S = D + \hat{S}$. Thus, we can value a European call or put by using the Black-Scholes formula replacing S with $\hat{S} = S - D$.

HW4

- A stock is currently priced at \$75 and has a σ of 0.35.
- It will pay two \$1 dividends in 1 month and 4 months.
- If r = 6%, what is the value of a European put and call option with an exercise 8 price of \$65 maturing in 6 months?

$$D = \$1\, e^{-0.06\,(\frac{1}{12})} + \$1\, e^{-0.06\,(\frac{4}{12})} \cong \$1.9752.$$

Thus, $\hat{S} = S - D \cong \$75 - \$1.9752 \cong \$73.02.$

$$\begin{aligned}
d_1 &= \frac{\ln(\widehat{S}/X) + (\mathrm{r} + \frac{1}{2}\sigma^2)\,\tau}{\sigma\sqrt{\tau}} \\
&\cong \frac{\ln(73.02/65.00) + (0.06 + \frac{1}{2}0.35^2)\,\frac{6}{12}}{0.35\sqrt{\frac{6}{12}}} \\
&\cong 0.715 \\
d_2 &= d_1 - \sigma\sqrt{\tau} \\
&\cong 0.715 - 0.35\sqrt{\frac{6}{12}} \\
&\cong 0.468.
\end{aligned}$$

$N(-0.715) \cong 0.2373$ and $N(-0.468) \cong 0.3199$, so

$$\begin{aligned}
p &= Xe^{-\mathrm{r}\,\tau}\,N(-d_2) - \widehat{S}\,N(-d_1) \\
&\cong \$65\, e^{-0.06\,(\frac{6}{12})}\,(0.3199) - \$73.02\,(0.2373) \\
&\cong \$20.179 - \$17.328 \\
&\cong \$2.85.
\end{aligned}$$

Q and A