Overfitting to evaluate Linear Regression Model and Non-linear Regression

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INTRODUCTION

Regression Definition:

• A regression is a statistical analysis assessing the association between two variables. It is used to find the relationship between two variables.

Regression Formula: (another formla produces the same result)

```
Regression Equation(y) = a + bx

Slope(b) = (N\Sigma XY - (\Sigma X)(\Sigma Y)) / (N\Sigma X^2 - (\Sigma X)^2)

Intercept(a) = (\Sigma Y - b(\Sigma X)) / N
```

Where:

x and y are the variables.

b = The slope of the regression line

a = The intercept point of the regression line and the y axis.

N = Number of values or elements

X = First Score

Y = Second Score

EXY = Sum of the product of first and Second Scores

ΣX = Sum of First Scores

 $\Sigma Y = Sum of Second Scores$

 ΣX^2 = Sum of square First Scores

DESIGN

Linear Regression using Least Square Method

- This formula is called <u>The Normal Equation</u> which is based on <u>Linear Regression using Least Square Method</u>.
- To find the Simple/Linear Regression of given data X and Y
- First find slope, intercept and use it to form regression equation.
 - 1. Step 1: Count the number of values. N = 10
 - Step 2:
 Find X * Y, X²

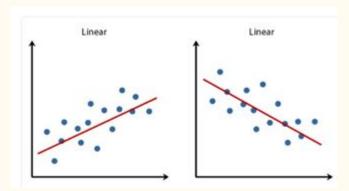
X Value	Y Value	X*Y	X*X
1	1.8	1*1.8=1.8	1*1=1
2	2.4	2*2.4=4.8	2*2=4
3.3	2.3	3.3*2.3=7.59	3.3*3.3=10.89
4.3	3.8	4.3*3.8=16.34	4.3*4.3=18.49
5.3	5.3	5.3*5.3=28.09	5.3*5.3=28.09
1.4	1.5	1.4*1.5=2.1	1.4*1.4=1.96
2.5	2.2	2.5*2.2=5.5	2.5*2.5=6.25
2.8	3.8	2.8*3.8=10.64	2.8*2.8=7.84
4.1	4.0	4.1*4.0=16.4	4.1*4.1=16.81
5.1	5.4	5.1*5.4=27.54	5.1*5.1=26.01

Linear Regression using Least Square Method

3 Step 3:

Find ΣX , ΣY , ΣXY , ΣX^2 .

- \circ $\Sigma X = 31.8$
- \circ $\Sigma Y = 32.5$
- \circ $\Sigma XY = 120.8$
- \circ $\Sigma X^2 = 121.34$



Step 4:

Substitute in the above slope formula given.

```
Slope (b) = (N\Sigma XY - (\Sigma X)(\Sigma Y)) / (N\Sigma X^2 - (\Sigma X)^2)
= ((10)*(120.8)-(31.8)*(32.5))/((10)*(121.34)-(31.8)^2)
= (1208 - 1033.5)/(1213.4 - 1011.24)
= 174.5/202.16
= 0.86
```

Linear Regression using Least Square Method

Step 5:

Now, again substitute in the above intercept formula given.

```
Intercept(a) = (\Sigma Y - b(\Sigma X)) / N
= (32.5 - 0.86(31.8))/10
= (32.5 - 27.34)/10
= 51.52/10
= 0.5152
```

Step 6:

Then substitute Intercept(a) and Slope(b) in regression equation formula

Linear Regression Equation(y) = a + bx

$$= 0.51 + 0.86x$$
.

Linear Regression using Least Square Method

Step 7:

Suppose if we want to know the approximate y value for the variable x = 64. Then we can substitute the value in the above equation.

Regression Equation(y) = a + bx

- = -8.098 + 0.19(64).
- = -8.098 + 12.16
 - = 4.06

Non-linear Regression:

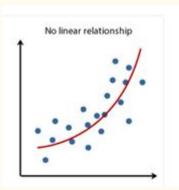
Regression Equation(y) =
$$a + b^2$$

We can still use <u>Linear Regression formula</u>

Slope(b) =
$$(N\Sigma \underline{P}Y - (\Sigma \underline{P})(\Sigma Y)) / (N\Sigma \underline{P}^2 - (\Sigma \underline{P})^2)$$

Intercept(a) =
$$(\Sigma Y - b(\Sigma P)) / N$$

Where
$$\underline{P} = X * X$$



Non-linear Regression:

To find the Simple Non-Linear Regression of given data X and Y We can simply create \underline{X} from X

To find regression equation, we will first find slope, intercept and use it to form regression equation.

- o Step 1:
 - Count the number of values. N = 10
- o Step 2:

Find $X * Y, X^2$

X Value	Y Value	<u>X</u> =X*X			
1	1.8	1*1=1			
2	2.4	2*2=4			
3.3	2.3	3.3*3.3=10.89			
4.3	3.8	4.3*4.3=18.49			
5.3	5.3	5.3*5.3=28.09			
1.4	1.5	1.4*1.4=1.96			
2.5	2.2	2.5*2.5=6.25			
2.8	3.8	2.8*2.8=7.84			
4.1	4.0	4.1*4.1=16.81			
5.1	5.4	5.1*5.1=26.01			

Non-linear Regression:

Step 3:

Find $\Sigma \underline{X}$, ΣY , $\Sigma \underline{X}Y$, $\Sigma \underline{X}^2$.

$$\Sigma \underline{X} = 121.34$$
 $\Sigma Y = 32.5$
 $\Sigma \underline{X}Y = 509.5$
 $\Sigma \underline{X}^2 = 2329.92$

Step 4:

Substitute in the above slope formula given.

X Value	Y Value	<u>X</u> *Y	<u>X*X</u>
1	1.8	1*1.8=1.8	1*1=1
4	2.4	4*2.4=9.6	4*4=16
10.89	2.3	10.89*2.3=25.04	10.89*10.89=118.59
18.49	3.8	18.49*3.8=70.02	18.49*18.49=341.88
28.09	5.3	28.09*5.3=148.87	28.09*28.09=789
1.96	1.5	1.96*1.5=2.94	1.96*1.96=3.84
6.25	2.2	6.25*2.2=13.75	6.25*6.25=39.06
7.84	3.8	7.84*3.8=29.79	7.84*7.84=61.46
16.81	4.0	16.81*4.0=67.24	16.81*16.81=282.57
26.01	5.4	26.01*5.4=140.45	26.01*26.01=676.52

```
Slope (b) = (N\Sigma\underline{X}Y - (\Sigma\underline{X})(\Sigma Y)) / (N\Sigma\underline{X}^2 - (\Sigma\underline{X})^2)

= ((10)*(509.5) - (121.34)*(32.5)) / ((10)*(2329.92) - (121.34)

= (5095 - 3943.55) / (23299.2 - 14723.39)

= 1151.45/8575.80

= 0.1346
```

• Step 5:

Now, again substitute in the above intercept formula given.

```
Intercept (a) = (\Sigma Y - b(\Sigma X)) / N
= (32.5 - 0.00021(121.34))/10
= (32.5 - 0.0254)/10
= 32.474/10
= 1.6168
```

Step 6:

Then substitute these values in regression equation formula Regression Equation(y) = $\frac{a}{a} + \frac{b}{b}x^2$

$$= 1.6168 + 0.1346x^{2}$$

Step 7:

Suppose if we want to know the approximate y value for the variable x = 64. Then we can substitute the value in the above equation.

Regression Equation(y) = a + bx² = 1.6168 + 0.1346 (64 * 64) =1.6168 + 551.32 = 552.93

Mean Square Error:

Mean Squared Error (MSE)

- The Mean Squared Error (MSE) is a measure of how close a fitted line is to data points.
 - · The smaller the MSE, the closer the fit is to the data.
- If \(\hat{Y} \) is a vector of n predictions, and \(Y \) is the vector of the true values, then the (estimated) \(\text{MSE} \) of the predictor is:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (\hat{Y}_i - Y_i)^2.$$

	Tr	ain	ing Se	t		Validation Set					
Real Data		Me	odel 1	Model 2		Real	Data	Model 1		Model 2	
x	y	x	ŷ	x	ŷ	x	y	x	ŷ	x	ŷ
1	1.8	1	1.38	1	1.75	1	1.7	1	1.81	1	1.91
2	2.4	2	2.24	2	2.15	2	2.7	2	3.014	2	2.74
3.3	2.3	3.3	3.358	3.3	3.08	3.3	2.5	3.3	3.702	3.3	3.45
4.3	3.8	4.3	4.218	4.3	4.10	4.3	2.8	4.3	4.562	4.3	4.59
5.3	5.3	5.3	5.078	5.3	5.39	5.3	5.5	5.3	4.902	5.3	5.11

Training Set

Model 1

- MSE
- = $[(1.38-1.8)^2 + (2.24-2.4)^2 + (3.358-2.3)^2 + (4.218-3.8)^2 + (5.078-5.3)^2]/5$
- (31313 313) = 1,7
- = 0.3347

Model 2

• MSE

```
[(1.75-1.8)^2 + (2.15-2.4)^2 + (3.08-2.3)^2 + (4.10-3.8)^2 + (5.39-5.3)^2]/5
```

= 0.17712

Validation Set

Model 1

• MSE

```
= [(1.81-1.7)^2 + (3.014-2.7)^2 + (3.70-2.5)^2 + (4.56-2.8)^2 + (4.90-5.5)^2]/5
= 1.0035
```

Model 2

• MSE

```
= [(1.91-1.7)^2 + (2.74-2.7)^2 + (3.45-2.5)^2 + (4.59-2.8)^2 + (5.11-5.5)^2]/5
```

= 0.86

- Compare Model 1 and Model 2
 - Mode1

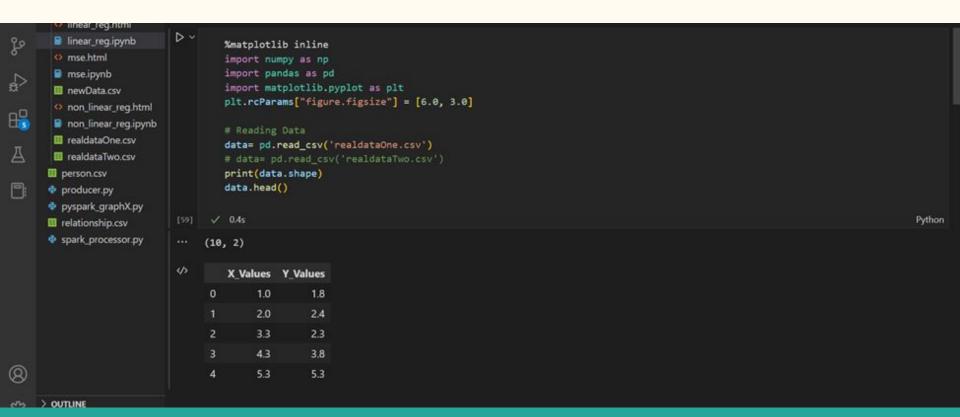
$$1.003 / 0.334 = 3.0$$

Model 2

$$0.860 / 0.177 = 4.86$$

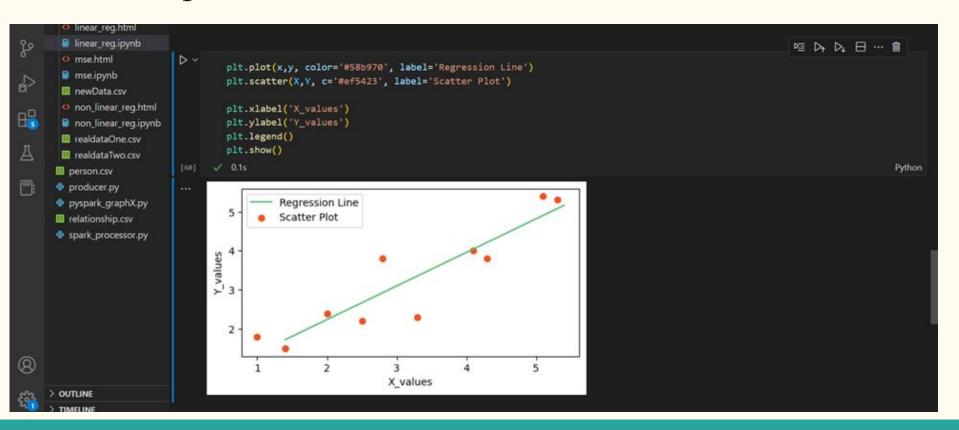
- Conclusion
 - Model 1 is a better model

IMPLEMENTATION: Linear Regression

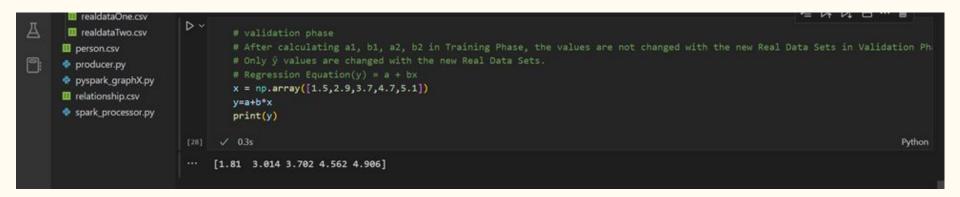


## ## ## ## ## ## ## ## ## ## ## ## ##			
<pre>X_values=np.array(X) print('Sum of all the X_values: ", round(x_values.sum(),2)) y_values=np.array(Y) print('Sum of all the Y_values in xip(X, Y)] xy = [x_values * y_values for x_values, y_values in xip(X, Y)] xy_values=np.array(x) print('Sum of all the X_values: ", round(xy_values.sum(),2)) xx = [x_values * x_values for x_values, y_values in xip(X, Y)] xy_values=np.array(x) print('Sum of all the X_values: ", round(xy_values.sum(),2)) xx = [x_values * x_values for x_values, y_values.sum(),2)) print('Sum of all the X_values: ", xx_values.sum()) Sum of all the X_values: 31.8 Sum of all the X_values: 32.5 Sum of all the X_values: 120.8 Sum</pre>		<pre>X= data['X_Values'].values Y= data['Y_Values'].values</pre>	Python
X_valuesnp.array(X)	20		
print("Sum of all the Y_Values: ", y_values in zip(X, Y)] xy = [x_values * y_values for x_values, y_values in zip(X, Y)] xy_values=np.array(xy) print("Sum of all the XY_Values: ", round(xy_values.sum(),2)) xx = [x_values * x_values for x_values, x_values in zip(X, X)] xx_values=np.array(xx) print("Sum of all the XY_values: ", xx_values in zip(X, X)] xx_values=np.array(xx) print("Sum of all the X_values: ", xx_values.sum()) Python Sum of all the X_values: 31.8 Sum of all the X_values: 120.8 Sum of all the X_values: 120.8 Sum of all the X_values: 121.34 *Using the formula to calculate slope(b) n= len(X) b = round(((n* xy_values.sum())-(x_values.sum()*y_values.sum()))/ ((n*xx_values.sum())-(x_values.sum()**2)),2) print("slope(b) is: 0.86 *Using the formula to calculate intercept(a) a = round((y_values.sum() - (b*x_values.sum()))/n ,2) print("intercept(a): ", a) Python Python			
<pre>xy_walues=np.array(xy) print("Sum of all the XY_Values: ", round(xy_values.sum(),2))</pre>			
XX_values=np.array(XX)		xy_values=np.array(xy)	
Python Sum of all the X_values: 31.8 Sum of all the Y_values: 32.5 Sum of all the XX_values: 120.8 Sum of all the XX_values: 121.34 # Using the formula to calculate slope(b) n=len(X) b = round(((n* xy_values.sum())-(x_values.sum()))/ ((n*xx_values.sum())-(x_values.sum()**2)),2) print("slope(b) is: ", b) (141)		xx_values=np.array(xx)	
Sum of all the Y_Values: 32.5 Sum of all the XY_Values: 120.8 Sum of all the XX_Values: 121.34 Pusing the formula to calculate slope(b) n= len(X) b = round((n* xy_values.sum())-(x_values.sum()))/ ((n*xx_values.sum())-(x_values.sum()**2)),2) print("slope(b) is: ", b) Python But the XY_Values: 121.34 # Using the formula to calculate intercept(a) a = round((y_values.sum() - (b*x_values.sum()))/n ,2) print("intercept(a): ", a) Python	[145]		Python
b = round(((n* xy_values.sum())-(x_values.sum()))/ ((n*xx_values.sum())-(x_values.sum()**2)),2) print("slope(b) is: ", b) Python Slope(b) is: 0.86 # Using the formula to calculate intercept(a) a = round((y_values.sum() - (b*x_values.sum()))/n ,2) print("intercept(a): ", a) Python	8	Sum of all the Y_Values: 32.5 Sum of all the XY_Values: 120.8	
<pre># Using the formula to calculate intercept(a) a = round((y_values.sum() - (b*x_values.sum()))/n ,2) print("intercept(a): ", a)</pre> <pre>Python</pre>		<pre>n= len(X) b = round(((n* xy_values.sum())-(x_values.sum()*y_values.sum()))/ ((n*xx_values.sum())-(x_values.sum()**2)),2)</pre>	
<pre># Using the formula to calculate intercept(a) a = round((y_values.sum() - (b*x_values.sum()))/n ,2) print("intercept(a): ", a)</pre> <pre>Python</pre>	[141]		Python
<pre>a = round((y_values.sum() - (b*x_values.sum()))/n ,2) print("intercept(a): ", a) [146]</pre> Python		slope(b) is: 0.86	
		<pre>a = round((y_values.sum()) - (b*x_values.sum()))/n ,2) print("intercept(a): ", a)</pre>	Python

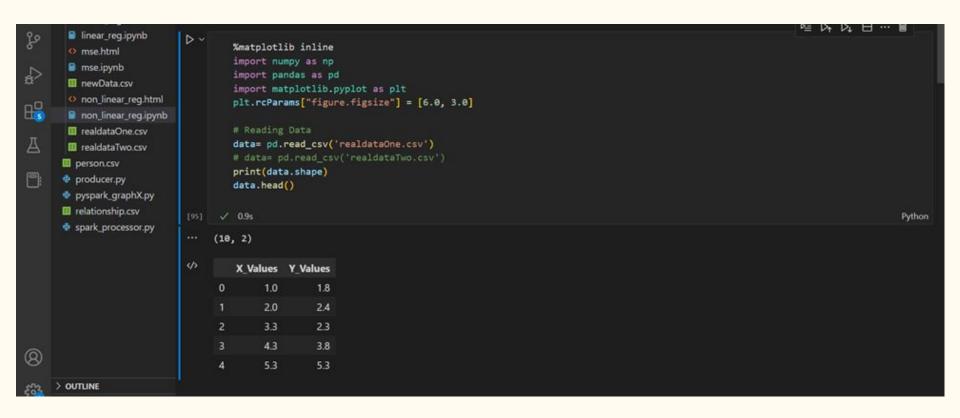
Linear Graph



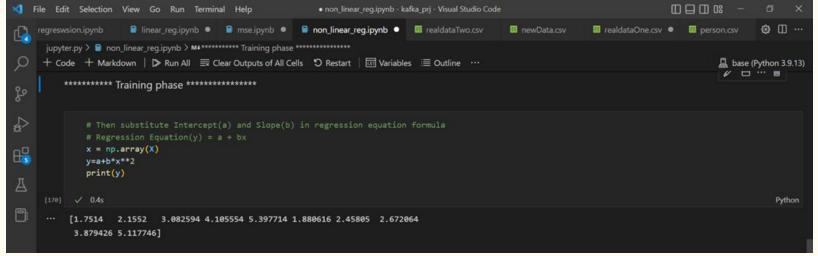
Validation phase:



IMPLEMENTATION: NonLinear Regression



```
linear_reg.ipynb
       mewData.csv
                                     #Collecting X and Y
       non_linear_reg.ipynb
                                     X= data['X Values'].values
       III values.csv
                                     Y= data['Y_Values'].values
      person.csv
                                                                                                                                                              Python
      producer.py
B.
      pyspark_graphX.py
      relationship.csv
                                     y_values=np.array(Y)
                                     print("Sum of all the Y_Values: ", y_values.sum())
      spark_processor.py
                                     xx = [x_values * x_values for x_values, x_values in zip(X, X)]
xx_values=np.array(xx)
                                     print("Sum of all the XX_Values: ", xx_values.sum())
                                                                                                                                                              Python
                                 Sum of all the Y_Values: 32.5
                                 Sum of all the XX Values: 121.34
               xy = [xx_values * y_values for xx_values, y_values in zip(xx, Y)]
               xy_values=np.array(xy)
               print("Sum of all the XY_Values: ", round(xy_values.sum(),2))
$
```



NonLinear Graph:

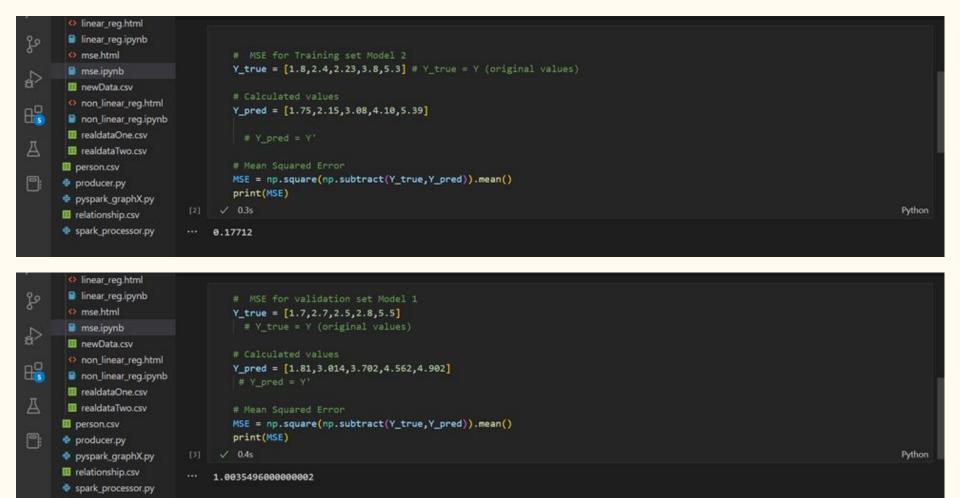


Validation Phase

```
m realdataOne.csv
m realdataTwo.csv
                               # validation phase
                               # After calculating al, b1, a2, b2 in Training Phase, the values are not changed with the new Real Data Sets in Validation Ph
m person.csv
                               # Only ŷ values are changed with the new Real Data Sets.
producer.py
                               # Regression Equation(y) = a + bx
pyspark_graphX.py
                               x = np.array([1.5, 2.9, 3.7, 4.7, 5.1])
relationship.csv
                               y=a+b*x**2
spark_processor.py
                               print(y)
                                                                                                                                                           Python
                           [1.91965 2.748786 3.459474 4.590114 5.117746]
```

Mean Square Error(MSE):





```
mse.html
                                      Y true = [1.7,2.7,2.5,2.8,5.5] # Y true = Y (original values)
        mse.ipynb
                                       # Calculated values
       mewData.csv
                                      Y_pred = [1.91, 2.74, 3.45, 4.59, 5.11]
        o non_linear_reg.html
8
                                         # Y pred = Y'
        non_linear_reg.ipynb
       m realdataOne.csv
                                      # Mean Squared Error
       m realdataTwo.csv
                                      MSE = np.square(np.subtract(Y_true,Y_pred)).mean()
                                      print(MSE)
       person.csv
       producer.py
                                                                                                                                                                    Python
       pyspark_graphX.py
                                   0.8608800000000001
       marelationship.csv
       spark processor.pv
         linear_reg.html
         linear_reg.ipynb
                                        # max(Training Set MSE, Validation Set MSE) / min(Training Set MSE, Validation Set MSE)
         mse.html
                                        # Compare Model 1 and Model 2
         mse.ipynb
                                        Training_Set_MSE = 0.3347568
         III newData.csv
                                        Validation_Set_MSE =1.00354960000000002
         o non linear reg.html
 ModelOne =Validation_Set_MSE/Training_Set_MSE
         non_linear_reg.ipynb
                                        print("Model 1 MSE :", round(ModelOne,2))
         m realdataOne.csv
 A
         III realdataTwo.csv
        m person.csv
                                        # Model 2
 Training_Set_MSE =0.17712
        producer.py
                                        Validation Set MSE = 0.8608800000000001
        pyspark_graphX.py
                                        ModelTwo =Validation_Set_MSE/Training_Set_MSE
        relationship.csv
                                        print("Model 2 MSE :", round(ModelTwo,2))
        spark processor.py
                                        # Compare to find better model
                                        if ( ModelTwo < ModelOne ):
                                           print("Model 2 is a better model")
                                        else:
                                           print("Model 1 is a better model")
                                                                                                                                                                    Python
                                    Model 1 MSE : 3.0
 (8)
                                    Model 2 MSE : 4.86
                                    Model 1 is a better model
```

MSE for validation set Model 2

linear_reg.ipynb

Test Phase:



	Training Ph	nase	7	alidation P	hase	Test Phase					
Real Data Set 1 50% of the collcted data	Model 1: Linear Regression	Model 2: Non- Linear Regression	Real Data Set 2 25% of the collcted data	Regression	Model 2: Non- Linear Regression	Real Data Set 3 25% of the collcted data	The better model (Model 1 or Model 2) selected from the Validation Phase based on the analysis of overfitting will be used to calculate ŷ				
(After calculating a1, b1, a2, b2 in Training Phase, the values are not changed with the new Real Data Sets in Validation Phase and Test Phase. Only ŷ values are changed with the new Real Data Sets. 										

Only y values are changed with the new Near Data Sets.

x	у	ŷ=a1 + b1 * x	$\hat{y}=a2+b2$ * x^2	x	y	ŷ=a1 + b1 * x	$ \hat{y}=a2+b2 $ * x^2	x	$ \hat{y}=a1 + b1 * x $ or $ \hat{y}=a2 + b2 * x^2 $
1	1.8	1.38	1.75	1.5	1.7	1.81	1.91	1.4	1.724
2	2.4	2.24	2.15	2.9	2.7	3.014	2.74	2.5	2.67
3.3	2.3	3.358	3.08	3.7	2.5	3.702	3.45	3.6	3.616
4.3	3.8	4.218	4.10	4.7	2.8	4.562	4.59	4.5	4.39
5.3	5.3	5.078	5.39	5.1	5.5	4.902	5.11	5.4	5.164
1.4	1.5	1.724	1.88	X	X	X	X	X	X
2.5	2.2	2.67	2.45	X	X	X	X	X	X
2.8	3.8	2.928	2.67	X	X	X	X	X	X
4.1	4.0	4.046	3.87	X	X	X	X	X	X
5.1	5.4	4.906	5.11	X	X	X	X	X	X

Enhancement Ideas:

- This data set is used to minimize overfitting.
 - To verify that any increase in accuracy over the training data set actually yields an increase in accuracy over a data set that has not been shown to the model before, or at least the model hasn't trained on it (i.e. validation data set).
 - If the accuracy over the training data set increases, but the accuracy over then validation data set stays the same or decreases, then you're overfitting your model and you should stop training.
 - A test set (i.e., validation set) is a set of data that is independent of the training data, but that follows the same probability distribution as the training data.
 - If a model fit to the training set also fits the test set well, minimal overfitting has taken place.
 - If the model fits the training set much better than it fits the test set, overfitting is likely the cause.
- In order to avoid overfitting, when any classification parameter needs to be adjusted, it is necessary to have a validation set in addition to the training and test sets.
 - For example if the most suitable classifier for the problem is sought,
 - Training Data is used to train the candidate algorithms.
 - Validation Data is used to compare their performances and decide which one to take.
 - Test Data is used to obtain the performance characteristics such as accuracy, sensitivity, specificity.

Conclusion:

• If the accuracy over the training data set increases, but the accuracy over the validation data set stays the same or decreases, then you're overfitting your model and you should stop training.

 If the model fits the training set much better than it fits the test set (i.e., validation set), overfitting is likely the cause.

References:

Prof Chang's class material

Linear regression Algorithm

Non Linear Regression Algorithm

Thank you