University of Tehran

Clustering-Based Activity Detection Algorithms for Grant-Free Random Access in Cell-Free Massive MIMO

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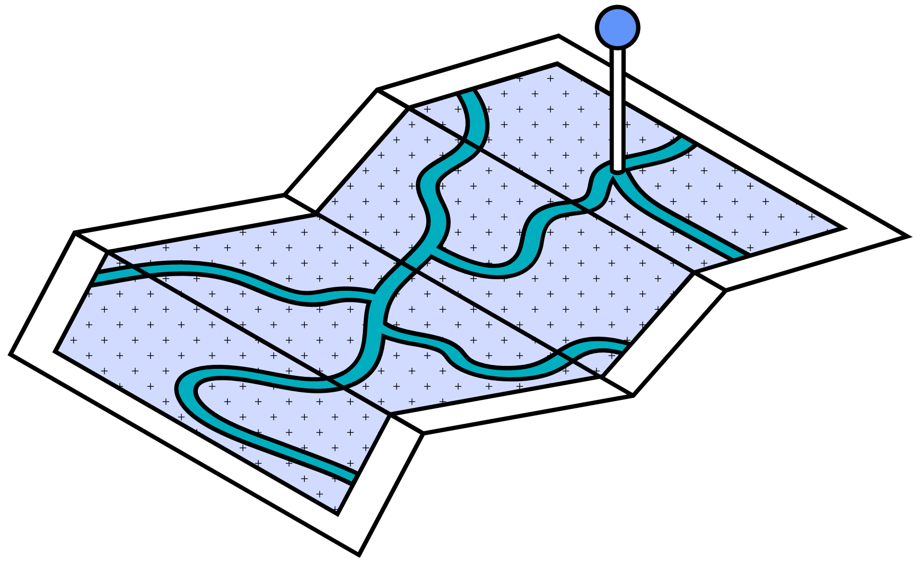

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Introduction

- eMBB / mMTC / URLLC
- Conventional Grant-Based Massive Random Access
- Cell-Free Massive MIMO
- Cluster-Based Detection Algorithm

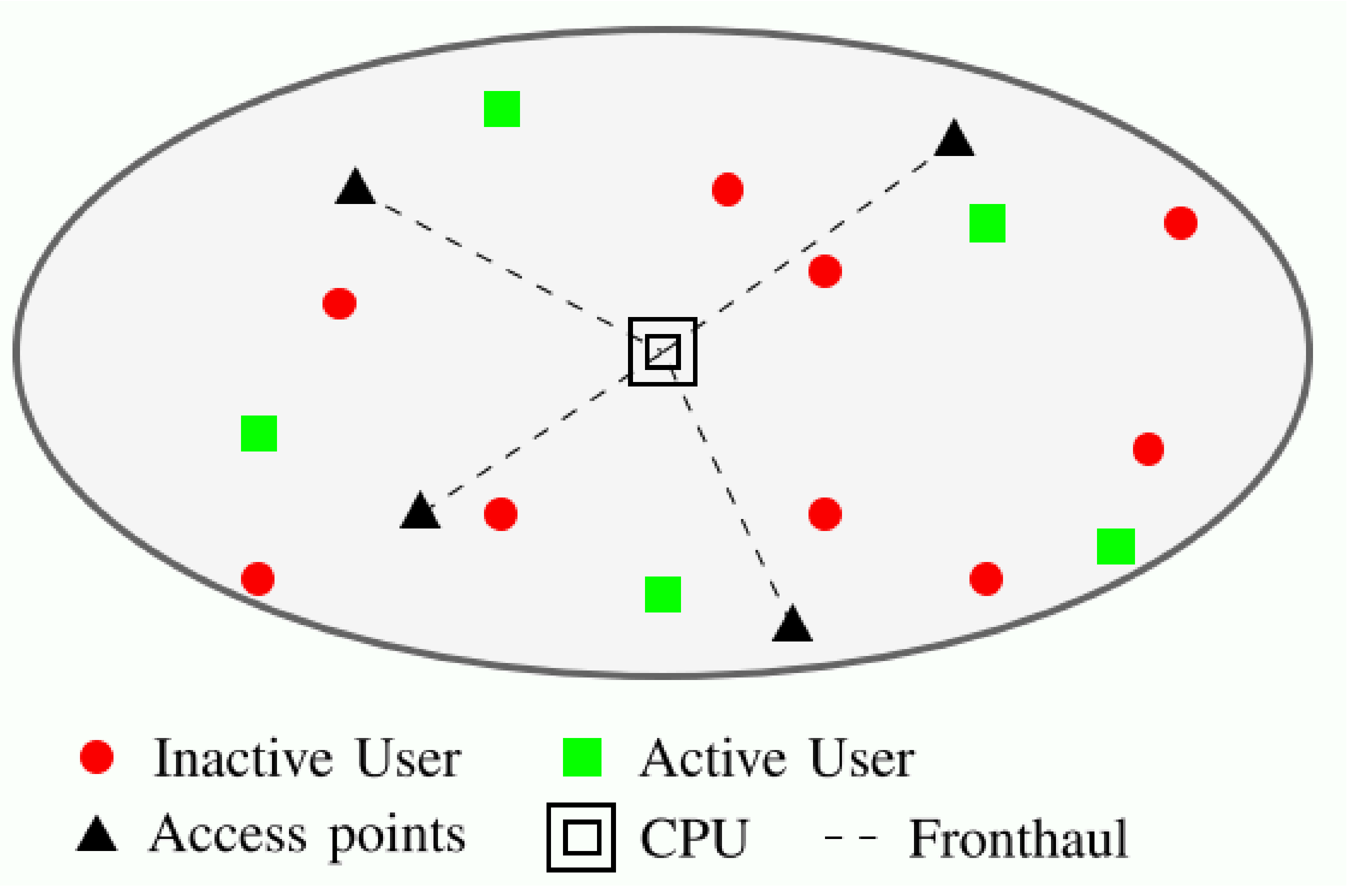


Fig1: Cell-Free Network Model for mMTC-

Problem Formulation

$$g_{mnk} = \beta_{mk}^{\frac{1}{2}} h_{mnk}$$

$$\begin{aligned}\mathbf{y}_{mn} &= \sum_{k=1}^{K} a_k \rho_k^{\frac{1}{2}} g_{mnk} \mathbf{s}_k + \mathbf{w}_{mn} \\ &= \mathbf{S}\mathbf{D}_{\mathrm{a}}\mathbf{D}_{\rho}^{\frac{1}{2}} \mathbf{g}_{mn} + \mathbf{w}_{mn},\end{aligned}$$

$$\mathbf{Y}_m = \mathbf{S}\mathbf{D}_{\mathrm{a}}\mathbf{D}_{\rho}^{\frac{1}{2}} \mathbf{G}_m + \mathbf{W}_m,$$

$\longrightarrow$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \mathbf{Y}_M \end{bmatrix} = \begin{bmatrix} \mathbf{S}\mathbf{D}_{\mathrm{a}}\mathbf{D}_{\rho}^{\frac{1}{2}}\mathbf{G}_1 \\ \mathbf{S}\mathbf{D}_{\mathrm{a}}\mathbf{D}_{\rho}^{\frac{1}{2}}\mathbf{G}_2 \\ \vdots \\ \mathbf{S}\mathbf{D}_{\mathrm{a}}\mathbf{D}_{\rho}^{\frac{1}{2}}\mathbf{G}_M \end{bmatrix} + \mathbf{W} \qquad \mathbf{Y}(:,i) \sim \mathcal{CN}(\mathbf{0}_{LM}, \mathbf{Q}),$$

$$= \begin{bmatrix} \mathbf{S} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{S} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{D}_{\mathrm{a}}\mathbf{D}_{\rho}^{\frac{1}{2}} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{\mathrm{a}}\mathbf{D}_{\rho}^{\frac{1}{2}} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{D}_{\mathrm{a}}\mathbf{D}_{\rho}^{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \\ \vdots \\ \mathbf{G}_M \end{bmatrix} + \mathbf{W},$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{S}\mathbf{D}_{\gamma}\mathbf{D}_{\beta_1}\mathbf{S}^{\mathrm{H}} & \mathbf{0}_L & \dots & \mathbf{0}_L \\ \mathbf{0}_L & \mathbf{S}\mathbf{D}_{\gamma}\mathbf{D}_{\beta_2}\mathbf{S}^{\mathrm{H}} & \dots & \mathbf{0}_L \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_L & \mathbf{0}_L & \dots & \mathbf{S}\mathbf{D}_{\gamma}\mathbf{D}_{\beta_M}\mathbf{S}^{\mathrm{H}} \end{bmatrix} + \sigma^2 \mathbf{I}_{LM},$$

$$\begin{aligned} p(\mathbf{Y}|\gamma) &= \prod_{m=1}^{M} \prod_{n=1}^{N} \frac{1}{|\pi \mathbf{Q}_m|} \exp\left(-\mathbf{y}_{mn}^{\mathrm{H}} \mathbf{Q}_m^{-1} \mathbf{y}_{mn}\right) \\ &= \prod_{m=1}^{M} \frac{1}{|\pi \mathbf{Q}_m|^N} \exp(-\operatorname{Tr}(\mathbf{Q}_m^{-1} \mathbf{Y}_m \mathbf{Y}_m^{\mathrm{H}})), \end{aligned}$$

$\longrightarrow$

$$\begin{aligned} \gamma^* &= \arg\min_{\gamma} \sum_{m=1}^{M} \log|\mathbf{Q}_m| + \operatorname{Tr}\left(\mathbf{Q}_m^{-1} \frac{\mathbf{Y}_m \mathbf{Y}_m^{\mathrm{H}}}{N}\right) \\ &\text{subject to } \gamma \geq \mathbf{0}_K. \end{aligned}$$

Device Activity Detection

$$f(\gamma) = \sum_{m=1}^{M} \log|\mathbf{Q}_m| + \mathrm{Tr}\left(\mathbf{Q}_m^{-1}\frac{\mathbf{Y}_m\mathbf{Y}_m^{\mathrm{H}}}{N}\right)$$

$$f^m(\gamma) = \log|\mathbf{Q}_m| + \mathrm{Tr}\left(\mathbf{Q}_m^{-1}\frac{\mathbf{Y}_m\mathbf{Y}_m^{\mathrm{H}}}{N}\right)$$

$$\begin{aligned}\mathbf{Q}_m(\gamma) &= \mathbf{S}\mathbf{D}_\gamma\mathbf{D}_{\boldsymbol{\beta}_m}\mathbf{S}^{\mathrm{H}} + \sigma^2\mathbf{I}_L \\ &= \sum_{k=1}^{K}\gamma_k\beta_{mk}\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}} + \sigma^2\mathbf{I}_L,\end{aligned}$$

Sherman-Morrison rank-1

$$f_k^m(d) = f^m(\gamma + d\mathbf{e}_k),$$

$$\left(\mathbf{Q}_m + d\beta_{mk}\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}}\right)^{-1} = \mathbf{Q}_m^{-1} - d\beta_{mk}\frac{\mathbf{Q}_m^{-1}\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}}{1 + d\beta_{mk}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}\mathbf{s}_k}.$$

$$|\mathbf{Q}_m + d\beta_{mk}\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}}| = (1 + d\beta_{mk}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}\mathbf{s}_k)|\mathbf{Q}_m|.$$

$$\xrightarrow{\star}$$

$$f_k(d) = c + \sum_{m=1}^{M}\left(\log(1 + d\beta_{mk}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}\mathbf{s}_k) - d\beta_{mk}\frac{\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}\mathbf{Q}_{\mathbf{Y}_m}\mathbf{Q}_m^{-1}\mathbf{s}_k}{1 + d\beta_{mk}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}\mathbf{s}_k}\right),$$

Dominant AP-Based Activity Detection

$$m' = \underset{m}{\operatorname{argmax}}\{\beta_{mk}\}$$

$$f_{k,m'}(d) = \log(1 + d\beta_{m'k}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k) - d\beta_{m'k}\frac{\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{Q}_{\mathbf{Y}_{m'}}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k}{1 + d\beta_{m'k}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k}.$$

$$d^* = \frac{\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{Q}_{\mathbf{Y}_{m'}}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k - \mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k}{\beta_{m'k}(\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k)^2}.$$

preserve the non-negativity $\longrightarrow$ $\max\{d^*, -\gamma_k\}$

Algorithm 1 Coordinate Descend Algorithm for Estimating γ

Input: Observations $\mathbf{Y}_m, \forall m = 1, 2, \ldots M$, $\beta_{mk}, \forall m = 1, 2, \ldots M, k = 1, 2, \ldots K$

Initialize: $\mathbf{Q}_m^{-1} = \sigma^{-2}\mathbf{I}_L, \forall m = 1, 2, \ldots M$, $\hat{\gamma}^0 = \mathbf{0}_K$

Compute $\mathbf{Q}_{\mathbf{Y}m} = \frac{1}{N}\mathbf{Y}_m\mathbf{Y}_m^{\mathrm{H}}, \forall m = 1, 2, \ldots M$

for $i = 1, 2, \ldots, I$ **do**

  Select an index set $\mathcal{K}$ from the random permutation of set $\{1, 2, \ldots, K\}$

  for $k \in \mathcal{K}$ **do**

    Find the strongest link or AP for device k, i.e., $m' = \underset{m}{\operatorname{argmax}}\{\beta_{mk}\}$

    $\delta = \max\left\{\frac{\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{Q}_{\mathbf{Y}m'}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k - \mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k}{\beta_{m'k}(\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_{m'}^{-1}\mathbf{s}_k)^2}, -\hat{\gamma}_k\right\}$

    $\hat{\gamma}_k^i = \hat{\gamma}_k^{i-1} + \delta$

    for $m = 1, 2, \ldots, M$ **do**

      $\mathbf{Q}_m^{-1} \leftarrow \mathbf{Q}_m^{-1} - \delta\frac{\beta_{mk}\mathbf{Q}_m^{-1}\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}}{1+\delta\beta_{mk}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}\mathbf{s}_k}$

    end for

  end for

  if $f(\hat{\gamma}^i) \geq f(\hat{\gamma}^{i-1})$ **then**

    $\hat{\gamma} = \hat{\gamma}^{i-1}$

    break

  end if

  $\hat{\gamma} = \hat{\gamma}^i$

end for

return $\hat{\gamma}$

Clustering Based Activity Detection

- Algorithm 1 uses data from one dominant AP per device
- optimal method would be using all APs

$$\mathcal{M}_k = \underset{m,T}{\text{indmax}} \{\beta_{mk}\},$$

$$a_m = \beta_{mk}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}\mathbf{s}_k$$
$$b_m = \beta_{mk}\mathbf{s}_k^{\mathrm{H}}\mathbf{Q}_m^{-1}\mathbf{Q}_{\mathbf{Y}_m}\mathbf{Q}_m^{-1}\mathbf{s}_k.$$

$$f_{k,T}(d) = \sum_{m\in\mathcal{M}_k}\left(\log(1+da_m) - \frac{db_m}{1+da_m}\right)$$

$$\sum_{m\in\mathcal{M}_k}\Big(((a_m+b_m)+a_m^2 d)\prod_{m'\in\mathcal{M}_k\setminus\{m\}}(1+2a_{m'}d + a_{m'}^2 d^2)\Big) = 0$$

$$\mathcal{D} = \{d : f'_{k,T}(d) = 0, \Im(d) = 0, \Re(d) \geq -\gamma_k\} \cup \{-\gamma_k\},$$

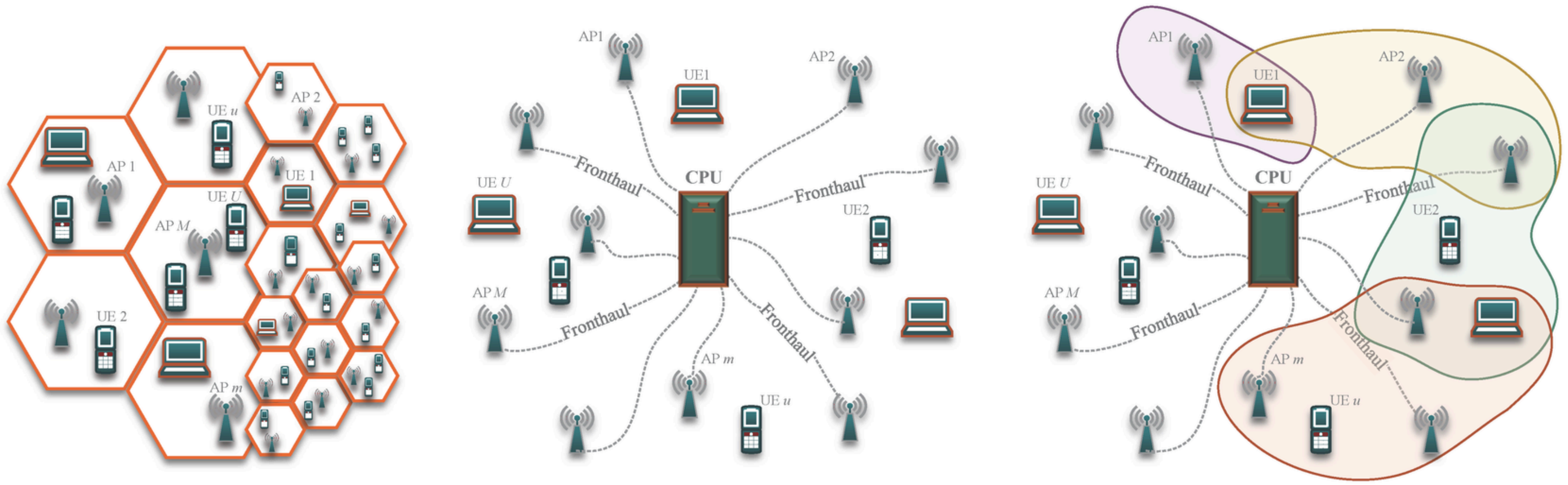
D : set of roots

$$\xrightarrow{} \quad \delta = \underset{d\in\mathcal{D}}{\text{argmin}}\, f_{k,T}(d).$$

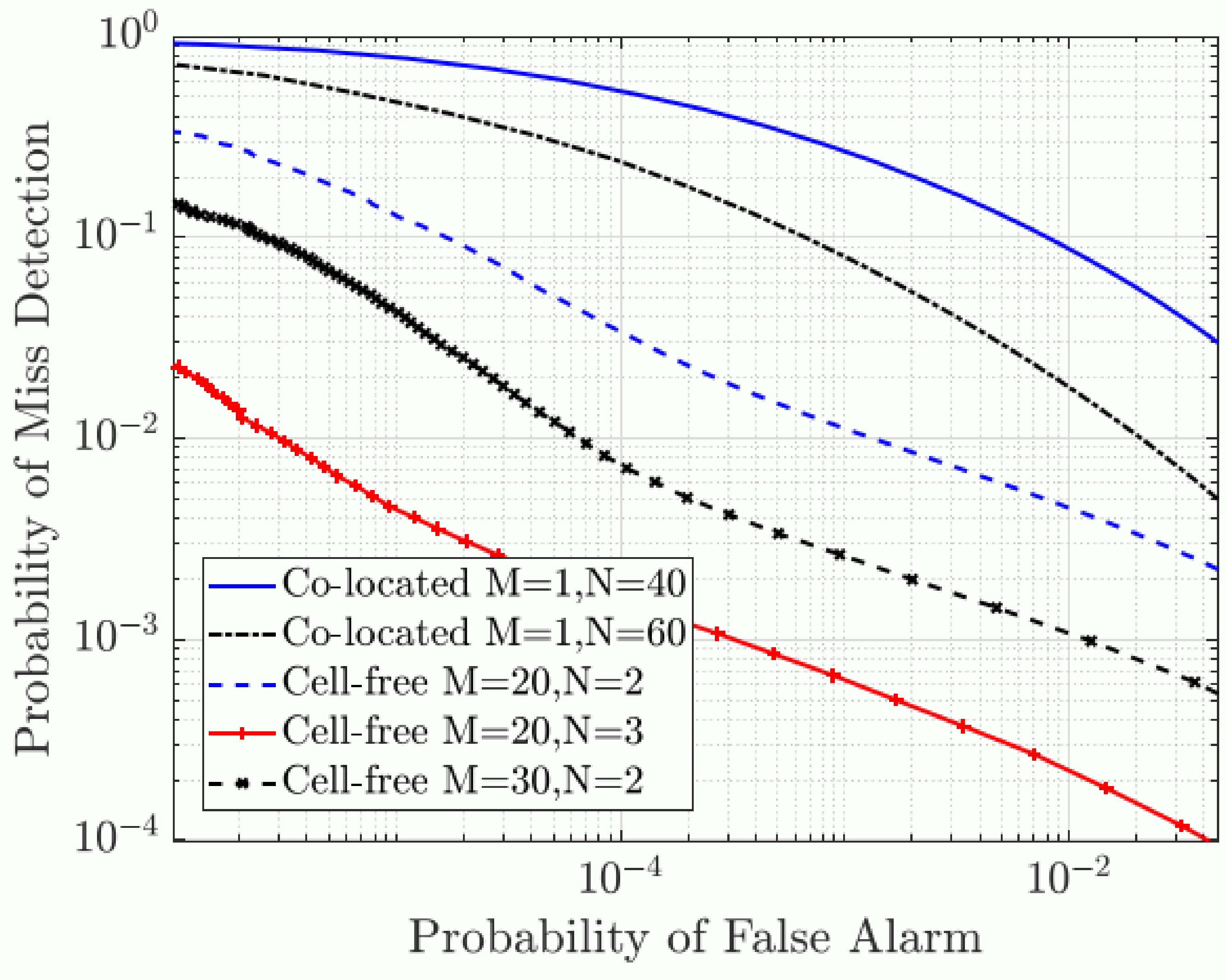
Parallel Architecture of Algorithms

- Update each user sequentially irrespective of whether the user is active or not
- sub-covariance matrices in Algorithm 2 do not change much

$$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 \cup \cdots \cup \mathcal{K}_G.$$

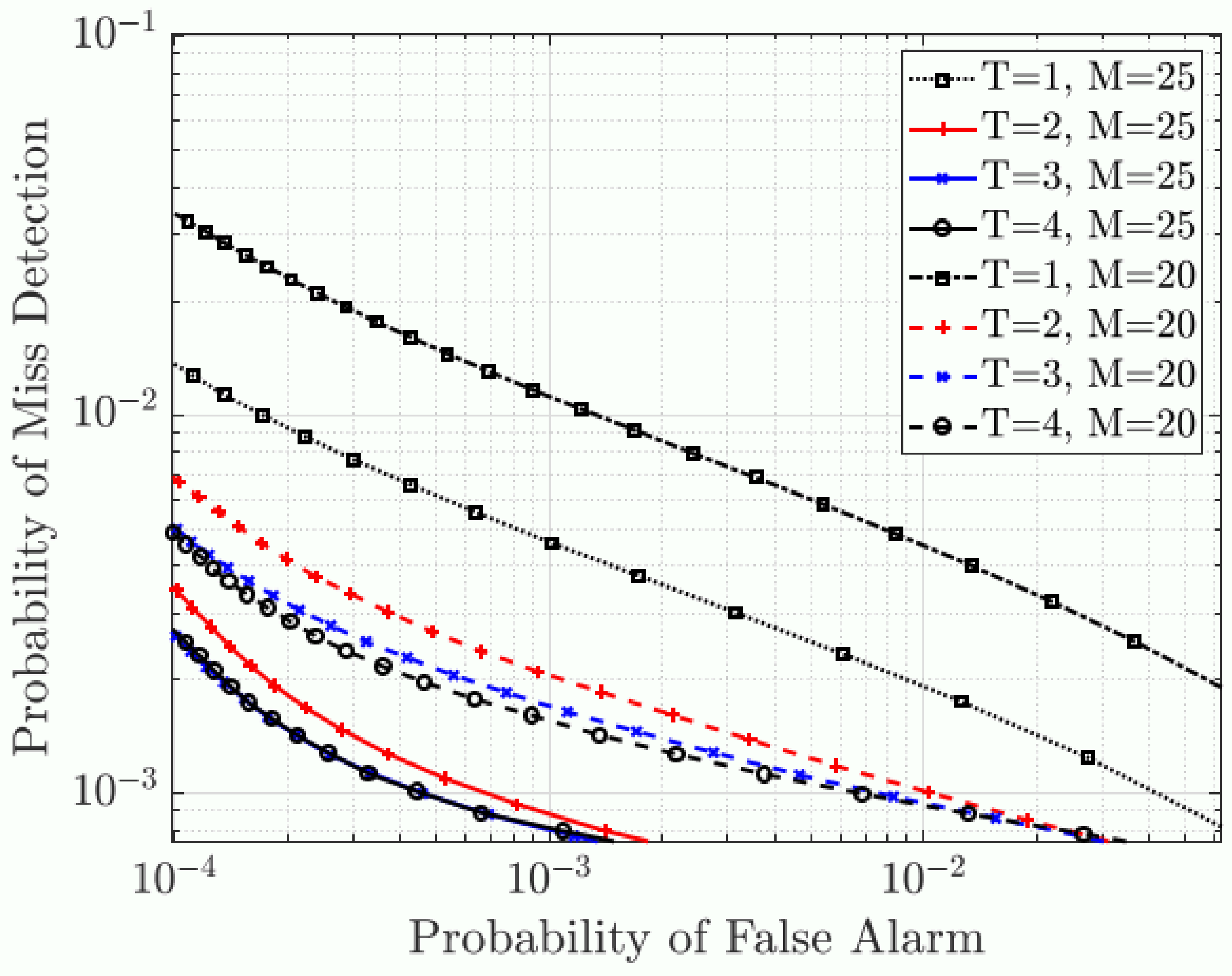


Simulation Results



Performance in 2×2 km^2 cell area. $K = 400$ users, activation probability $\epsilon = 0.1$, sequence length $L = 40$.

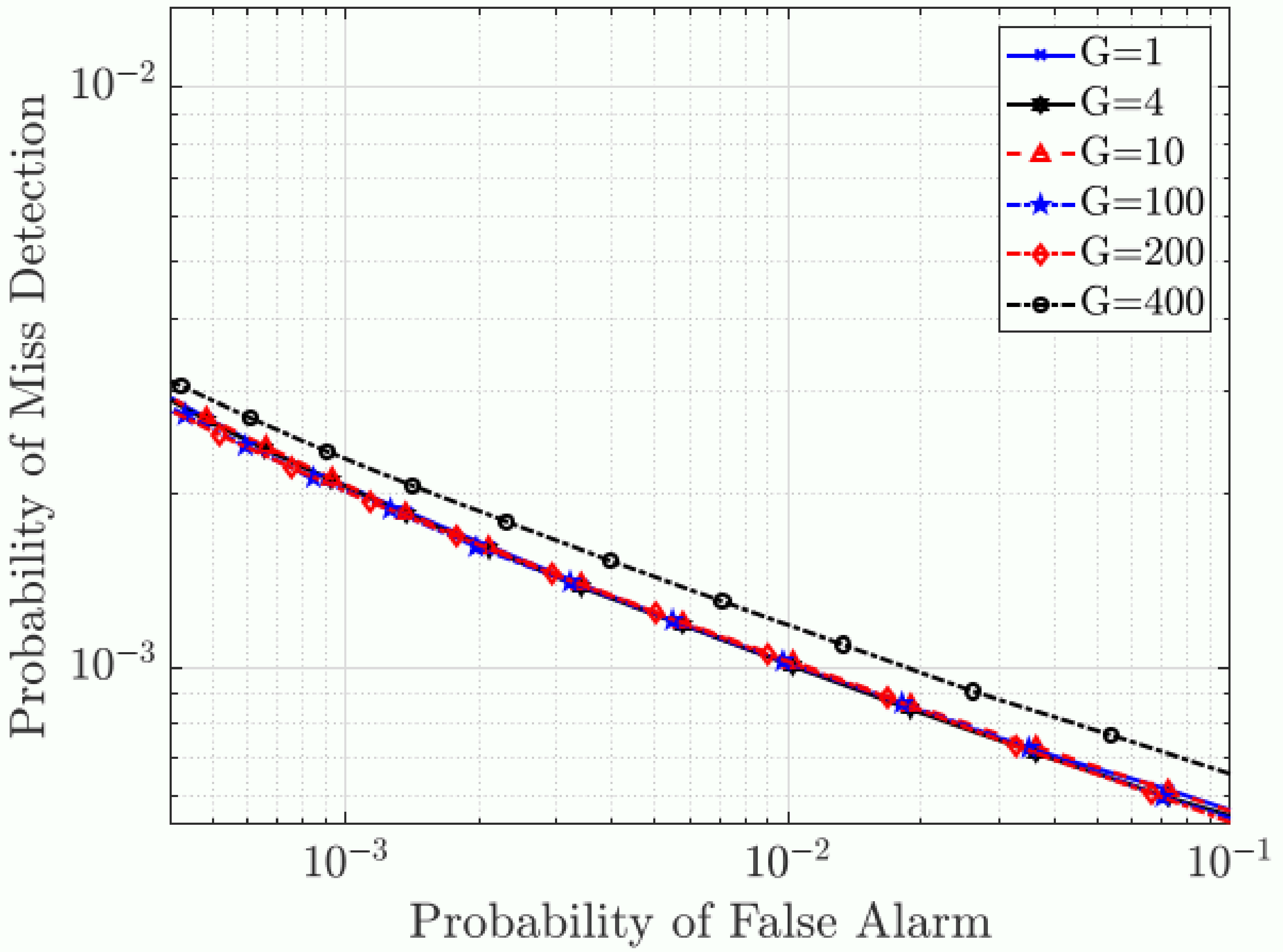
Fig2: Algorithm1 Performance-



Activity detection performance with 2×2 km^2 cell area, $N = 2$, $K = 400$, $L = 40$, $\epsilon = 0.1$.

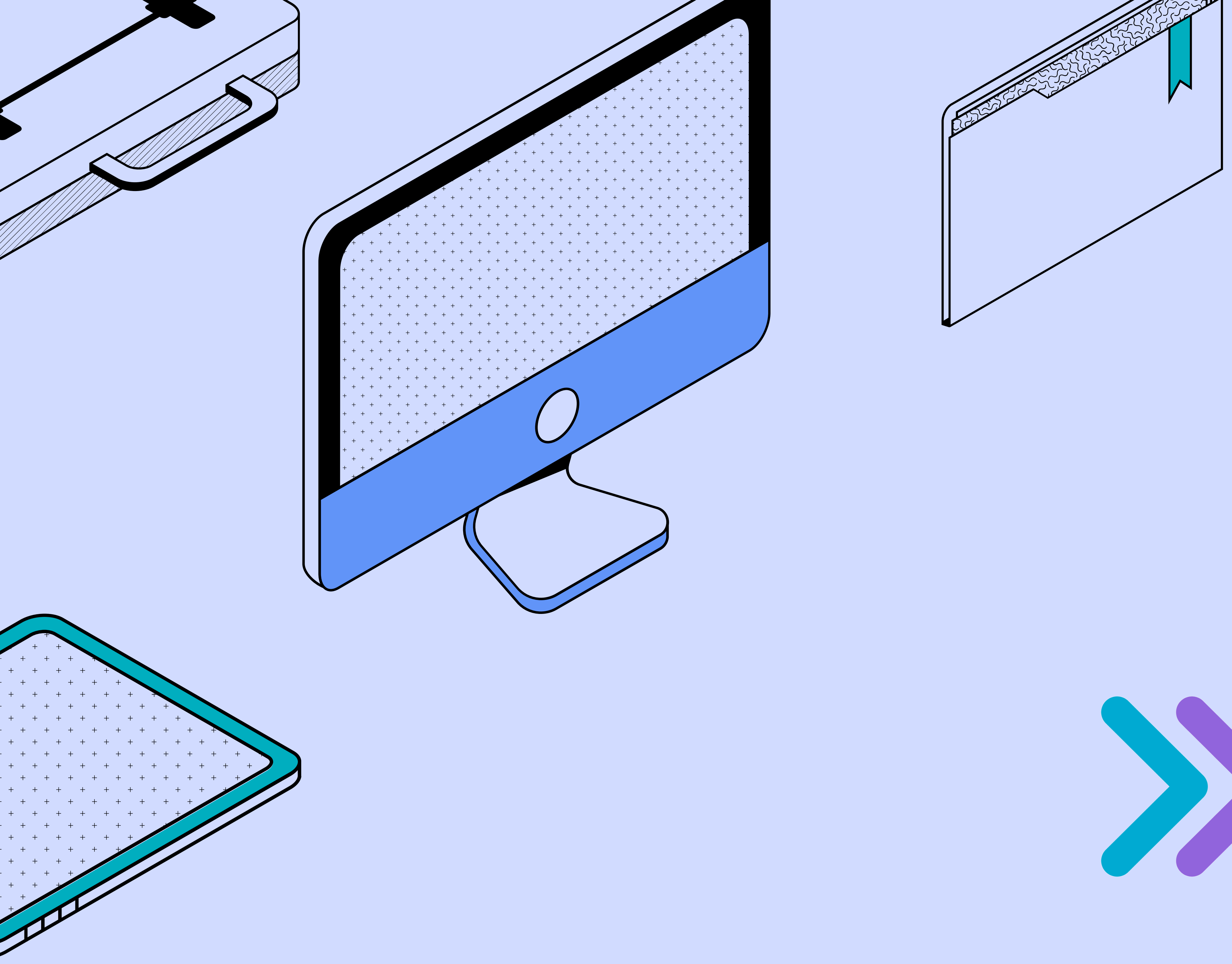
Fig3: Algorithm2 Performance-

Simulation Results



Activity detection performance with Algorithm 3 for 2×2 km^2 cell area, $M = 20$, $N = 2$, $K = 400$, $L = 40$, $\epsilon = 0.1$, $T = 2$.

Fig4: Algorithm3 Performance-



Conclusion