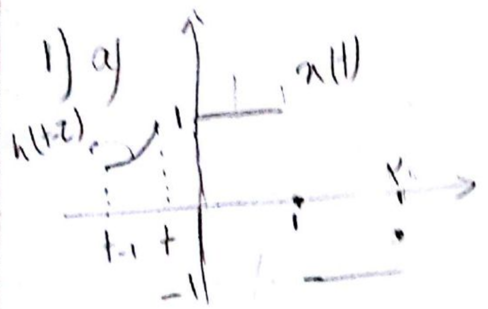
1) a)



$t<0 \rightarrow \int h(t-\tau)\cdot x(\tau)d\tau = ۰$

$۰<t<۱ \quad \int_{۰}^{t} e^{-t+\tau}\, d\tau = e^{-t+t} - e^{-t+۰} = ۱ - e^{-t}$

$۱<t<۲ \quad \int_{۱}^{t} -e^{-t+\tau}\, d\tau + \int_{t-۱}^{۱} e^{-t+\tau}\, d\tau = -e^{۱-t} + e^{۰} + e^{۱-t} - e^{-۱}$

$۲<t<۳ \quad \int_{t-۱}^{۲} -e^{-t+\tau}\, d\tau = -e^{۲-t} + e^{-۱}$

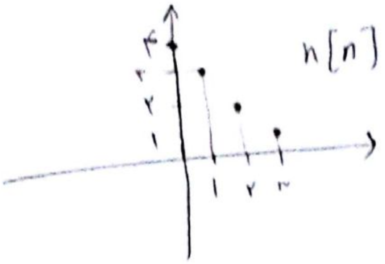
b) $x(t) = e^{-t}u(t) = e^{-t}, t>۰$

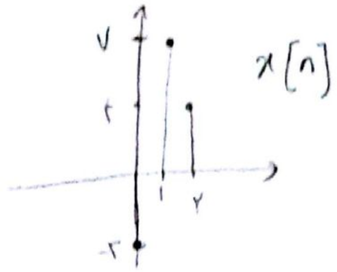
$h(t) = \delta(t) + ۰/۵\, \delta(t-۱) + ۰/۳\, \delta(t-۲) + ۰/۲\, \delta(t-۳)$

$y(t) = x(t) * h(t) = e^{-t} * \delta(t) + ۰/۵\, \delta(t-۱) * e^{-t} + ۰/۳\, \delta(t-۲) * e^{-t} + ۰/۲\, \delta(t-۳) * e^{-t}$

$y(t) = ۱ + ۰/۵\, e^{-۱} + ۰/۳\, e^{-۲} + ۰/۲\, e^{-۳}$

2) a)





$n=0 \quad \sum_{k=0}^{k=0} h[-k]\times x[k] = -3\times 4 = -12$

$n=1 \quad \sum_{k=0}^{k=1} h[1-k]\times x[k] = 4\times 7 + 3\times -3 = 19$

$n=2 \quad \sum_{k=0}^{k=2} h[2-k]\times x[k] = 4\times 4 + 3\times 7 + 2\times -3 = 31$

$n=3 \quad \sum_{k=0}^{k=4} h[3-k]\times x[k] = 3\times 4 + 2\times 7 + 1\times -3 = 23$

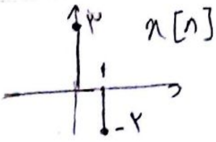
$n=4 \quad \sum_{k=1}^{k=2} h[4-k]\times x[k] = 1\times 7 + 2\times 4 = 15$

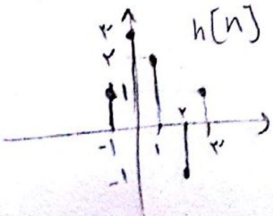
$n=5 \quad \sum_{k=2}^{k=2} h[5-k]\times x[k] = 1\times 4 = 4$

$y[n] = \{-12, 19, 31, 23, 15, 4\}$

b) $y[n] = \sum_{k=0}^{n} 0.8^k = \frac{1-(0.8)^{n+1}}{1-0.8} = 5 - 5(0.8)^{n+1}$

3) a)





$n=-1 \quad x[0]\times h[-1] = 3$

$n=0 \quad \sum_{k=0}^{k=1} x[k]\times h[-k] = 3\times 3 + -1\times -2 = 11$

$n=1 \quad \sum_{k=0}^{k=1} x[k]\times h[1-k] = 2\times 3 + -2\times 3 = 0$

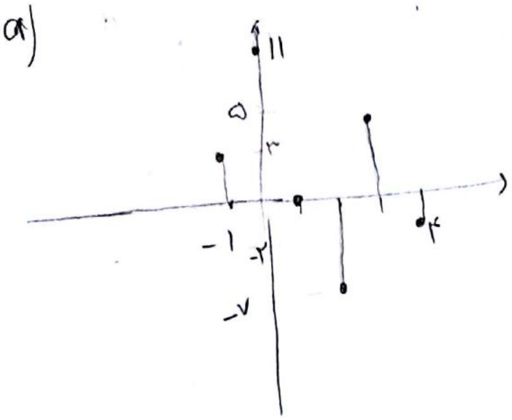
$n=2 \quad \sum_{k=0}^{k=1} x[k]\times h[2-k] = -3 + -4 = -7$

$n=3 \quad \sum_{0}^{1} x[k]\times h[3-k] = 3 + 2 = 5$

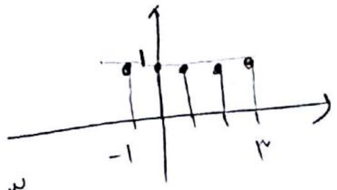
$n=4 \rightarrow$
$y[n] = 1\times -2 = -2$

تمرین

۳) a)



b)



$n=-2 \quad y[-2] \xrightarrow{k=-1} h[-1]\times x[-1] = 1$

$n=-1 \quad y[-1] = \sum_{k=-1}^{k=0} h[-1-k]\times x[k] = 3 + 1 = 4$

$n=3 \quad \sum_{k=0}^{3} h[3-k]\times x[k] = 2+3=5$

$n=0 \quad y[0] = \sum_{k=1}^{k=1} h[-k]\times x[k] = 1+3+2=6$

$n=4 \quad y[4] = \sum_{k=1}^{3} h[4-k]x[k] = 2$

$n=1 \quad y[1] = \sum_{k=-1}^{2} h[1-k]\times x[k] = -1+2+3+1 = 5$

$n=5 \quad y[5] = \sum_{k=2}^{3} h[5-k]\times x[k] = 1+-1=0$

$n=2 \quad y[2] = \sum_{k=-1}^{3} h[2-k]\times x[k] = 1-1+1+2+3=6$

$n=6 \quad y[6] = 1$

۴)

$$y(t) = x(t) * h_1(t) + x(t) * h_2(t) * h_3(t) + x(t) * h_2(t) * h_4(t)$$

$$y(t) = x(t) * \big(h_1(t) + h_2(t) * (h_3(t) + h_4(t))\big)$$

$$h_{eq}(t) = e^{-t}u(t) + \underbrace{u(t)-u(t-1) * u(t)-u(t-1)}_{A} + \underbrace{(-u(t-1)+u(t)) * \delta(t-1)}_{0}$$

$$A = \begin{cases} t & 0\le t\le 1 \\ 2-t & 1\le t\le 2 \end{cases} \qquad h_{eq}(t) = \begin{cases} t+e^{-t} & 0\le t\le 1 \\ 2-t+e^{-t} & 1\le t\le 2 \\ e^{-t} & t\ge 2 \end{cases}$$

$$y_1(t) = u(t) * e^{-t}u(t) = \int_0^{+\infty} e^{-t} = -e^{-t}$$

$$w(t) = [u(t) - u(t-1)] * [u(t) - u(t-1)] = \begin{cases} t & ۰<t<۱ \\ ۲-t & ۱<t<۲ \end{cases}$$

$$y_2(t) = w(t) * h_2(t) = \begin{cases} \frac{t^۲}{۲} & ۰<t<۱ \\ \frac{۱}{۲} - \frac{(t-1)^۲}{۲} + ۲t - \frac{۵}{۲} - \frac{t^۲}{۲} & ۱<t<۲ \\ ۲-۲t + \frac{(t-1)^۲}{۲} & ۲<t<۳ \end{cases}$$

$$y_4(t) = w(t) * h_4(t) = w(1) = 1$$

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۵) a) $h(t)$, $-۱<t<۱ \Rightarrow$ noncausal

$h(t) = u(t+1) - u(t-1) \Rightarrow$ memory

$\int_{-\infty}^{+\infty} |h(t)| = ۲ < \infty \Rightarrow$ stable

---

b) $h(t)$, $۰<t \Rightarrow$ non causal

$h(t) = u(t) - ۲u(t-1) \Rightarrow$ memory

$\int_{-\infty}^{+\infty} |h(t)| = \infty \Rightarrow$ unstable

c) $h(t) = e^{-۲|t|}$ $-\infty<t<+\infty \Rightarrow$ memory

$\Rightarrow$ noncausal

$\int_{-\infty}^{+\infty} e^{-۲|t|} = -۲e^{-۲t}\Big|_0^{+\infty} + ۲e^{+۲t}\Big|_{-\infty}^{0} = ۰ + ۲ + ۲ - ۰ = ۴$ stable

---

d) $h(t) = \cos(\pi t)u(t)$, $t>۰ \Rightarrow$ non causal

$\Rightarrow$ memory

$\int_0^{+\infty} \cos(\pi t) = \pi \sin \pi t \neq \infty \Rightarrow$ stable

e) $h[n] = 2^n u[-n]$, $n \leq 0 \rightarrow$ causal

$\rightarrow$ memory

$$\sum_{n=-\infty}^{0} |2^n u[-n]| = 2 \rightarrow \text{stable}$$

f) $h[n] = e^{2n} u[n-1]$, $n > 1 \Rightarrow$ non causal , memory

$$\sum_{1}^{+\infty} e^{2n} = \infty \Rightarrow \text{unstable}$$

g) $h[n] = (\frac{1}{2})^n u(n]$, $n > 0 \rightarrow$ noncausal , memory

$$\sum_{n=0}^{+\infty} 2^{-n} = 2 \rightarrow \text{stable}$$

h) $h[n] = \cos(\frac{\pi}{2} n) u[n+2]$, $n > -2 \rightarrow$ memory , non causal

$$\sum_{n=-2}^{+\infty} |h[n]| < \infty \rightarrow \text{stable}$$

4) $y(t) = x(t) + \alpha_1 x(t-\tau_1) + \alpha_2 x(t-\tau_2)$

$x'(t) = ax(t) \Rightarrow y'(t) = ax(t) + \alpha_1 a x(t-\tau_1) + \alpha_2 a x(t-\tau_2) = ay(t)$ ✓

$x(t) = x_1(t) + x_2(t) \rightarrow y(t) = x_1(t) + x_2(t) + \alpha_1 (x_1(t-\tau_1) + x_2(t-\tau_2)) + \alpha_2 (x_1(t-\tau_2) + x_2(t-\tau_2))$

$y_1(t) = x_1(t) + \alpha_1 x_1(t-\tau_1) + \alpha_2 x_1(t-\tau_2)$

$y_2(t) = x_2(t) + \alpha_1 x_2(t-\tau_1) + \alpha_2 x_2(t-\tau_2)$ $\rightarrow y(t) = y_1(t) + y_2(t)$ ✓

linear

$t \rightarrow t > 0$, $t > \tau_1$, $t > \tau_2 \rightarrow$ non causal

اگر مثبت باشند

if $x(t) < A \rightarrow y(t) < A + \alpha_1 A + \alpha_2 A$ ✓ stable !

$x(t-t_0) + \alpha_1 x(t-t_0-\tau_1) + \alpha_2 x(t-t_0-\tau_2) = y(t-t_0) \rightarrow$ time invariant ✓

۷) $g[n] = u[n] * h[n]$

a) $g[n] = \sum_{n=0}^{n=+\infty} \frac{1}{۲}^{n} = ۲\, u[n]$

b) $g(t) = \int_{0}^{+\infty} e^{-t}\, dt + \int_{-\infty}^{0} e^{t}\, dt = -e^{-t}\Big|_{0}^{+\infty} + e^{t}\Big|_{-\infty}^{0} = 1 + 1 - 0 = ۲$

c) $g(t) = u(t) * (\delta(t) - \delta(t-1)) = -u(1) + u(0) = 0$

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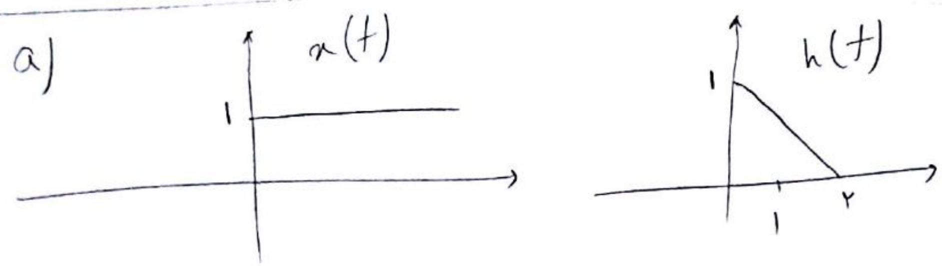
۹) a)



$t > 0 \to y(t) \quad \int_{0}^{t} h(t-\tau)\, d\tau = \int_{0}^{t} \frac{1}{۲}(\tau - ۲)\, d\tau = \frac{t^{۲}}{۴} - t$

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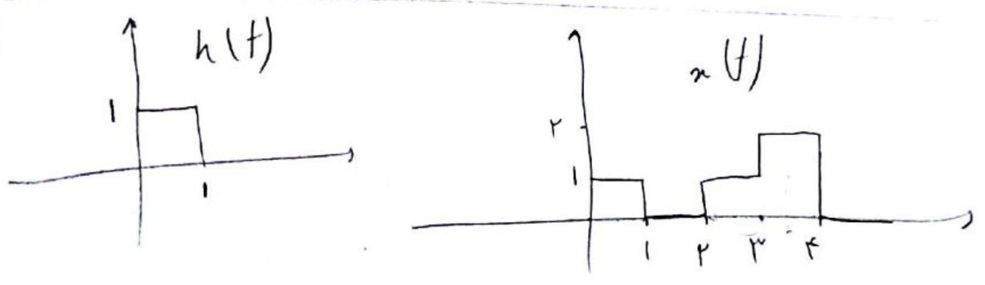
b)



$۰ < t < ۱ \quad \int_{0}^{t} 1\, d\tau = t \quad \Big| \quad ۱ < t < ۲ \quad \int_{t-1}^{1} d\tau = 1 - t$

$۲ < t < ۳ \quad \int_{۲}^{t} d\tau = t - ۲ \quad \Big\{ \quad ۳ < t < ۴ \quad \int_{۳}^{t} ۲\, d\tau + \int_{t-1}^{۳} d\tau = ۲t - ۶ + ۳ - (t-1) = t - ۲$

$۴ < t < ۵ \quad \int_{t-1}^{۴} ۲\, d\tau = ۸ - ۲t + ۲ = ۱۰ - ۲t$

۸) $x(t) = u(t-۱,۵) - u(t-۲,۵)$

$$y(t) = \left(u(t-۱) - u(t-۲)\right) \cdot \frac{t}{۲} + u(t-۲)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} \left(u(\tau-۱,۵) - u(\tau-۲,۵)\right) h(t-\tau)\, d\tau =$$

$$\int_{۱,۵}^{۲,۵} h(t-\tau)\, d\tau = u(t-۱) + \left(۱-\frac{t}{۲}\right) u(t-۲) \Longrightarrow$$

$$\boxed{h(t) = u(t-۱) + \left(۱-\frac{t}{۲}\right) u(t-۲)}$$