

# Quantitative Economics Project: Replicating Aiyagari; Uninsured Idiosyncratic Risk & Savings

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## **Abstract**

This study replicates S. Rao Aiyagari's 1994 paper on uninsured idiosyncratic risk and aggregate saving. Aiyagari's model fundamentally altered our understanding of macroeconomic savings by integrating precautionary saving motives and liquidity constraints into a standard neoclassical growth framework. Our replication revisits these dynamics using modern computational techniques and extended simulation periods, reinforcing Aiyagari's findings that idiosyncratic shocks to individual labor endowments significantly increase aggregate savings through precautionary behavior. Employing the Rouwenhorst method for better accuracy in approximating stochastic processes and simulating 50,000 households over 2,000 periods, we provide a robust analysis of the model's predictions. Our results affirm the original conclusions, highlighting that higher income variability and persistence in earnings elevate precautionary savings and aggregate capital accumulation. This replication not only validates Aiyagari's influential work but also underscores the persistent relevance of idiosyncratic risk in shaping macroeconomic outcomes, offering deeper insights for economic policy.

# 1 Introduction

The pioneering work of S. Rao Aiyagari (1994) addresses a fundamental question in macroeconomics: how does uninsured idiosyncratic risk influence aggregate saving behavior? Aiyagari's paper, "*Uninsured Idiosyncratic Risk and Aggregate Saving*", integrates the precautionary saving motive and liquidity constraints into a standard growth model, providing a comprehensive qualitative and quantitative analysis. By introducing idiosyncratic shocks to individual agents' labor endowments, Aiyagari demonstrates how these shocks lead to increased aggregate saving due to precautionary motives [Aiyagari \(1994\)](#).

The significance of Aiyagari (1994) lies in its departure from the traditional representative agent framework, which assumes complete markets and aggregate uncertainty. Instead, Aiyagari's model acknowledges the incomplete nature of real-world markets, where individuals face personal risks that are not insurable. This approach allows for a more accurate representation of the observed disparities in consumption, wealth, and income distributions. The model's implications extend to various economic phenomena, including asset pricing, monetary policy, business cycles, and taxation, making it a cornerstone for subsequent research in these areas.

The objective of this replication study is to revisit Aiyagari's influential model, verifying its results and exploring its robustness. By replicating the original code and conducting a detailed analysis, this study aims to enhance the understanding of the mechanisms driving aggregate saving in the presence of idiosyncratic risk.

In the following sections, this report is structured as follows: Section 2 provides a comprehensive literature review, summarizing key contributions and the significance of Aiyagari (1994) in the context of idiosyncratic risk and aggregate savings. Section 3 details the methodology employed, including a description of the original model and the replication process, highlighting any modifications made. Section 4 presents the results, comparing them with those from the original paper and offering an in-depth analysis of the findings. Finally, Section 5 concludes the report, summarizing the key insights and discussing potential

directions for future research and improvements to the model.

## 2 Literature Review

The study of idiosyncratic risk and its impact on aggregate economic variables has garnered significant attention in the field of macroeconomics. [Aiyagari \(1994\)](#) seminal paper on uninsured idiosyncratic risk and aggregate saving represents a critical advancement in this area. By incorporating individual-level shocks and market incompleteness into the standard growth model, Aiyagari challenges the traditional representative agent models, which often fail to capture the observed heterogeneity in wealth and income distributions.

Aiyagari's work builds on the foundational models of [Bewley \(1986\)](#), who introduced the concept of precautionary saving in the face of uninsured risks. Bewley's model highlighted how individuals accumulate excess capital as a buffer against future income uncertainties, a theme that Aiyagari expands upon by integrating borrowing constraints and examining their macroeconomic implications. Additionally, the influence of [Brock and Mirman \(1972\)](#) growth model is evident in Aiyagari's framework, particularly in the treatment of capital accumulation and the role of precautionary motives.

Subsequent research has further explored the themes introduced by Aiyagari. For instance, [Huggett \(1993\)](#) and [Krusell and Smith \(1998\)](#) developed models that account for both idiosyncratic and aggregate shocks, providing deeper insights into the interaction between individual risks and macroeconomic outcomes. These studies underscore the importance of asset trading and market frictions in determining the overall saving rate and wealth distribution in an economy.

The empirical relevance of Aiyagari's model has been supported by various studies that document significant precautionary saving behavior among households. For example, [Carroll and Samwick \(1998\)](#) provide empirical evidence showing that households with higher income uncertainty tend to save more, consistent with the predictions of models incorporat-

ing precautionary motives. Similarly, [Dynan et al. \(2004\)](#) find that precautionary saving is a key driver of wealth accumulation, particularly among households facing substantial income volatility.

The importance of studying idiosyncratic risk and aggregate savings extends beyond theoretical contributions. Policymakers can draw valuable insights from these models when designing social insurance programs and financial regulations aimed at mitigating the adverse effects of income uncertainty. By understanding the mechanisms driving precautionary saving, policymakers can better anticipate the potential impacts of economic policies on wealth distribution and aggregate economic stability.

In summary, the literature on uninsured idiosyncratic risk and aggregate saving highlights the critical role of individual risks and market incompleteness in shaping macroeconomic outcomes. [Aiyagari \(1994\)](#) paper remains a cornerstone of this literature, providing a robust framework for analyzing the effects of idiosyncratic shocks on aggregate saving behavior. This replication study seeks to validate and extend Aiyagari’s findings, contributing to the ongoing discourse on the interplay between individual risks and macroeconomic stability.

### 3 Methodology

The model presented in Aiyagari (1994) incorporates precautionary savings motives and liquidity constraints into a standard neoclassical growth model. Below is a detailed description of the various components and the overall structure of the model.

#### 3.1 Agents and Preferences

The economy consists of a continuum of infinitely-lived agents indexed by  $i \in [0, 1]$ . Each agent maximizes the expected present value of utility from consumption:

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \right], \tag{1}$$

where  $0 < \beta < 1$  is the discount factor,  $c_{i,t}$  is consumption at time  $t$ , and  $u(\cdot)$  is the period utility function. The utility function is assumed to exhibit constant relative risk aversion (CRRA):

$$u(c) = \frac{c^{1-\mu}}{1-\mu}, \quad (2)$$

where  $\mu$  is the coefficient of relative risk aversion.

### 3.2 Endowments and Income

Agents receive idiosyncratic labor endowment shocks,  $l_{i,t}$ , which follow a stochastic process. The labor income for agent  $i$  at time  $t$  is given by:

$$y_{i,t} = wl_{i,t}, \quad (3)$$

where  $w$  is the wage rate. For labor endowment shocks, we employ a Markov chain specification with seven states to match the following first-order autoregressive process for the logarithm of the labor endowment shock (or earnings):

$$\log(l_{i,t}) = \rho \log(l_{i,t-1}) + \sigma(1 - \rho^2)^{\frac{1}{2}} \epsilon_{i,t}, \quad (4)$$

where  $\epsilon_{i,t} \sim \mathcal{N}(0, 1)$ .

### 3.3 Assets and Borrowing Constraints

Agents can save in the form of a single asset, capital,  $a_{i,t}$ , which earns a return  $r$ . There is a borrowing constraint that prevents agents from borrowing more than a certain limit,  $\phi$ . The budget constraint for agent  $i$  at time  $t$  is:

$$c_{i,t} + a_{i,t+1} = y_{i,t} + (1 + r)a_{i,t}, \quad (5)$$

with the constraint  $a_{i,t+1} \geq -\phi$ , where  $\phi$  is defined as:

$$\phi = \min \left\{ b, \frac{wl_{\min}}{r} \right\}. \quad (6)$$

Here,  $b$  is a fixed borrowing limit, and  $l_{\min}$  is the minimum labor endowment shock.

### 3.4 Production

The production side of the economy is represented by a neoclassical production function with capital,  $K_t$ , and labor,  $L_t$ . The aggregate production function is given by:

$$Y_t = F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}, \quad (7)$$

where  $\alpha$  is the capital share of income. The wage rate and the return on capital are determined by the marginal products of labor and capital, respectively:

$$w = (1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha, \quad (8)$$

$$r = \alpha \left( \frac{K_t}{L_t} \right)^{\alpha-1} - \delta, \quad (9)$$

where  $\delta$  is the depreciation rate of capital.

### 3.5 Total Resources

$\hat{a}_t$  is defined as follows:

$$\hat{a}_t = a_t + \phi, \quad (10)$$

The total resources of an agent at time  $t$ ,  $z_{i,t}$ , are defined as:

$$z_{i,t} = wl_{i,t} + (1 + r)\hat{a}_{i,t} - r\phi. \quad (11)$$

So we have:

$$c_t + \hat{a}_{t+1} = z_t \quad (12)$$

$$z_{t+1} = wl_{t+1} + (1 + r)\hat{a}_{t+1} - r\phi. \quad (13)$$

### 3.6 Value Function and Policy Functions

Let  $V(z_t, b, w, r)$  be the value function of the agent with total resources  $z_t$ . The Bellman equation characterizing the optimal decision problem is:

$$V(z_t, b, w, r) = \max_{\hat{a}_{t+1}} \{U(z_t - \hat{a}_{t+1}) + \beta \mathbb{E}[V(z_{t+1}, b, w, r)]\}, \quad (14)$$

subject to the budget and borrowing constraints. The policy functions  $\hat{a}_{t+1}$  is derived from this optimization problem as follows:

$$\hat{a}_{t+1} = A(z_t, b, w, r) \quad (15)$$

Substituting 15 into 13, transition law for  $z_t$  is obtained:

$$z_{t+1} = wl_{t+1} + (1 + r)A(z_t, b, w, r) - r\phi. \quad (16)$$

#### 3.6.1 Replication of Figure 1a of paper

Figure 1 illustrates the dynamics of consumption, assets, and total resources in the model presented by Aiyagari (1994). While the original paper provided estimated drawings of these relationships to convey the predictions of the model, the figures presented here are based on actual data plots, providing a more precise visualization of the model's dynamics. The

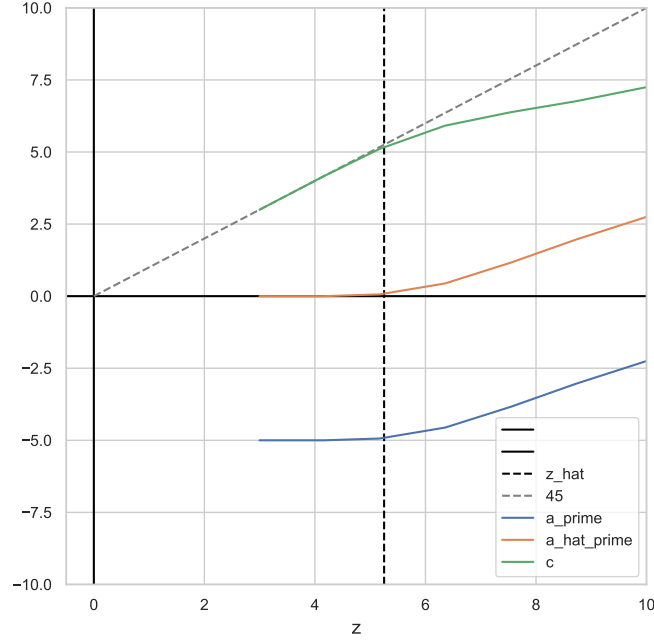
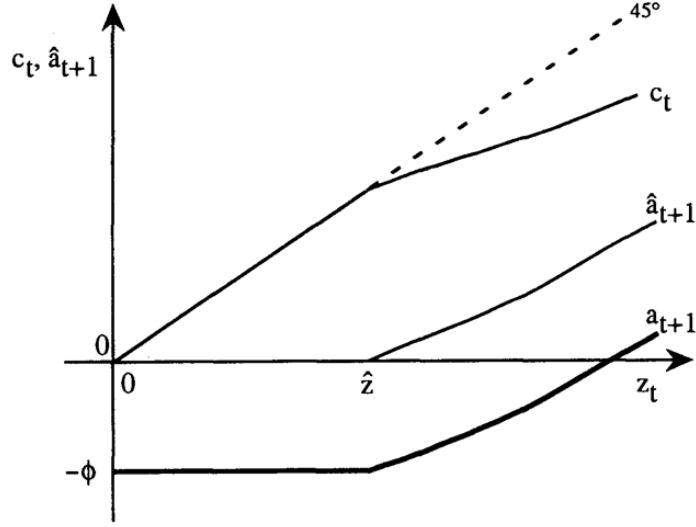


Figure 1: Consumption And Assets as Functions of Total Resources (Figure Ia Replication)

replication of these figures with confirms the predictions, showing how agents' consumption and asset holdings respond to changes in total resources and highlighting the importance of precautionary savings in the presence of income uncertainty.

Figure 1 depicts the relationship between total resources  $z$ , consumption  $c$ , and future assets  $\hat{a}$ . The curves  $a$ ,  $\hat{a}$ , and  $c$  indicate the optimal choices of future assets and consumption for each level of current total resources. The figure highlights how agents allocate their resources between consumption and savings, taking into account the precautionary savings motive and the borrowing constraints. As depicted in the figure, There exists a positive value  $\hat{z}$  such that whenever  $z_t \leq \hat{z}$ , it is optimal to consume all the total resources, set  $c_t = z_t$ , and set  $\hat{a}_{t+1}$  to its lowest permissible value, which is zero, and the agent's debt equals to the borrowing limit,  $\phi$ . As total resources increase beyond  $\hat{z}$ , both consumption ( $c_t$ ) and future assets  $\hat{a}_{t+1}$  rises with a slope less than unity, indicating that the agent starts saving and the borrowing constraint becomes non-binding.





**FIGURE 1a**  
**Consumption and Assets as Functions**  
**of Total Resources**

Figure 2: By Aiyagari (1994)

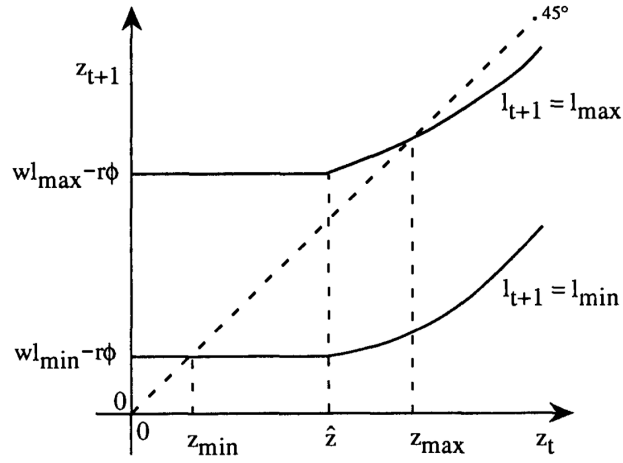
### 3.6.2 Replication of Figure 1b of paper

Figure 4 shows the evolution of total resources over time. The 45-degree line represents the steady-state where the current total resources equal the future total resources where  $z = z'$ . When  $z \leq \hat{z}$ ,  $\hat{a}$  would be zero, so  $z_{t+1}$  would be equal to  $wl_{t+1} - r\phi$ , which is between  $z_{min} \equiv wl_{min} - r\phi$  and  $wl_{max} - r\phi$ , as shown in figure. When  $z \geq \hat{z}$ ,  $\hat{a}_{t+1}$  rise with a slope less than unity, therefore maximum  $z_{t+1}$  will cross the 45-degree line in  $z_{max}$  value.

## 3.7 Endogenous Heterogeneity and Aggregation

There exists a unique, stable stationary distribution for  $z_t$ , which reflects the endogenous heterogeneity. Long run average assets using 15 and 10 is given by:

$$Ea_w = E\{A(z, b, w, r)\} - \phi, \quad (17)$$



**FIGURE Ib**  
**Evolution of Total Resources**

Figure 3: By Aiyagari (1994)

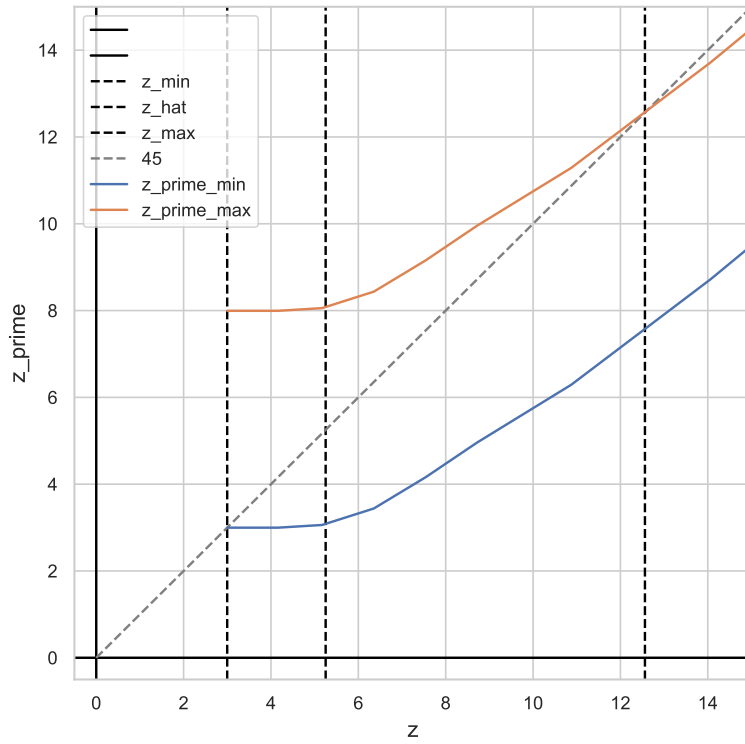
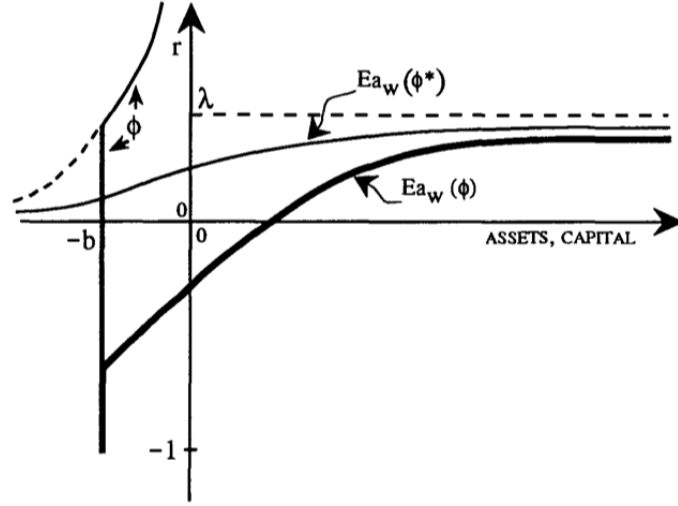


Figure 4: Evolution of Total Resources (Figure Ib Replication)



**FIGURE IIa**  
**Interest Rate versus Per Capita Assets**

Figure 5: By Aiyagari (1994)

### 3.7.1 Replication of Figure 2a of paper

Figure 6 shows the relationship between the interest rate  $r$  and  $Ea_w$ . The dashed line represents the time preference rate ( $\lambda$ ). As the interest rate approaches the time preference rate from below,  $Ea_w$  tends to infinity. If  $r$  equals or exceeds  $\lambda$ , individuals prefer to postpone consumption.

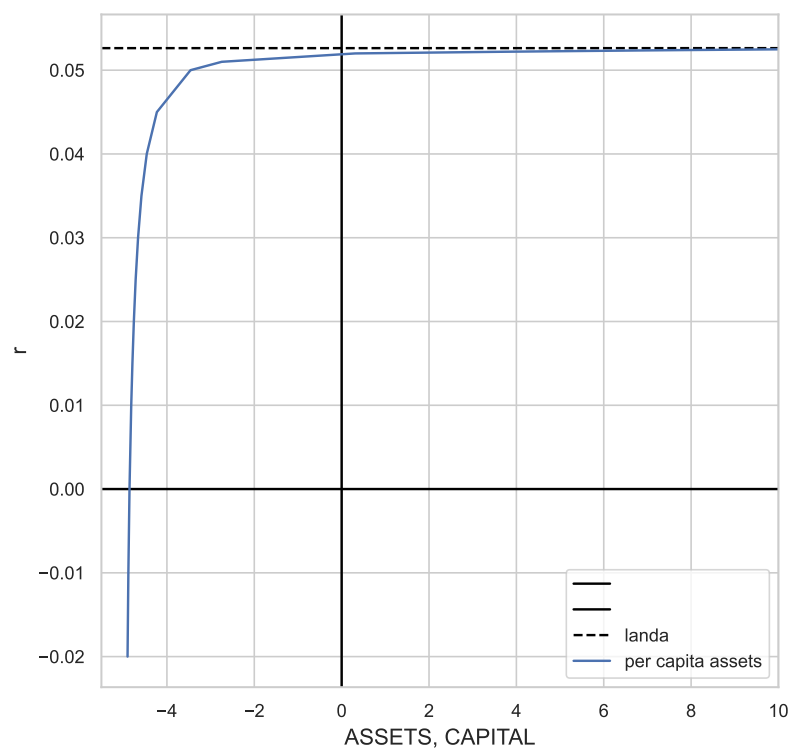


Figure 6: Interest Rate vs per Capita Assets (Figure IIa Replication)

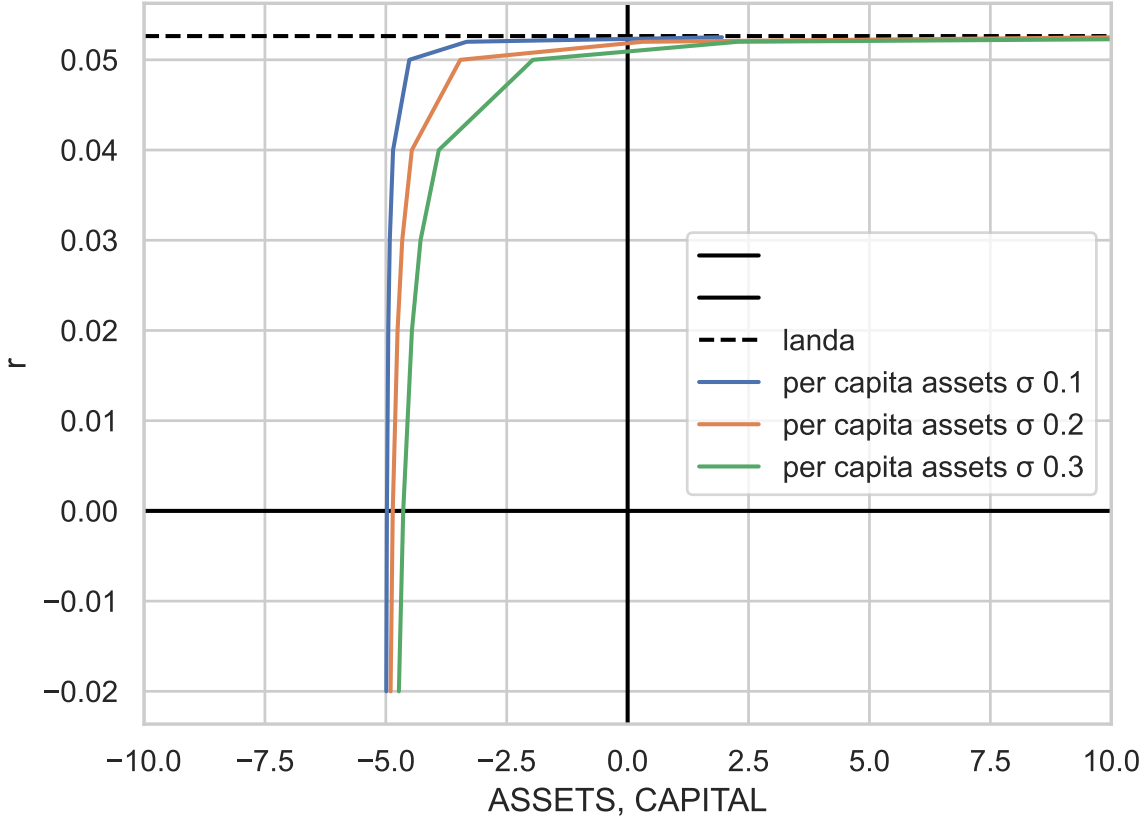


Figure 7: Interest Rate vs per Capita Assets with different levels of uncertainty

For interest rates below the time preference rate,  $Ea_w$  is higher under uncertainty compared to a scenario with certain earnings, regardless of whether marginal utility is convex or concave. This is due to the borrowing constraint. In a steady state with incomplete markets, agents with different histories of labor endowment shocks will have varying levels of total resources. Those with low resources remain liquidity constrained, while those with high resources accumulate more assets. Consequently, per capita assets in aggregate exceed their levels under certainty. Figure 7 shows  $Ea_w$  for different values of  $\sigma$ . As  $\sigma$  increases, level of uncertainty gets higher. Therefore, the amount of assets in a given  $r$  increases.

## 3.8 General Equilibrium

### 3.8.1 Replication of Figure 2b of paper

Figure 8 illustrates how uninsured idiosyncratic shocks and borrowing constraints lead to higher aggregate saving. The crucial features are that  $Ea_w$  is finite only if the interest rate  $r$  is less than the time preference rate  $\rho$ , and tends to infinity as  $r$  approaches  $\rho$  from below. This happens because, in steady state, the capital demanded by firms (determined by producer profit maximization; equation 9;  $K(r)$ ) must equal the capital supplied by households. Since  $w$  can be expressed as a function of  $r$ , For a given  $r$ , let  $Ea$  denote the value of  $Ea_w$  when  $w$  equals  $w(r)$  and let NI be the graph of  $Ea$  versus  $r$ . A steady state of this economy is where  $Ea(r) = K(r)$ .

Dashed line represents individual's desired asset holdings as a function of  $r$  when there is no uncertainty, (Full Insurance). The figure shows that the steady state in the economy with idiosyncratic shocks and borrowing constraints is characterized by a higher capital stock and lower interest rate compared to the full insurance case.

### 3.8.2 Effects of varying the borrowing limit $\phi$

Figure 10 represents  $Ea_w$  for different values of  $\phi$ . As figure shows, permitting a higher borrowing limit serves to lower aggregate capital and raise the interest rate toward the time preference rate. The intuition behind this conclusion is that when borrowing is permitted individuals need not rely solely on holdings of capital to buffer earnings variation. Borrowing can also be used to buffer these shocks and, hence, leads to smaller holdings of capital.

## 3.9 Calibration and Computational Method

The model period is set to one year, with the utility discount factor  $\beta$  set at 0.96. The production function  $f(\cdot)$  is characterized by a capital share parameter (denoted  $\alpha$ ) of 0.36. The capital depreciation rate ( $\delta$ ) is set at 0.08. Results are presented for three different values

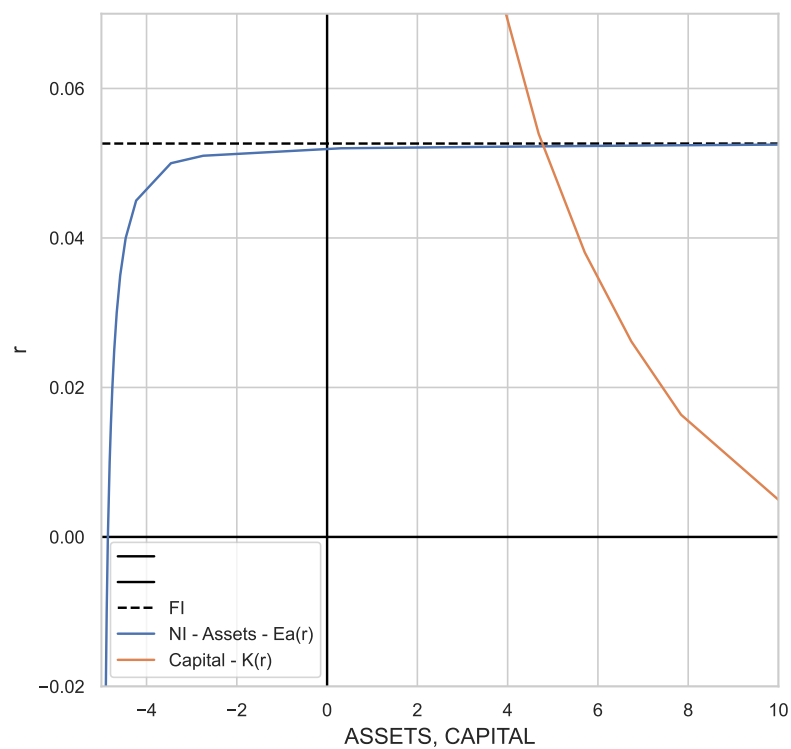
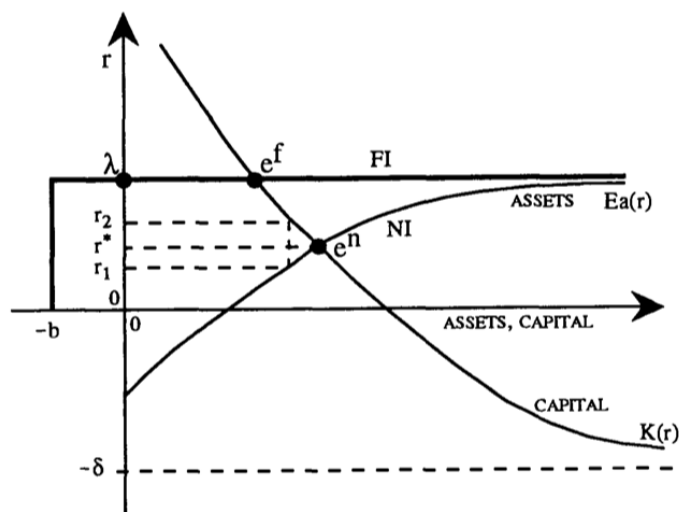


Figure 8: Steady State Determination (Figure IIb Replication)



**FIGURE IIb**  
**Steady-State Determination**

Figure 9: By Aiyagari (1994)

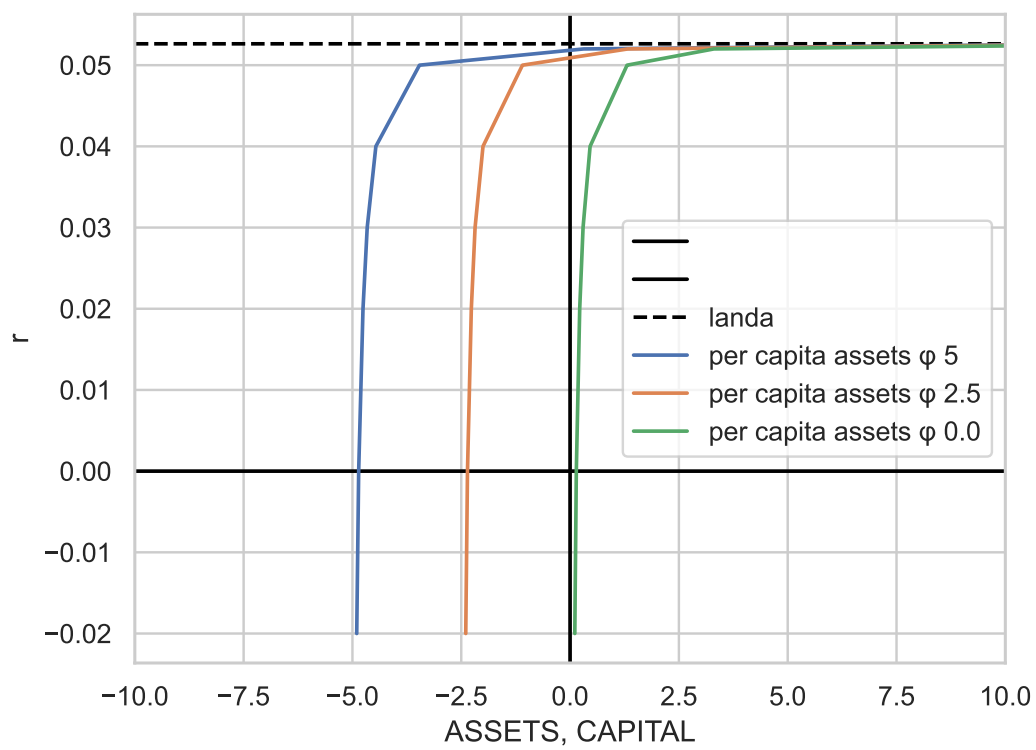


Figure 10: Interest Rate vs per Capita Assets with different values of  $\phi$



of  $\mu \in 1, 3, 5$ . The selected technology and preference specifications, along with parameter values, are consistent with aggregate features of the postwar U.S. economy and are commonly employed in models of economic growth and business cycles.

Shocks are characterized by  $\sigma \in \{0.2, 0.4\}$ , and  $\rho \in \{0, 0.3, 0.6, 0.9\}$ . Where the coefficient of variation is equal to  $\sigma$  and the serial correlation coefficient is  $\rho$ .

The paper then follows the procedure outlined by Deaton and Tauchen to approximate the autoregressive process using a seven-state Markov chain. However, we employ **the Rouwenhorst method** instead, which is more accurate for highly persistent processes compared to the Tauchen method. This method provides better approximations with fewer grid points and handles the tails of the distribution more effectively.

Lastly, the borrowing limit  $b$  is established at zero, meaning borrowing is not allowed. As discussed in the previous section, allowing some level of borrowing would result in even smaller impacts on the overall saving rate.

The paper employs the original bisection method to determine the steady-state interest rate (the rate at which the demand and supply of capital are equal). However, we utilize intuitive methods to enhance the bisection method. Additionally, while the original paper simulates a single agent over 10,000 periods, we simulate 50,000 households over 2,000 periods. This approach yields more accurate results.

After obtaining the simulated series, we can calculate various statistical measures including the mean, median, standard deviation, coefficient of variation, skewness, and serial correlation coefficient for labor income, asset (capital) holdings, net income, gross income, gross saving, and consumption. Additionally, we can assess inequality for each of these variables by computing measures such as the Gini coefficient and constructing Lorenz curves.

## 4 Results

Table 1 is the replication of results of paper, as shown in Figure 11. This presents the net return to capital in percent and the saving rates in percent for  $\sigma$  equal to 0.2 and 0.4 and various values of  $\rho$  (serial correlation in earnings) and  $\mu$  (the relative risk aversion coefficient).

It is evident that there is a high degree of similarity, with only minor differences between these two tables. The general trends in the steady-state interest rates and aggregate saving rates are consistent across both sets of tables. Specifically, for both the original and replicated tables, as  $\rho$  or  $\mu$  or  $\sigma$  increase, the steady-state interest rates decrease and aggregate saving rates tend to increase. This consistency demonstrates the robustness of the replication process and the accuracy of the methods employed.

However, there are minor numerical deviations in some of the values. For instance in Table A, for  $\rho = 0.3$  and  $\mu = 1$ , my replicated values are 4.1289/23.57, compared to the original value of 4.1365/23.73. Such differences are small and fall within an acceptable range for replication studies, indicating a successful replication overall.

The differences become more noticeable for higher values of  $\rho$  and  $\mu$ . For example, in Table B for  $\rho = 0.9$  and  $\mu = 5$ , my replication shows 0.7133/33.072, whereas the original values are -0.3456/37.63. These larger deviations can be attributed to several factors, including the methods used for approximating the stochastic processes, differences in simulation lengths and sample sizes and computational precision.

In this replication, we employed the Rouwenhorst method to approximate the autoregressive process, which is known for its accuracy with highly persistent processes. This contrasts with the original paper, which likely used the Tauchen method. Although both methods are valid, differences in approximation techniques can lead to slight variations in the results. Additionally, this replication involved simulating 50,000 households over 2,000 periods, compared to Aiyagari's simulation of 10,000 periods for a single agent. This difference in the simulation framework can introduce variability, particularly in scenarios with high persistence and high-risk aversion. Further, small discrepancies in the implementation

TABLE II

A. Net return to capital in %/aggregate saving rate in % ( $\sigma = 0.2$ )			
$\rho \backslash \mu$	1	3	5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36
B. Net return to capital in %/aggregate saving rate in % ( $\sigma = 0.4$ )			
$\rho \backslash \mu$	1	3	5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	-0.3456/37.63

Figure 11: Table by Aiyagari (1994)

details and computational precision could contribute to the observed differences.

Table 1: Replication of Results

A. Steady State Interest Rate/Aggregate Saving Rate ( $\sigma=0.2$ )			
$\rho/\mu$	1	3	5
0.0	4.1464/23.299	4.0906/23.398	4.0184/23.502
0.3	4.1289/23.57	4.0298/23.732	3.9055/23.934
0.6	4.1016/23.703	3.9352/24.079	3.7281/24.469
0.9	4.0316/23.952	3.6297/24.766	3.0916/25.947
B. Steady State Interest Rate/Aggregate Saving Rate ( $\sigma=0.4$ )			
$\rho/\mu$	1	3	5
0.0	4.071/23.876	3.8307/24.318	3.5458/24.959
0.3	4.0008/23.976	3.5917/24.842	3.1193/25.902
0.6	3.847/24.287	3.051/26.032	2.211/28.189
0.9	3.5789/24.832	2.1434/28.346	0.7133/33.072

Overall, the replication study demonstrates a high degree of accuracy and successfully validates the key dynamics and trends described in Aiyagari's model. The minor differences observed are primarily due to methodological variations and computational specifics. This robust replication confirms the reliability of the original findings regarding the impact of uninsured idiosyncratic risk on aggregate saving behavior, providing confidence in the

model's conclusions.

Table 2: Steady State Capital for Different Combinations of  $\rho$  and  $\mu$

<b>Steady State Capital: (<math>\sigma=0.2</math>)/(<math>\sigma=0.4</math>)</b>			
$\rho/\mu$	1	3	5
0.0	5.47/5.53	5.5/5.69	5.53/5.92
0.3	5.5/5.56	5.55/5.88	5.61/6.28
0.6	5.5/5.67	5.62/6.32	5.76/7.16
0.9	5.55/5.87	5.85/7.23	6.29/9.19

Table 2 represents steady state capital for different values of  $\sigma$ ,  $\mu$  and  $\rho$ . The results indicate that higher income variability ( $\sigma = 0.4$ ) leads to higher steady-state capital due to increased precautionary savings, particularly for higher persistence of shocks ( $\rho$ ) and higher risk aversion ( $\mu$ ).

Table 3: Precautionary Savings Rate for Different Combinations of  $\rho$  and  $\mu$

<b>Precautionary Savings Rate: (<math>\sigma=0.2</math>)/(<math>\sigma=0.4</math>)</b>			
$\rho/\mu$	1	3	5
0.0	0.0002/0.00096	0.00076/0.00336	0.00148/0.00621
0.3	0.00038/0.00166	0.00137/0.00575	0.00261/0.01047
0.6	0.00065/0.0032	0.00231/0.01116	0.00439/0.01956
0.9	0.00135/0.00588	0.00537/0.02023	0.01075/0.03453

Table 3 represents Precautionary Savings Rates for different values of  $\sigma$ ,  $\mu$  and  $\rho$ . The precautionary saving rate can be defined as the difference between the saving rate under full insurance (certainty) and the saving rate in the presence of uncertainty. This rate captures the additional savings that individuals accumulate as a buffer against potential income shocks and uncertainties in the economy. Essentially, it reflects the extra savings driven by precautionary motives when individuals face idiosyncratic risks that cannot be insured. The results indicate that higher income variability ( $\sigma = 0.4$ ) leads to higher precautionary savings rates, particularly for higher persistence of shocks ( $\rho$ ) and higher risk aversion ( $\mu$ ).

Table 4 represents capital stock comparison for different values of  $\sigma$ ,  $\mu$  and  $\rho$ . For

Table 4: Capital Stock Comparison for Different Combinations of  $\rho$  and  $\mu$ 

<b>Precautionary Capital: (<math>\sigma=0.2</math>)/(<math>\sigma=0.4</math>)</b>			
$\rho/\mu$	1	3	5
0.0	0.41/1.47	0.94/4.42	1.45/8.74
0.3	0.97/2.16	1.83/7.98	2.98/15.21
0.6	0.98/4.13	3.26/16.06	5.66/31.43
0.9	1.91/7.86	7.38/32.67	15.48/68.76

example, for  $\rho = 0.3$  and  $\mu = 1$  and  $\sigma = 0.2$ , capital stock in incomplete markets is 0.97 percent higher than with complete markets. The trend shows that higher income variability ( $\sigma$ ) and greater persistence of shocks ( $\rho$ ) significantly increase precautionary savings, leading to higher capital accumulation.

The analysis of our results reveals significant policy implications related to the interplay between risk aversion ( $\mu$ ), income persistence ( $\rho$ ), and income variability ( $\sigma$ ). Specifically, when both  $\mu$  and  $\rho$  are relatively low, increases in  $\sigma$  have a more moderate impact on precautionary savings and aggregate capital accumulation. In contrast, when  $\mu$  and  $\rho$  are higher, the effect of increasing  $\sigma$  becomes markedly more pronounced, leading to substantial increases in savings and capital stock. This nuanced understanding underscores the importance of tailoring economic policies to the specific risk profiles and income dynamics of households. Policies aimed at mitigating income uncertainty, such as enhanced social insurance programs or targeted financial regulations, could be more effective if they account for the varying impacts of these parameters. Recognizing these differences can help policymakers design interventions that more precisely address the needs of households with different risk aversion levels and income shock persistencies, ultimately fostering greater economic stability.

## 5 Conclusion

This replication study confirms and extends S. Rao Aiyagari’s 1994 findings on uninsured idiosyncratic risk and aggregate saving. Utilizing modern computational techniques and an extended simulation framework, we validate the original model’s conclusion that individual-specific income shocks significantly increase precautionary savings and aggregate capital accumulation. Our results align closely with Aiyagari’s, demonstrating the critical impact of market incompleteness and income variability on macroeconomic behavior. The methodological enhancements, including the use of the Rouwenhorst method for stochastic process approximation, provide a robust framework that underscores the original model’s relevance and reliability.

Furthermore, this study highlights the importance of understanding individual risks in the context of broader economic stability. By reinforcing the significance of precautionary savings, our findings offer valuable insights for policymakers focused on designing effective social insurance programs and financial regulations. The consistent trends observed across different scenarios emphasize the robustness of Aiyagari’s model and its implications for economic policy. Future research should explore additional dimensions of market incompleteness and the effects of various policy interventions on savings behavior and wealth distribution, further contributing to the discourse on the interplay between individual risks and aggregate economic outcomes.

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