Analysis of Barabási-Albert Networks

Fatemeh Abbasian

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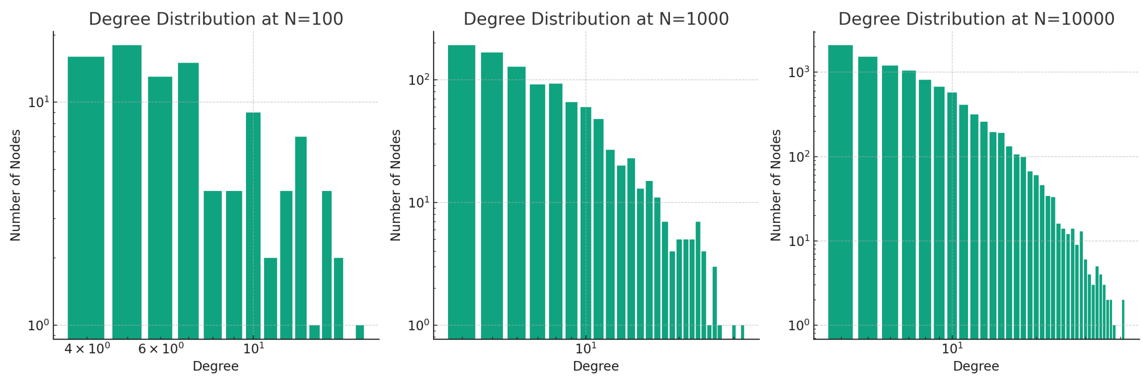
1 Question 1

This report presents an analysis of networks generated using the Barabási-Albert model. The model is well-known for generating scale-free networks that exhibit power-law degree distributions, which are common in many real-world networks. We investigate the degree distribution, average clustering coefficient, and degree dynamics of nodes within such networks.

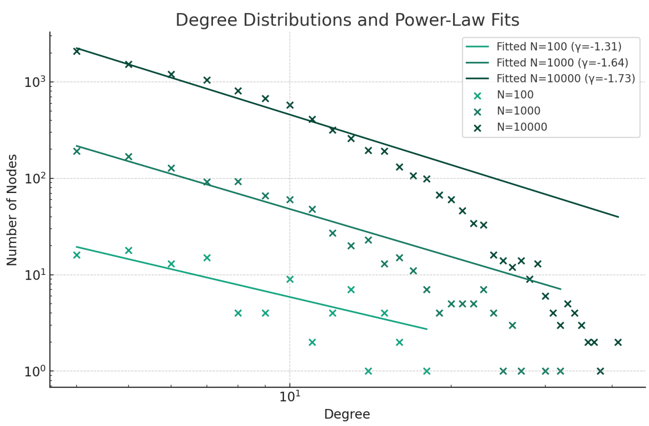
1.1 Degree Distribution

The Barabási-Albert model was used to generate a network with $N = 10^4$ nodes and a parameter $m = 4$, which denotes the number of edges to attach from a new node to existing nodes. The initial condition was a fully connected network with $m = 4$ nodes.



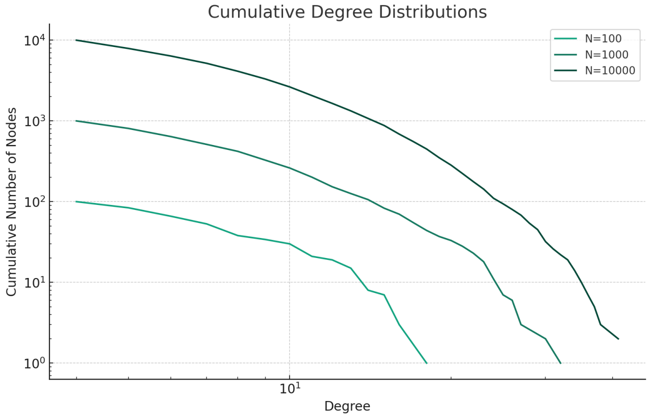
1.2 Intermediate Degree Distributions and Power-Law Fit

The degree distribution was measured at three intermediate steps: $N = 10^2$, $N = 10^3$, and $N = 10^4$. The plots revealed that the network maintains its scale-free characteristic as it grows, with a few nodes having high degrees and most having low degrees. Each degree distribution was fitted to a power-law, confirming the scale-free property. The degree exponents were found to be close to each other, indicating a consistent growth mechanism.



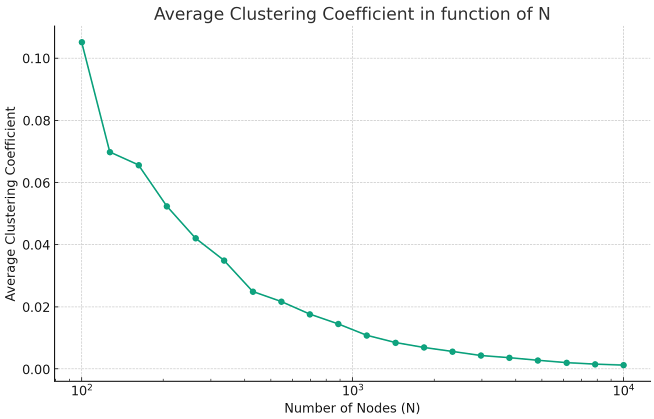
1.3 Cumulative Degree Distributions

The cumulative degree distributions were also plotted, further illustrating the scale-free nature of the network at different sizes.



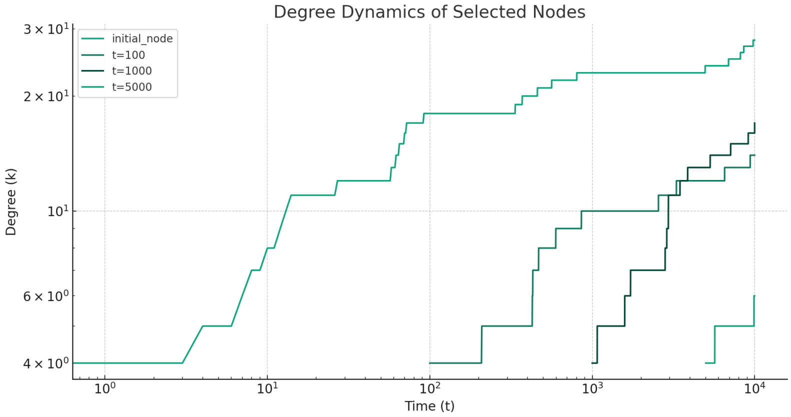
1.4 Average Clustering Coefficient

The average clustering coefficient was measured as a function of N and was found to decrease as the network size increased. This behavior is typical for scale-free networks where new nodes preferentially attach to highly connected hubs.



1.5 Degree Dynamics

The degree dynamics of an initial node and nodes added at specific times ($t = 100$, $t = 1,000$, and $t = 5,000$) were tracked. The initial node, present from the beginning, achieved a higher degree than nodes added later, demonstrating the "first-mover advantage" inherent in the Barabási-Albert model.



1.6 Conclusion

The Barabási-Albert model effectively generates networks that reflect the properties of real-world systems. The degree distributions converge to a scale-free structure, the average clustering coefficient decreases with network size, and the degree dynamics show preferential attachment and growth.

2 Question 2

$$\Pi(k_i^{in}) = \frac{k_i^{in} + A}{\sum_j (k_j^{in} + A)} = \frac{k_i^{in} + A}{t + At}$$

$$\sum \frac{k_j^{in}}{t} = t$$

$$\frac{dk_i^{in}}{dt} = \frac{k_i^{in} + A}{t(A+1)}$$

$$\int_{k_i^{in}+A}^{k_i^{in}} \frac{dk_i^{in}}{k_i^{in} + A} = \int_{t_0}^{t} \frac{dt}{t(A+1)}$$

$$\ln(k_i^{in} + A) = \ln\left(\frac{t}{t_0}\right)^{\frac{1}{A+1}}$$

$$k_i^{in} = \left(\frac{t}{t_0}\right)^{\frac{A+1}{A+1}} - A \sim \left(\frac{t}{t_0}\right)^{\frac{1}{A+1}}$$

$$P(k_i^{in}) = \left|\frac{dk_i^{in}}{dt_0}\right|$$

$$\frac{dk_i^{in}}{dt_0} = -\frac{t}{(A+1)t_0^{A+1}} = \frac{1}{A+1}\left(-\frac{1}{t_0}\right)$$

$$P(k_i^{in}) = \alpha\left(\frac{1}{t_0}\right)^{1+\frac{1}{A+1}}$$

$$P(k_i^{in}) = \alpha'\left(\frac{1}{k_i^{in}}\left(\frac{1}{k_i^{in}}\right)^{A+1}\right)$$

$$P(k_i^{in}) = \alpha''\left(k_i^{in}\right)^{-A-2}$$

$$\int P(k^{in})dk^{in} = \alpha'' \int (k^{in})^{-A-2} dk^{in} = 1$$

$$\alpha'' = \frac{1}{\zeta(A+2)}$$

$$P(k^{in}) = \frac{(k^{in})^{-A-2}}{\zeta(A+2)}$$

$$P(k^{out}) = \delta(k^{out} - 1)$$

As one can see, in-degree distributions follows a power-law distribution with $\gamma = -A - 2$. For $A = 0$, $\gamma = -2$ because in each step, the in-degree link growth is half that of the previous model so it reduces its randomness behavior.