5 Maximum Likelihood Estimation

A sample of N decision makers is obtained for the purpose of estimation.

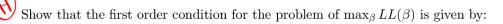
5.1 Log Likelihood function

Since the logit probabilities take a closed form, the traditional maximum-likelihood procedures can be applied. Show that the log likelihood function is then

$$LL(\beta) = \sum_{n=1}^{N} \sum_{i} y_{ni} \ln P_{ni}$$

$$\tag{2}$$

5.2 First order condition



$$\sum_{n=1}^{N} \sum_{i} (y_{ni} - P_{ni}) x_{ni} = 0$$
(3)

What is the interpretation of this equation? Show that the maximum likelihood estimates of β are those that make the predicted average of each explanatory variable equal to the observed average in the sample.

$$\mathcal{L} \mathcal{L}(\beta) = \sum_{n=1}^{N} \sum_{i=1}^{J} y_{ni} \ln P_{ni} \qquad \nabla \beta \qquad \sum_{n=1}^{N} \sum_{i=1}^{J} y_{ni} \nabla \beta \ln P_{ni} = 0$$

$$\sum_{i=1}^{J} P_{ni} = 0 \qquad \sum_{i=1}^{J} \nabla P_{ni} = 0 \qquad \sum_{i=1}^{J} \nabla \beta \ln P_{ni} = 0$$

$$\mathcal{L} \mathcal{L}(\beta) = \sum_{n=1}^{N} \sum_{i=1}^{J} y_{ni} \ln P_{ni} = 0$$

$$\sum_{n=1}^{N} \sum_{i=1}^{J} (y_{ni} - P_{ni}) \nabla \beta \ln P_{ni} = 0$$

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$$\mathcal{L}(\beta) = \sum_{n=1}^{N} y_{ni} \ln$$

$$\frac{P_{ni} = \frac{e^{x_{ni}\beta}}{\sum_{j} e^{x_{nj}'\beta}} \longrightarrow V_{\beta}P_{ni} = x_{ni}P_{ni} - \frac{P_{ni}}{\sum_{j} e^{x_{nj}'\beta}} \sum_{j} x_{nj} e^{x_{nj}'\beta} \longrightarrow V_{\beta}P_{ni} = x_{ni} - \frac{\sum_{j=1}^{J} x_{nj} e^{x_{nj}'\beta}}{\sum_{j=1}^{J} e^{x_{nj}'\beta}}$$

$$\underbrace{\frac{1}{2} \sum_{n=1}^{N} \frac{\sum_{i=1}^{J} (y_{ni} - P_{ni})}{\sum_{j=1}^{J} e^{2_{nj}^{J} \beta}} \left(\chi_{ni} - \sum_{j=1}^{J} \chi_{nj} e^{2_{nj}^{J} \beta} \right) = \sum_{n=1}^{N} \frac{\sum_{i=1}^{J} (y_{ni} - P_{ni})}{\sum_{j=1}^{J} e^{2_{nj}^{J} \beta}} = \delta 3$$

$$\sum_{n=1}^{N} \sum_{i=1}^{J} (Y_{ni} - P_{ni}) \sum_{j=1}^{J} \chi_{nj} P_{nj} = \sum_{n=1}^{N} \sum_{j=1}^{J} \chi_{nj} P_{nj} \sum_{i=1}^{J} (Y_{ni} - P_{ni}) = 0$$

Logit
$$\sum_{h=1}^{N} \sum_{i=1}^{J} (y_{hi} - P_{hi}) \chi_{hi} = 0$$

The fitted predicted average of each explanatory variable (55 Pni xni) equals to

the sample average of each
$$\chi_{ni}$$
 ($\sum_{n=1}^{\infty} Y_{ni} \chi_{ni}$).

5.3 Goodness of fit

The likelihood ratio index is defined as

$$\rho = 1 - \frac{LL(\hat{\beta})}{LL(0)}$$

In two sentences interpret this index. Is it similar to R^2 in linear regression? For instance, can you compare likelihood ratio index from two models on two datasets to each other? (Hint: the answer is NO! Explain why.)

$$L(\hat{\beta}) \langle 1 - LL(\hat{\beta}) \langle \log (1) = 0 \rangle$$

$$L(0) \langle 1 - L(0) \rangle \langle \log (1) = 0 \rangle$$

$$\text{if } \hat{\beta} \text{ is bester fit than } 0 \rightarrow L(\hat{\beta}) \rangle L(0) \rightarrow LL(\hat{\beta}) \rangle L(0) \stackrel{LL(\hat{\beta})}{\longrightarrow} \stackrel{LL(\hat{\beta})}{\longrightarrow} \langle 1 - \frac{LL(\hat{\beta})}{\longrightarrow} \rangle \rangle$$

$$\text{if } \hat{\beta} \text{ is bester fit than } 0 \rightarrow 1 \geqslant 1 - \frac{LL(\hat{\beta})}{\longrightarrow} \rangle \rangle$$

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$$\text{As } \hat{\beta} \text{ improves } \rangle 1 - \frac{LL(\hat{\beta})}{\longrightarrow} \text{ increases } \rho \text{ gets closer to } 1$$

this index is a comparison between $\hat{\beta}$ & $\beta=0$, so comparing 2 $\hat{\beta}$ within same model β data is possible with this index, but not possible between different models or data sets.