$$S \rightarrow AB$$
$$A \rightarrow CaA/a$$
$$B \rightarrow CbB/b$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$

Now the grammar is in CNF.

17. Convert the following grammar into CNF.

$$S \rightarrow aA/B/C/a$$
$$A \rightarrow aB/E$$
$$B \rightarrow aA$$
$$C \rightarrow cCD$$
$$D \rightarrow abd$$

Solution: The CFG is not simplified as it contains useless symbols and unit productions. First, we have to simplify the CFG and then it can be converted into CNF.

In the grammar, E is the useless symbol (non-generating symbol) as it does not produce any terminal symbol. So, the production $A \rightarrow E$ is removed. The modified grammar becomes

$$S \rightarrow aA/B/C/a$$
$$A \rightarrow aB$$
$$B \rightarrow aA$$
$$C \rightarrow cCD$$
$$D \rightarrow abd$$

In the grammar, except $S \rightarrow$ a, all other productions are not in CNF.

Consider four extra non-terminals C_a, C_b, C_c, C_d and two production rules $C_a \rightarrow a$ and $C_b \rightarrow b$,

$C_c \rightarrow c$ and $C_d \rightarrow d$. Replace 'a' by C_a and 'b' by C_b, c by C_c and d by C_d in the previous productions. The modified production rule becomes

$$S \rightarrow C_aA/C_cCD/a$$
$$A \rightarrow C_aB$$
$$B \rightarrow C_aA$$
$$C \rightarrow C_cCD$$
$$D \rightarrow C_aC_bC_d$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$
$$C_c \rightarrow c$$
$$C_d \rightarrow d$$

Here, $S \rightarrow C_c CD, C \rightarrow C_c CD, andD \rightarrow C_a C_b C_d$ are not in CNF. Introduce two production rules $X \rightarrow CDandY \rightarrow C_b C_d$ and replace CD by X and $C_b C_d$ by Y. The modified production rule becomes

$$S \rightarrow C_a A / C_c X / a$$
$$A \rightarrow C_a B$$
$$B \rightarrow C_a A$$
$$C \rightarrow C_c X$$
$$D \rightarrow C_a Y$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$
$$C_c \rightarrow c$$
$$C_d \rightarrow d$$
$$X \rightarrow CD$$
$$Y \rightarrow C_b C_d$$

Here, all the productions are in the specified format of CNF. The CFG is converted to CNF.

18. Convert the following grammar into CNF.

$$E \rightarrow E + T/T$$
$$T \rightarrow (E)/a$$

Solution: The grammar contains two non-terminal symbols E and T and four terminal symbols +, (,), and a. The grammar contains a unit production $E \rightarrow T$. First, the unit production has to be removed. After removing the unit production $E \rightarrow T$, the modified grammar becomes

$$E \rightarrow E + T/(E)/a$$
$$T \rightarrow (E)/a$$

In the previous grammar, except $E \rightarrow a$ and $T \rightarrow a$ all the other productions are not in CNF.

Introduce three non-terminals A, B, and C and three production rules $A \rightarrow +$, $B \rightarrow ($and $C \rightarrow)$ and an appropriate terminal by appropriate non-terminals. The modified production rules become

$$E \rightarrow EAT/BEC/a$$
$$T \rightarrow BEC/a$$
$$A \rightarrow +$$
$$B \rightarrow ($$
$$C \rightarrow)$$

In the previous grammar, $E \rightarrow EAT$, $E \rightarrow BEC$, and $T \rightarrow BEC$ are not in CNF. Consider two non-terminals X and Y and two production rules $X \rightarrow AT$ and $Y \rightarrow EC$. The modified production rules become

$$E \rightarrow EX/BY/a$$
$$T \rightarrow BY/a$$
$$A \rightarrow +$$
$$B \rightarrow ($$
$$C \rightarrow)$$
$$X \rightarrow AT$$
$$Y \rightarrow EC$$

Here, all the productions are in the specified format of CNF. The CFG is converted to CNF.

19. Convert the following grammar into CNF.

$$S \rightarrow ABb/a$$
$$A \rightarrow aaA/B$$
$$B \rightarrow bAb/b$$

Solution: The grammar contains unit production $A \rightarrow B$. After removing the unit production, the grammar is

$$S \rightarrow ABb/a$$
$$A \rightarrow aaA/bAb/b$$
$$B \rightarrow bAb/b$$

Except $S \rightarrow a$ and $B \rightarrow b$, the productions of the grammar are not in CNF.

Consider two non-terminals C_a and C_b and two production rules $C_a \rightarrow a$ and $C_b \rightarrow b$ and replace 'a' by Ca and 'b' by C_b in the appropriate production. The modified production rules become

$$S \rightarrow ABC_b/a$$
$$A \rightarrow C_aC_aA/C_bAC_b/b$$
$$B \rightarrow C_bAC_b/b$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$

Consider three non-terminals X, Y, and Z and three production rules $X \rightarrow BC_b$, $Y \rightarrow AC_b$, and $Z \rightarrow C_aA$. Replace the appropriate group of non-terminals in the production by an appropriate new non-terminal. The production rule becomes

$$S \rightarrow AX/a$$
$$A \rightarrow C_aZ/C_bY/b$$
$$B \rightarrow C_bY/b$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$
$$X \rightarrow BC_b$$
$$Y \rightarrow AC_b$$
$$Z \rightarrow C_aA$$

20. Convert the following CFG into an equivalent grammar in CNF.

$$S \rightarrow aAbB$$
$$A \rightarrow abAB/aAA/a$$
$$B \rightarrow bBaA/bBB/b$$

[WBUT 2007, 2010]

Solution: In CNF, all the productions will be in the form

Non-terminal$\rightarrow$String of exactly two non-terminals

Non-terminal$\rightarrow$single terminal

In the previous grammar, expect $A \rightarrow a$ and $B \rightarrow b$ all the productions are not in CNF.

Consider two productions $C_a \rightarrow a$ and $C_b \rightarrow b$.

The modified grammar becomes

$$S \rightarrow C_a A C_b B$$
$$A \rightarrow C_a C_b AB / C_a AA / a$$

$$B \rightarrow C_b B C_a A / C_b B B / b$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$

Replace $C_a A$ by X_1, $C_b B$ by X_2, $C_a C_b$ by X_3, and AB by X_4, The modified grammar becomes

$$S \rightarrow X_1 X_2$$
$$A \rightarrow X_3 X_4 / X_1 A / a$$
$$B \rightarrow X_2 X_1 / X_2 B / b$$
$$C_a \rightarrow a C_b \rightarrow b$$
$$X_1 \rightarrow C_a A$$
$$X_2 \rightarrow C_b B$$
$$X_3 \rightarrow C_a C_b$$
$$X_4 \rightarrow AB$$

The previous grammar is in CNF.

21. Convert the following grammar into CNF.

$$S \to AACD$$
$$A \to aAb/ \in$$
$$C \to aC/a$$
$$D \to aDa/bDb/ \in$$

[Gujrat Technical University 2010]

Solution: The grammar contains $\in$ production. First, we have to remove the $\in$ productions.

Removing the $\in$ productions, the modified grammar becomes

$$S \to AACD/ACD/CD/AAC/C/AC$$
$$A \to aAb/ab$$
$$C \to aC/a$$
$$D \to aDa/aa/bDb/bb.$$

The grammar contains the unit production $S \to C$. Removing the unit production, the grammar becomes

$$S \rightarrow AACD/ACD/CD/AAC/AC/aC/a$$
$$A \rightarrow aAb/ab$$
$$C \rightarrow aC/a$$
$$D \rightarrow aDa/bDb/aa/bb$$

Now, the grammar can be converted to CNF.

Introduce two new productions $C_a \rightarrow a$ and $C_b \rightarrow b$. Replacing Ca and Cb in appropriate positions in the grammar, the modified grammar becomes

$$S \rightarrow AACD/ACD/CD/AAC/AC/CaC/a$$
$$A \rightarrow CaACb/CaCb$$
$$C \rightarrow CaC/a$$
$$D \rightarrow CaDCa/CbDCb/CaCa/CbCb$$
$$C_a \rightarrow a$$
$$C_b \rightarrow b$$